Dr Oliver Mathematics
Mathematics
Coordinates Part 2
Past Examination Questions

This booklet consists of 23 questions across a variety of examination topics. The total number of marks available is 178.

1. The points A and B have coordinates $(5, -1)$ and $(13, 11)$ respectively.

   (a) Find the coordinates of the mid-point of AB. (2)

   Solution

   $$\left(\frac{5+13}{2}, \frac{-1+11}{2}\right) = \underline{\underline{(9, 5)}}.$$

   Given that AB is a diameter of the circle C,

   (b) find an equation for C. (4)

   Solution

   $$\begin{aligned} & (x-9)^2 + (y-5)^2 = (13-9)^2 + (11-5)^2 \\ \Rightarrow\ & \underline{\underline{(x-9)^2 + (y-5)^2 = 52}}. \end{aligned}$$

2. The circle C, with centre at the point A, has equation $x^2 + y^2 - 10x + 9 = 0$. Find

   (a) the coordinates of A, (2)

   Solution

   $$\begin{aligned} x^2 + y^2 - 10x + 9 = 0 &\Rightarrow x^2 - 10x + y^2 = -9 \\ &\Rightarrow x^2 - 10x + 25 + y^2 = 25 - 9 \\ &\Rightarrow (x-5)^2 + y^2 = 4^2; \end{aligned}$$

   the centre is $\underline{\underline{A(5, 0)}}$.

   (b) the radius of C, (2)

Solution

The radius is $\underline{\underline{4}}$.

(c) the coordinates of the points at which C crosses the x-axis. (2)

Solution

$$\begin{aligned} y = 0 &\Rightarrow (x-5)^2 = 4^2 \\ &\Rightarrow x - 5 = -4 \text{ or } x - 5 = 4 \\ &\Rightarrow x = 1 \text{ or } x = 9; \end{aligned}$$

the coordinates are $\underline{\underline{(1, 0)}}$ and $\underline{\underline{(9, 0)}}$.

Given that the line l with gradient $\frac{7}{2}$ is a tangent to C, and that l touches C at the point T,

(d) find an equation of the line which passes through A and T. (3)

Solution

$$\text{Gradient of the perpendicular} = -\tfrac{2}{7}$$

and the equation of the line is

$$\begin{aligned} y - 0 = -\tfrac{2}{7}(x-5) &\Rightarrow 7y = -2(x-5) \\ &\Rightarrow 7y = -2x + 10 \\ &\Rightarrow \underline{\underline{2x + 7y - 10 = 0}}. \end{aligned}$$

3. In Figure 1, $A(4, 0)$ and $B(3, 5)$ are the end points of a diameter of the circle C.

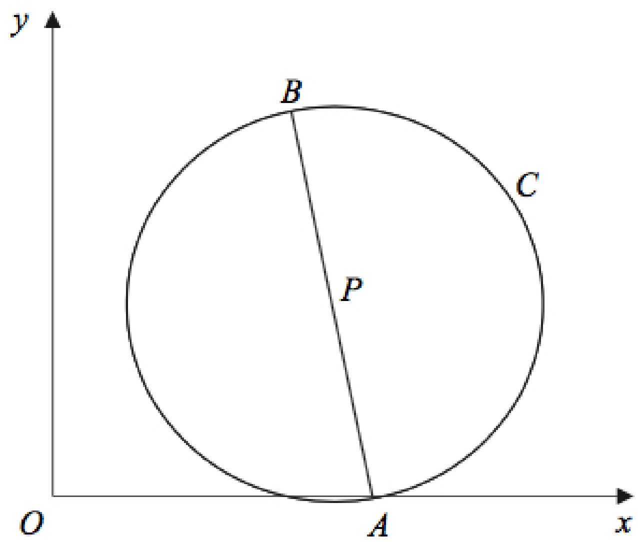

Figure 1: a circle C

Find

(a) the exact length of AB, (2)

Solution

$$\begin{aligned} AB &= \sqrt{(4-3)^2 + (0-5)^2} \\ &= \underline{\underline{\sqrt{26}}}. \end{aligned}$$

(b) the coordinates of the midpoint P of AB, (2)

Solution

$$\left(\frac{4+3}{2}, \frac{0+5}{2}\right) = \underline{\underline{(3\tfrac{1}{2}, 2\tfrac{1}{2})}}.$$

(c) an equation for the circle C. (3)

Solution

$$\begin{aligned} & (x - 3\tfrac{1}{2})^2 + (y - 2\tfrac{1}{2})^2 = [\tfrac{1}{2}\sqrt{26}]^2 \\ \Rightarrow\ & \underline{\underline{(x - 3\tfrac{1}{2})^2 + (y - 2\tfrac{1}{2})^2 = 6\tfrac{1}{2}}}. \end{aligned}$$

4. The line $y = 3x - 4$ is a tangent to the circle C, touching C at the point $P(2, 2)$, as shown in Figure 2.

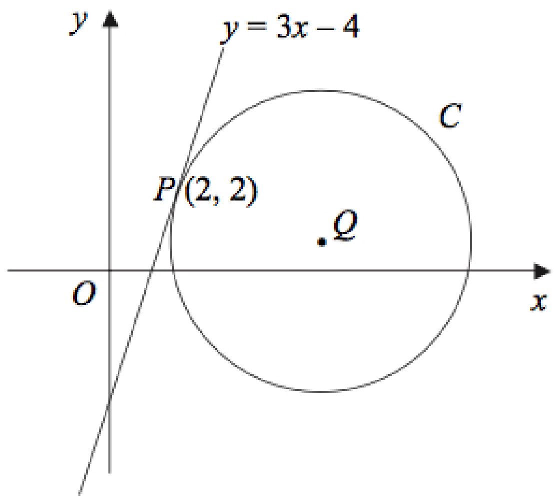

Figure 2: $y = 3x - 4$ is a tangent to the circle C

The point Q is the centre of C.

(a) Find an equation of the straight line through P and Q. (3)

Solution

$$\text{Gradient of the perpendicular} = -\tfrac{1}{3}$$

and the equation is

$$\begin{aligned} y - 2 = -\tfrac{1}{3}(x - 2) &\Rightarrow 3(y - 2) = -(x - 2) \\ &\Rightarrow 3y - 6 = -x + 2 \\ &\Rightarrow \underline{\underline{x + 3y - 8 = 0}}. \end{aligned}$$

Given that Q lies on the line $y = 1$,

(b) show that the x-coordinate of Q is 5, (1)

Solution

$$x + 3 - 8 = 0 \Rightarrow \underline{\underline{x = 5}}.$$

(c) find an equation for C. (4)

Solution

$$(x-5)^2+(y-1)^2 = (2-5)^2+(2-1)^2$$
$$\Rightarrow \quad \underline{\underline{(x-5)^2+(y-1)^2 = 10}}.$$

5. The line joining the points $(-1,4)$ and $(3,6)$ is a diameter of the circle C. Find an equation for C. (6)

Solution

$$\left(\frac{-1+3}{2}, \frac{4+6}{2}\right) = (1,5)$$

and the equation is

$$(x-1)^2+(y-5)^2 = (3-1)^2+(6-5)^2$$
$$\Rightarrow \quad \underline{\underline{(x-1)^2+(y-5)^2 = 5}}.$$

6. The points A and B lie on a circle with centre P, as shown in Figure 3.

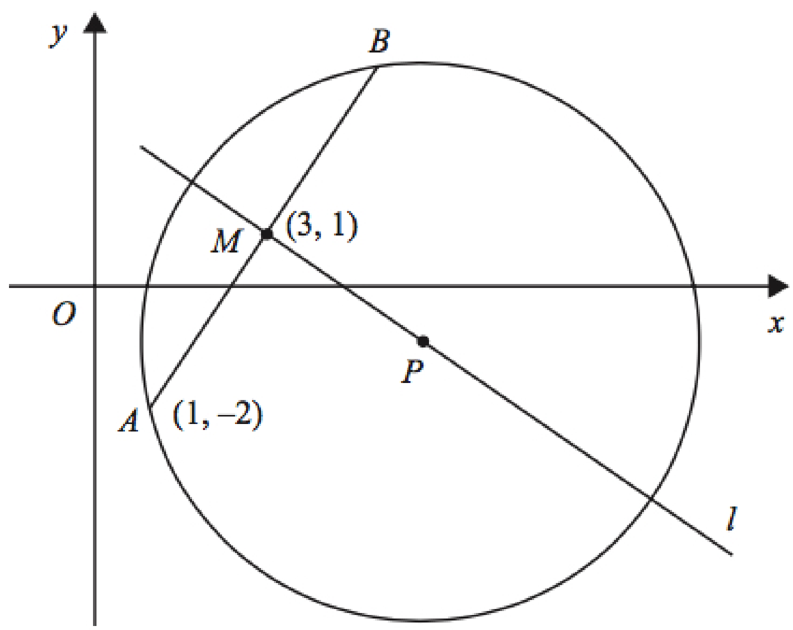

Figure 3: the points A and B

The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$. The line l passes through the points M and P.

(a) Find an equation for l. (4)

Solution

$$\text{Gradient of } AM = \frac{1-(-2)}{3-1} = \frac{3}{2}$$

and

$$\text{gradient of } MP = -\tfrac{2}{3}.$$

Finally,

$$\begin{aligned} y - 1 = -\tfrac{2}{3}(x-3) &\Rightarrow 3(y-1) = -2(x-3) \\ &\Rightarrow 3y - 3 = -2x + 6 \\ &\Rightarrow \underline{\underline{2x + 3y - 9 = 0}}. \end{aligned}$$

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is -1, (1)

Solution

$$x = 6 \Rightarrow 12 + 3y - 9 = 0 \Rightarrow 3y = -3 \Rightarrow \underline{\underline{y = -1}}.$$

(c) find an equation for the circle. (4)

Solution

$$\begin{aligned} & (x-6)^2 + (y+1)^2 = (6-1)^2 + (-1-(-2))^2 \\ \Rightarrow \quad & \underline{\underline{(x-6)^2 + (y+1)^2 = 26}}. \end{aligned}$$

7. A circle C has centre $M(6, 4)$ and radius 3. Write down the equation of the circle in the form (2)

$$(x-a)^2 + (y-b)^2 = r^2.$$

Solution

$$\underline{\underline{(x-6)^2 + (y-4)^2 = 9}}.$$

8. The circle C has centre $(3, 1)$ and passes through the point $P(8, 3)$.

   (a) Find an equation for C. (4)

   Solution

   $$\begin{aligned} & (x-3)^2 + (y-1)^2 = (8-3)^2 + (1-3)^2 \\ \Rightarrow \quad & \underline{\underline{(x-3)^2 + (y-1)^2 = 29}}. \end{aligned}$$

   (b) Find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (5)

   Solution

   $$\text{Gradient of the line} = \tfrac{3-1}{8-3} = \frac{2}{5}$$

   and

   $$\text{Gradient of the perpendicular} = -\tfrac{5}{2}.$$

   Finally,

   $$\begin{aligned} y - 3 = -\tfrac{5}{2}(x-8) &\Rightarrow 2(y-3) = -5(x-8) \\ &\Rightarrow 2y - 6 = -5x + 40 \\ &\Rightarrow \underline{\underline{5x + 2y - 46 = 0}}. \end{aligned}$$

9. The points $P(-3, 2)$, $Q(9, 10)$, and $R(a, 4)$ lie on the circle C, as shown in Figure 4.

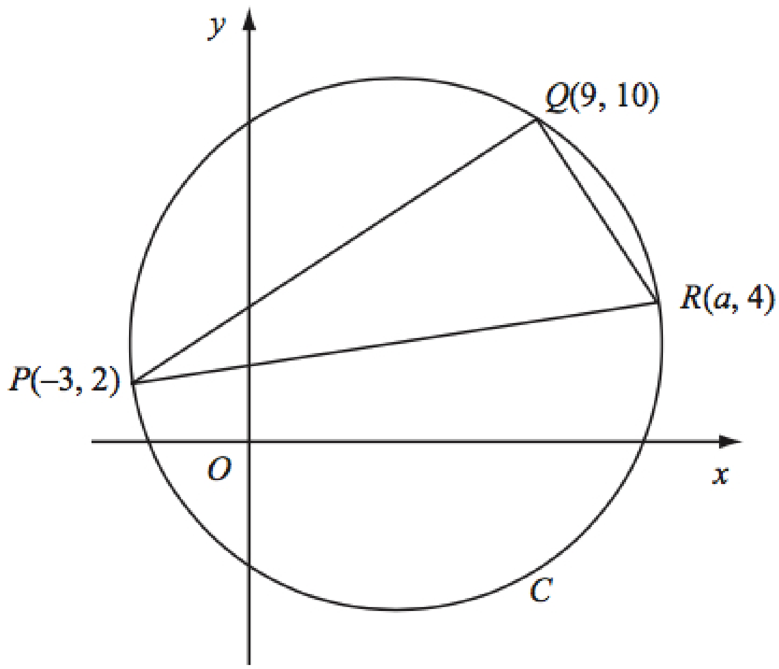

Figure 4: the points $P(-3,2)$, $Q(9,10)$, and $R(a,4)$

Given that PR is a diameter of C,

(a) show that $a = 13$, (3)

Solution

$$
\begin{aligned}
& PR^2 = PQ^2 + QR^2 \\
\Rightarrow\ & (a+3)^2 + (4-2)^2 = (9+3)^2 + (10-2)^2 + (a-9)^2 + (10-4)^2 \\
\Rightarrow\ & (a^2+6a+9) + 4 = 144 + 64 + (a^2 - 18a + 81) + 36 \\
\Rightarrow\ & 24a = 312 \\
\Rightarrow\ & \underline{\underline{a = 13}}.
\end{aligned}
$$

(b) find an equation for C. (5)

Solution

$$\left(\frac{13+(-3)}{2}, \frac{4+2}{2}\right) = (5,3)$$

and

$$\begin{aligned} & (x-5)^2+(y-3)^2 = (9-5)^2+(10-3)^2 \\ \Rightarrow\ & \underline{\underline{(x-5)^2+(y-3)^2 = 65}}. \end{aligned}$$

10. The circle C has equation

$$x^2+y^2-6x+4y=12.$$

(a) Find the centre and the radius of C. (5)

Solution

$$\begin{aligned} x^2+y^2-6x+4y=12 &\Rightarrow (x^2-6x+9)+(y^2+4y+4)=9+4+12 \\ &\Rightarrow (x-3)^2(y+2)^2=25; \end{aligned}$$

the centre is $\underline{\underline{(3,-2)}}$ and the radius is $\underline{\underline{5}}$.

The point $P(-1,1)$ and the point $Q(7,-5)$ both lie on C.

(b) Show that PQ is a diameter of C. (2)

Solution

$$PQ=\sqrt{(7+1)^2+(1+5)^2}=10=2\times 5;$$

hence, $\underline{\underline{PQ \text{ is a diameter of } C}}$.

The point R lies on the positive y-axis and the angle $PRQ=90°$.

(c) Find the coordinates of R. (4)

Solution

$$\begin{aligned} x=0 &\Rightarrow y^2+4y=12 \\ &\Rightarrow y^2+4y-12=0 \\ &\Rightarrow (y+6)(y-2)=0 \\ &\Rightarrow y=-6 \text{ or } y=2; \end{aligned}$$

hence, $\underline{\underline{R(0,2)}}$.

11. Figure 5 shows a sketch of the circle C with centre N and equation

$$(x-2)^2 + (y+1)^2 = \tfrac{169}{4}.$$

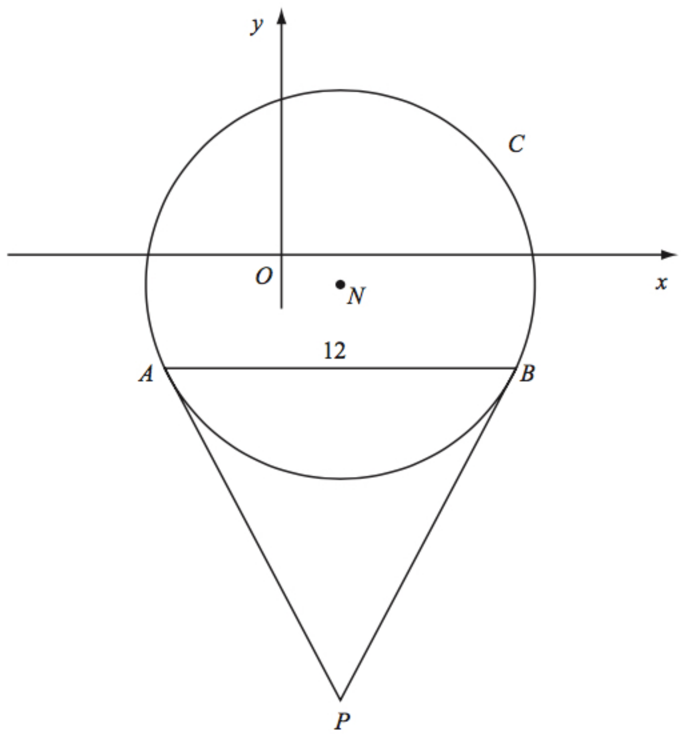

Figure 5: $(x-2)^2 + (y+1)^2 = \frac{169}{4}$

(a) Write down the coordinates of N. (2)

Solution
$\underline{\underline{N(2,-1)}}$.

(b) Find the radius of C. (1)

Solution
Radius is $\underline{\underline{\tfrac{13}{2}}}$.

The chord AB of C is parallel to the x-axis, lies below the x-axis and is of length 12 units, as shown in Figure 5.

(c) Find the coordinates of A and the coordinates of B. (5)

Solution

Let D be the midpoint of AB. Then

$$\begin{aligned} AN^2 = AD^2 + DN^2 &\Rightarrow \tfrac{169}{4} = 36 + DN^2 \\ &\Rightarrow DN^2 = \tfrac{25}{4} \\ &\Rightarrow DN = \tfrac{5}{2}. \end{aligned}$$

Now, the y-coordinate of D is $-1 - \frac{5}{2} = -3\frac{1}{2}$ and

$$\begin{aligned} (x-2)^2 + (-3\tfrac{1}{2} + 1)^2 = \tfrac{169}{4} &\Rightarrow (x-2)^2 = 36 \\ &\Rightarrow x - 2 = -6 \text{ or } x - 2 = 6; \end{aligned}$$

hence $\underline{\underline{A(-4, -3\frac{1}{2})}}$ and $\underline{\underline{B(8, -3\frac{1}{2})}}$

12. The circle C has centre $A(2, 1)$ and passes through the point $B(10, 7)$.

(a) Find an equation for C. (4)

Solution

$$\begin{aligned} & (x-2)^2 + (y-1)^2 = (10-2)^2 + (7-1)^2 \\ \Rightarrow\ & \underline{\underline{(x-2)^2 + (y-1)^2 = 100}}. \end{aligned}$$

The line l_1 is the tangent to C at the point B.

(b) Find an equation for l_1. (4)

Solution

$$\text{Gradient of } AB = \tfrac{7-1}{10-2} = \frac{3}{4}$$

and

$$\text{gradient of } l_1 = -\tfrac{4}{3}.$$

Finally,

$$\begin{aligned} y - 7 = -\tfrac{4}{3}(x - 10) &\Rightarrow 3(y-7) = -4(x-10) \\ &\Rightarrow 3y - 21 = -4x + 40 \\ &\Rightarrow \underline{\underline{4x + 3y - 61 = 0}}. \end{aligned}$$

The line l_2 is parallel to l_1 and passes through the mid-point of AB. Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form. (3)

Solution

$$\begin{aligned} PQ &= 2\sqrt{10^2 - 5^2} \\ &= 2\sqrt{75} \\ &= \underline{\underline{10\sqrt{3}}}. \end{aligned}$$

13. The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively. Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates $(3, 6)$, (1)

Solution

$$\left(\frac{(-2)+8}{2}, \frac{11+1}{2}\right) = \underline{\underline{(3, 6)}}.$$

(b) find an equation for C. (4)

Solution

$$\begin{aligned} & (x-3)^2 + (y-6)^2 = (8-3)^2 + (1-6)^2 \\ \Rightarrow\ & \underline{\underline{(x-3)^2 + (y-6)^2 = 50}}. \end{aligned}$$

(c) Verify that the point $(10, 7)$ lies on C. (1)

Solution

$$x = 10, y = 7 \Rightarrow (x-3)^2 + (y-6)^2 = 7^2 + 1^2 = 50$$

and hence $\underline{\underline{\text{the point } (10,7) \text{ lies on } C}}$.

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants. (4)

Solution

$$\text{Gradient of the radius} = \frac{7-6}{10-3} = \frac{1}{7}$$

and

$$\text{gradient of the perpendicular} = -7.$$

Finally,

$$\begin{aligned} y - 7 = -7(x - 10) &\Rightarrow y - 7 = -7x + 70 \\ &\Rightarrow \underline{\underline{y = -7x + 77}}. \end{aligned}$$

14. The circle C has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0.$$

Find

(a) the coordinates of the centre of C, (2)

Solution

$$\begin{aligned} x^2 + y^2 + 4x - 2y - 11 = 0 &\Rightarrow x^2 + 4x + y^2 - 2y = 11 \\ &\Rightarrow (x^2 + 4x + 4) + (y^2 - 2y + 1) = 4 + 1 + 11 \\ &\Rightarrow (x + 2)^2 + (y - 1)^2 = 16; \end{aligned}$$

the centre of C is $\underline{\underline{(-2, 1)}}$.

(b) the radius of C, (2)

Solution

And radius is $\underline{\underline{4}}$.

(c) the coordinates of the points where C crosses the y-axis, giving your answers as simplified surds. (4)

Solution

$$\begin{aligned} x = 0 &\Rightarrow 4 + (y - 1)^2 = 16 \\ &\Rightarrow (y - 1)^2 = 12 \\ &\Rightarrow y - 1 = \pm 2\sqrt{3} \\ &\Rightarrow y = 1 \pm 2\sqrt{3}; \end{aligned}$$

hence, the coordinates are $\underline{\underline{(0, 1 - 2\sqrt{3})}}$ and $\underline{\underline{(0, 1 + 2\sqrt{3})}}$.

15. A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C. (4)

Solution

$$(x+1)^2 + (y-7)^2 = (0+1)^2 + (0-7)^2$$
$$\Rightarrow \underline{\underline{(x+1)^2 + (y-7)^2 = 50}}.$$

16. The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0.$$

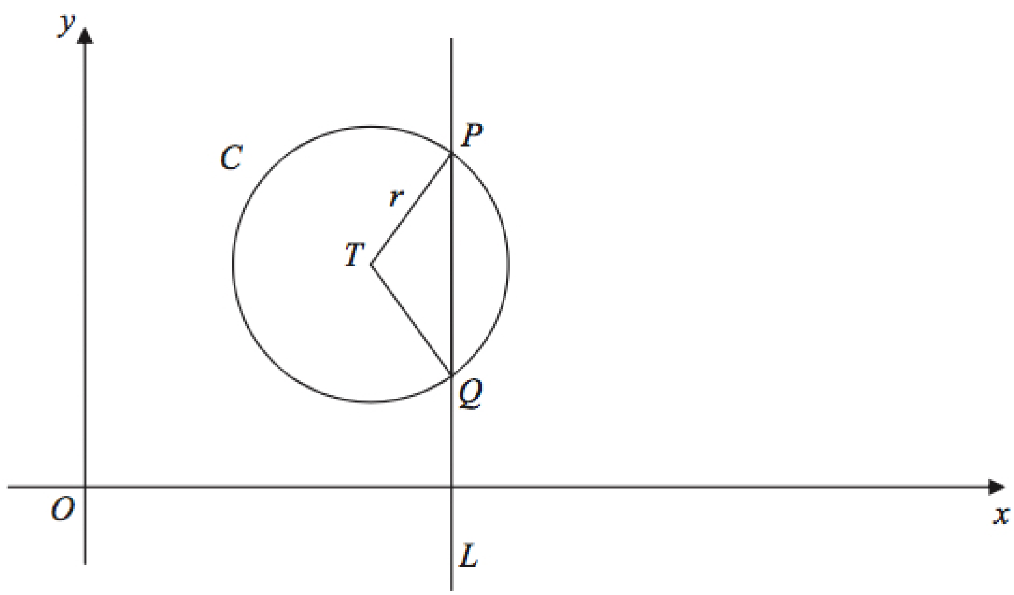

Figure 6: the circle C with centre T and radius r

(a) Find the coordinates of the centre of C. (3)

Solution

$$x^2 + y^2 - 20x - 16y + 139 = 0$$
$$\Rightarrow x^2 - 20x + y^2 - 16y = -139$$
$$\Rightarrow (x^2 - 20x + 100) + (y^2 - 16y + 64) = 100 + 64 - 139$$
$$\Rightarrow (x-10)^2 + (y-8)^2 = 25;$$

the centre of C is $\underline{\underline{(10, 8)}}$.

(b) Show that $r = 5$. (2)

Solution

$$r = \sqrt{25} = \underline{\underline{5}}.$$

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 6.

(c) Find the y-coordinate of P and the y-coordinate of Q. (3)

Solution

$$\begin{aligned} x = 13 &\Rightarrow 9 + (y-8)^2 = 25 \\ &\Rightarrow (y-8)^2 = 16 \\ &\Rightarrow y - 8 = -4 \text{ or } y - 8 = 4 \\ &\Rightarrow y = 4 \text{ or } y = 12; \end{aligned}$$

hence, the y-coordinate of P is $\underline{\underline{4}}$ and the y-coordinate of Q is $\underline{\underline{12}}$.

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,

(d) find the perimeter of the sector PTQ. (3)

Solution

$$\begin{aligned} \text{Perimeter of the sector} &= PT + TQ + QP \\ &= 5 + 5 + 5 \times 1.855 \\ &= \underline{\underline{19.275}}. \end{aligned}$$

17. The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0.$$

The centre of C is at the point M.

(a) Find (5)

(i) the coordinates of the point M,

Solution

$$\begin{aligned} & x^2 + y^2 - 20x - 24y + 195 = 0 \\ \Rightarrow \quad & x^2 - 20x + y^2 - 24y = -195 \\ \Rightarrow \quad & (x^2 - 20x + 100) + (y^2 - 24y + 144) = 100 + 144 - 195 \\ \Rightarrow \quad & (x-10)^2 + (y-12)^2 = 49; \end{aligned}$$

the centre of C is $\underline{\underline{(10, 12)}}$.

(ii) the radius of the circle C.

Solution

And radius is $\underline{\underline{7}}$.

N is the point with coordinates $(25, 32)$.

(b) Find the length of the line MN. (2)

Solution

$$\begin{aligned} \text{Length of } MN &= \sqrt{(25-10)^2 + (32-12)^2} \\ &= \underline{\underline{25}}. \end{aligned}$$

The tangent to C at a point P on the circle passes through point N.

(c) Find the length of the line NP. (2)

Solution

$$\begin{aligned} \text{Length of } NP &= \sqrt{25^2 - 7^2} \\ &= \underline{\underline{24}}. \end{aligned}$$

18. The circle C has radius 5 and touches the y-axis at the point $(0, 9)$, as shown in Figure 7.

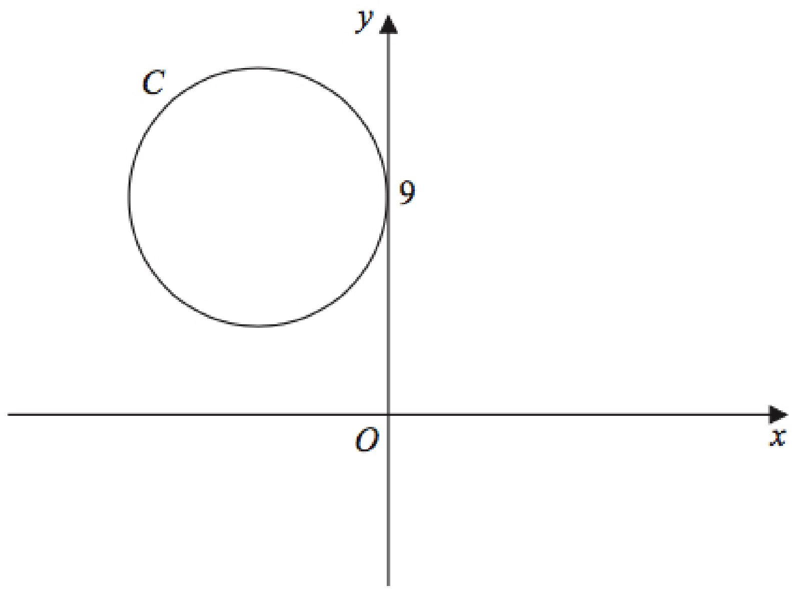

Figure 7: the circle C has radius 5

(a) Write down an equation for the circle C, that is shown in Figure 7. (3)

> **Solution**
>
> $$\underline{\underline{(x+5)^2 + (y-9)^2 = 25}}.$$

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T.

(b) Find the length of PT. (3)

> **Solution**
>
> Let S be the centre of the circle. Then
>
> $$\begin{aligned}\text{length of } SP &= (8+5)^2 + (-7-9)^2 \\ &= 425\end{aligned}$$
>
> and
>
> $$\begin{aligned}\text{length of } PT &= \sqrt{425 - 5^2} \\ &= \underline{\underline{20}}.\end{aligned}$$

19. Figure 8 shows a circle C with centre Q and radius 4 and the point T which lies on C.

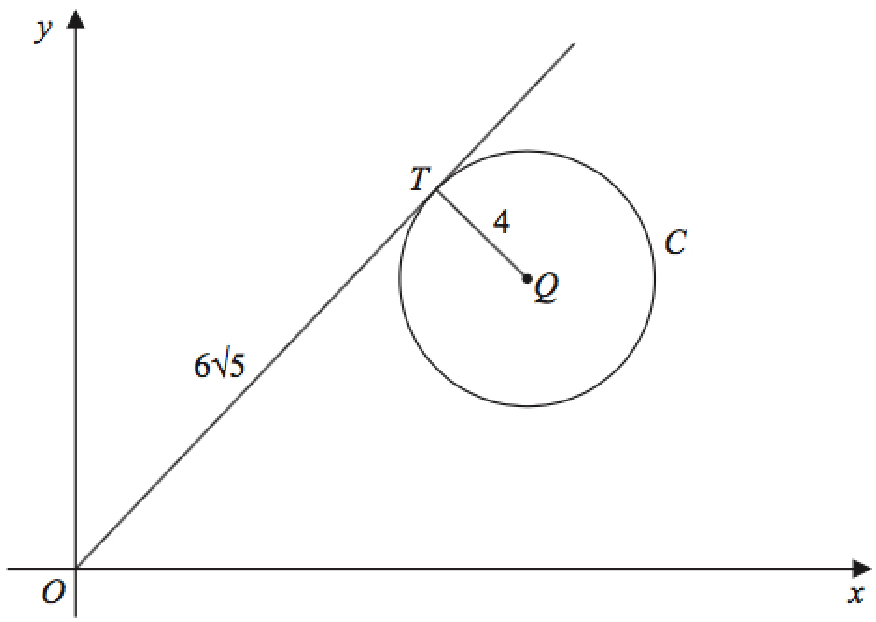

Figure 8: a circle C with centre Q and radius 4

The tangent to C at the point T passes through the origin O and $OT = 6\sqrt{5}$. Given that the coordinates of Q are $(11, k)$, where k is a positive constant,

(a) find the exact value of k, (3)

Solution
Let $S(11, 0)$. Then

$$\begin{aligned} OQ &= \sqrt{4^2 + (6\sqrt{5})^2} \\ &= 14 \end{aligned}$$

and

$$\begin{aligned} SQ &= \sqrt{14^2 - 11^2} \\ &= \underline{\underline{5\sqrt{3}}}. \end{aligned}$$

(b) find an equation for C. (2)

Solution

$$\underline{\underline{(x - 11)^2 + (y - 5\sqrt{3})^2 = 16}}.$$

20. The circle C, with centre A, passes through the point P with coordinates $(-9, 8)$ and the point Q with coordinates $(15, -10)$. Given that PQ is a diameter of the circle C,

(a) find the coordinates of A, (2)

Solution

$$\left(\frac{(-9)+15}{2}, \frac{8+(-10)}{2}\right) = \underline{\underline{(3, -1)}}.$$

(b) find an equation for C. (3)

Solution

$$(x-3)^2 + (y+1)^2 = \tfrac{1}{4}[(-9-15)^2 + (8+10)^2]$$
$$\Rightarrow \quad \underline{\underline{(x-3)^2 + (y+1)^2 = 225}}.$$

A point R also lies on the circle C. Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR. Give your answer as a surd in its simplest form. (2)

Solution

$$\begin{aligned}\text{Shortest distance} &= \sqrt{15^2 - 10^2}\\ &= \underline{\underline{5\sqrt{5}}}.\end{aligned}$$

(d) Find the size of the angle ARQ, giving your answer to the nearest 0.1 of a degree. (2)

Solution

$$\sin\angle ARQ = \frac{10}{15} \Rightarrow \angle ARQ = 41.810\,314\,9^\circ \text{ (FCD)}$$
$$\Rightarrow \angle ARQ = \underline{\underline{41.8^\circ \text{ (1 dp)}}}.$$

21. A circle C with centre at the point $(2, -1)$ passes through the point A at $(4, -5)$.

(a) Find an equation for the circle C. (3)

Solution

$$\begin{aligned} (x-2)^2 + (y+1)^2 &= (4-2)^2 + (-5+1)^2 \\ \Rightarrow \quad (x-2)^2 + (y+1)^2 &= 20. \end{aligned}$$

(b) Find an equation of the tangent to the circle C at the point A, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient} = \frac{-5+1}{4-2} = -2$$

and

$$\text{gradient of the perpendicular} = \tfrac{1}{2}.$$

Finally,

$$\begin{aligned} y + 5 = \tfrac{1}{2}(x-4) &\Rightarrow 2(y+5) = x - 4 \\ &\Rightarrow 2y + 10 = x - 4 \\ &\Rightarrow x - 2y - 14 = 0. \end{aligned}$$

22. The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$, as shown in Figure 9.

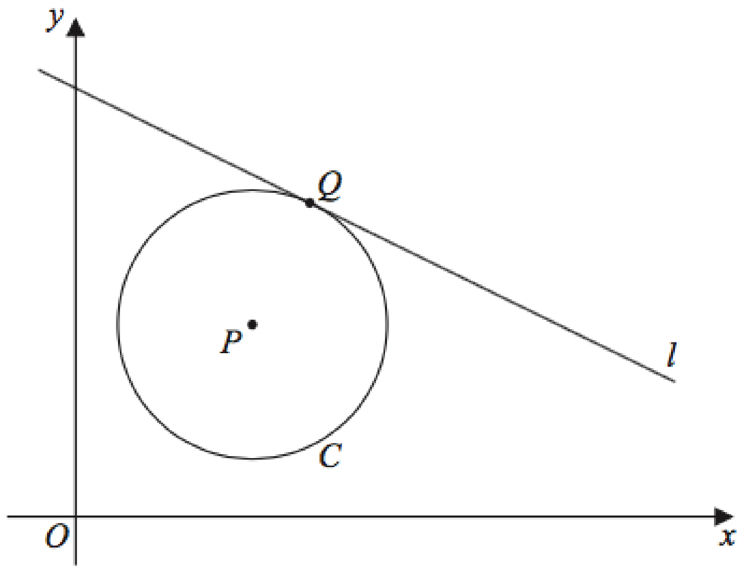

Figure 9: the circle C has centre $P(7, 8)$

(a) Find the length PQ, giving your answer as an exact value. (2)

Solution

$$\text{Length of } QP = \sqrt{(10-7)^2 - (8-13^2}$$
$$= \underline{\underline{\sqrt{34}}}.$$

(b) Hence write down an equation for C. (2)

Solution

$$\underline{\underline{(x-10)^2 + (y-8)^2 = 34}}.$$

The line l is a tangent to C at the point Q, as shown in Figure 9.

(c) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b, and c are integers. (4)

Solution

$$\text{Gradient of } PQ = \frac{13-8}{10-7} = \frac{5}{3}$$

and

$$\text{gradient of the perpendicular} = -\tfrac{3}{5}.$$

Finally,

$$\begin{aligned} y - 13 = -\tfrac{3}{5}(x - 10) &\Rightarrow 5(y - 13) = -3(x - 10) \\ &\Rightarrow 5y - 65 = -3x + 30 \\ &\Rightarrow \underline{\underline{3x + 5y - 95 = 0}}. \end{aligned}$$

23. The circle C has equation

$$x^2 + y^2 - 10x + 6y + 30 = 0.$$

Find

(a) the coordinates of the centre of C, (2)

Solution

$$\begin{aligned} x^2 + y^2 - 10x + 6y + 30 = 0 &\Rightarrow x^2 - 10x + y^2 + 6y = -30 \\ &\Rightarrow (x^2 - 10x + 25) + (y^2 + 6y + 9) = 25 + 9 - 30 \\ &\Rightarrow (x - 5)^2 + (y + 3)^2 = 4; \end{aligned}$$

the centre of C is $\underline{\underline{(5, -3)}}$.

(b) the radius of C, (2)

Solution

And radius is $\underline{\underline{2}}$.

(c) the y-coordinates of the points where the circle C crosses the line with equation $x = 4$, giving your answers as simplified surds. (3)

Solution

$$\begin{aligned} x = 4 &\Rightarrow 1 + (y + 3)^2 = 4 \\ &\Rightarrow (y + 3)^2 = 3 \\ &\Rightarrow y + 3 = -\sqrt{3} \text{ or } y + 3 = \sqrt{3} \\ &\Rightarrow \underline{\underline{y = -3 - \sqrt{3}}} \text{ or } \underline{\underline{y = -3 + \sqrt{3}}}. \end{aligned}$$