Dr Oliver Mathematics
GCSE Mathematics
2024 June Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Here are the first four terms of an arithmetic sequence: (2)

$$1 \qquad 5 \qquad 9 \qquad 13.$$

Find an expression, in terms of n, for the nth term of this sequence.

Solution

Let the

$$n\text{th term} = an + b.$$

Well,

$$\begin{array}{ccccccc} 1 & & 5 & & 9 & & 13 \\ & 4 & & 4 & & 4 & \end{array}$$

$$\begin{array}{ccccccc} a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:

$$a = 4$$

and

$$\begin{aligned} a + b = 1 &\Rightarrow 4 + b = 1 \\ &\Rightarrow b = -3. \end{aligned}$$

Hence,

$$n\text{th term} = \underline{\underline{4n - 3}}.$$

2. (a) Work out (2)

$$3\tfrac{4}{5} - 1\tfrac{2}{3}.$$

Solution
Well,

$$\begin{aligned} 3\tfrac{4}{5} - 1\tfrac{2}{3} &= 3 + \tfrac{4}{5} - 1 - \tfrac{2}{3} \\ &= (3-1) + (\tfrac{4}{5} - \tfrac{2}{3}) \end{aligned}$$

we make a common denominator:

$$\begin{aligned} &= 2 + (\tfrac{12}{15} - \tfrac{10}{15}) \\ &= 2 + \tfrac{2}{15} \\ &= \underline{\underline{2\tfrac{2}{15}}}. \end{aligned}$$

Kevin was asked to work out

$$2\tfrac{1}{3} \times \tfrac{5}{8}.$$

Here is his working and his answer:

$$\begin{aligned} 2\tfrac{1}{3} \times \tfrac{5}{8} &= \tfrac{7}{3} \times \tfrac{5}{8} \\ &= \tfrac{35}{24} \\ &= 1\tfrac{9}{24}. \end{aligned}$$

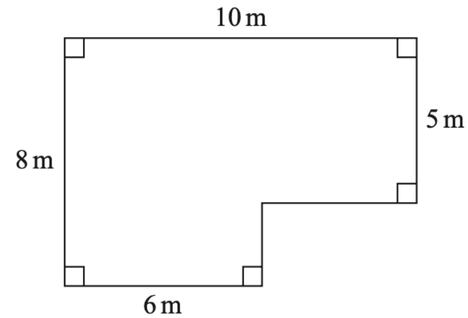
Kevin's answer is wrong.

(b) What mistake has Kevin made? (1)

Solution
On the last line, it should read:

$$\tfrac{35}{24} = \tfrac{24+11}{24} = \underline{\underline{1\tfrac{11}{24}}}.$$

3. The diagram shows a plan of a floor.



Petra is going to cover the floor with paint.

- Petra has 3 tins of paint.
- There are 2.5 litres of paint in each tin.

Petra thinks 1 litre of paint will cover 10 m^2 of floor.

(a) Assuming Petra is correct, does she have enough paint to cover the floor? (4)
You must show all your working.

Solution
Well,

$$\begin{aligned}\text{area of the floor} &= \text{big rectangle} - \text{smaller rectangle} \\ &= (10 \times 8) - [(10 - 6) \times (8 - 5)] \\ &= 80 - (4 \times 3) \\ &= 80 - 12 \\ &= 68 \text{ m}^2.\end{aligned}$$

She has

$$3 \times 2.5 = 7.5 \text{ litres.}$$

Now,

$$7.5 \times 10 = 75;$$

hence, she have enough paint to cover the floor.

Actually, 1 litre of paint will cover 11 m^2 of floor.

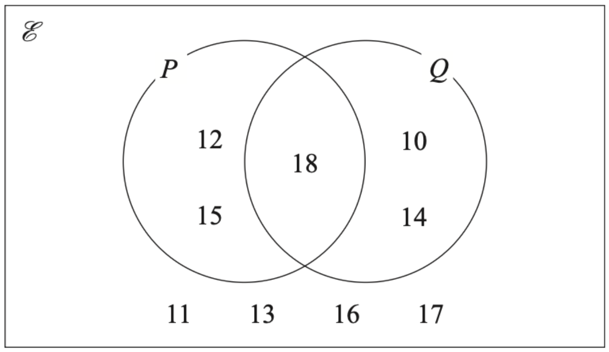
(b) Does this affect your answer to part (a)? (1)
You must give a reason for your answer.

Solution
No effect:

$$7.5 \times 11 = 75 + 7.5 = 82.5;$$

hence, she have even more paint to cover the floor.

4. Here is a Venn diagram.



(a) Write down the numbers that are in set P'. (1)

Solution
$\underline{\underline{P' = \{10, 11, 13, 14, 16, 17\}}}$.

A number is chosen at random from the universal set, $\mathscr{E}$.

(b) Find the probability that this number is in the set $P \cup Q$. (2)

Solution
Well, there are nine numbers and five are in $P \cup Q$. Hence,

$$P \cup Q = \underline{\underline{\tfrac{5}{9}}}.$$

5. Sophie drives a distance of 513 kilometres on a motorway in France.
She pays 0.81 euros for every 10 kilometres she drives.

(a) Work out an estimate for the total amount that Sophie pays. (3)

Solution

Well, round everything to 1 significant figure:

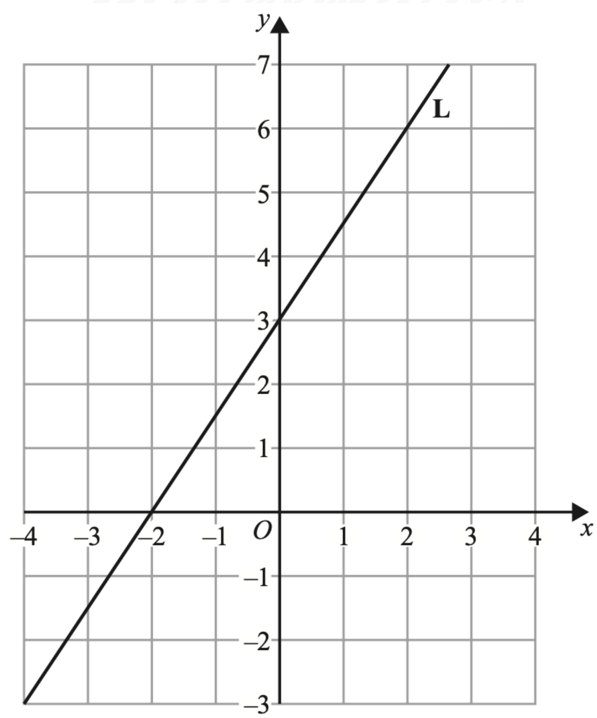
$$\begin{aligned} 0.81 \times \frac{513}{10} &\approx 0.8 \times \frac{500}{10} \\ &= 0.8 \times 50 \\ &= \underline{\underline{40 \text{ euros}}}. \end{aligned}$$

(b) Is your answer to part (a) an underestimate or an overestimate? (1)
Give a reason for your answer.

Solution

We rounded down both 0.81 (0.8) and 513 (500) so it is an $\underline{\underline{\text{underestimate}}}$.

6. Here is a straight line **L** drawn on a grid.



(a) Find an equation for **L**. (3)

Solution

It goes through the points $(-2, 0)$ and $(0, 3)$ and it has gradient

$$\begin{aligned} m &= \frac{3-0}{0-(-2)} \\ &= \tfrac{3}{2}. \end{aligned}$$

Hence, using $(0, 3)$, an equation for $\mathbf{L}$ is

$$y - 3 = \tfrac{3}{2}(x - 0) \Rightarrow \underline{\underline{y = \tfrac{3}{2}x + 3}}.$$

$\mathbf{M}$ is a different straight line with equation $y = 5x$.

(b) Write down the equation of a straight line parallel to $\mathbf{M}$. (1)

Solution
E.g., $\underline{\underline{y = 5x + 1}}$, $\underline{\underline{y = 5x - 17}}$, etc.

7. Kasim has some small jars, some medium jars, and some large jars. (5)
He has a total of 400 jars.

$\frac{3}{8}$ of the 400 jars are empty.

For the empty jars,

$$\begin{aligned} \text{number of small jars} : \text{number of medium jars} &= 3 : 4, \\ \text{number of medium jars} : \text{number of large jars} &= 1 : 2. \end{aligned}$$

Work out the percentage of Kasim's jars that are empty small jars.

Solution
Well, we increase the medium and large jars by a factor of four:

$$\text{number of medium jars} : \text{number of large jars} = 1 : 2 = 4 : 8$$

and now

$$\text{number of small jars} : \text{number of medium jars} : \text{number of large jars} = 3 : 4 : 8.$$

Now,

$$\begin{aligned}\text{number that are empty} &= \tfrac{3}{8} \times 400 \\ &= 3 \times 50 \\ &= 150,\end{aligned}$$

so

$$\begin{aligned}\text{number of small jars} &= \tfrac{3}{15} \times 150 \\ &= 30.\end{aligned}$$

Finally,

$$\begin{aligned}\text{empty small jars} &= \tfrac{30}{400} \times 100 \\ &= 30 \times \tfrac{1}{4} \\ &= \underline{\underline{7.5\%}}.\end{aligned}$$

8. Len has 8 parcels. (3)

- The mean weight of the 8 parcels is 2.5 kg.
- The mean weight of 3 of the parcels is 2 kg.

Work out the mean weight of the other 5 parcels.

Solution

Well,

$$\begin{aligned}\frac{(\text{mean weight} \times 5) + (2 \times 3)}{8} = 2.5 &\Rightarrow (\text{mean weight} \times 5) + (2 \times 3) = 2.5 \times 8 \\ &\Rightarrow 5\,\text{mean weight} + 6 = 20 \\ &\Rightarrow 5\,\text{mean weight} = 14 \\ &\Rightarrow \text{mean weight} = \underline{\underline{2.8 \text{ kg}}}.\end{aligned}$$

9. In a sale, the normal price of a coat is reduced by $R\%$. (1)

Given that

$$\text{sale price} = 0.7 \times \text{normal price},$$

find the value of R.

Solution

Well,

$$1 - 0.7 = 0.3,$$

so the the value of $\underline{\underline{R = 30}}$.

10. Solve the simultaneous equations: (4)

$$\begin{aligned} 5x - 2y &= 23, \\ 2x - 3y &= 18. \end{aligned}$$

Solution

Well,

$$\begin{aligned} 5x - 2y &= 23 \quad (1) \\ 2x - 3y &= 18 \quad (2). \end{aligned}$$

Do $3 \times (1)$ and $2 \times (2)$:

$$\begin{aligned} 15x - 6y &= 69 \quad (3) \\ 4x - 6y &= 36 \quad (4). \end{aligned}$$

Do $(3) - (4)$:

$$11x = 33 \Rightarrow \underline{\underline{x = 3}}.$$

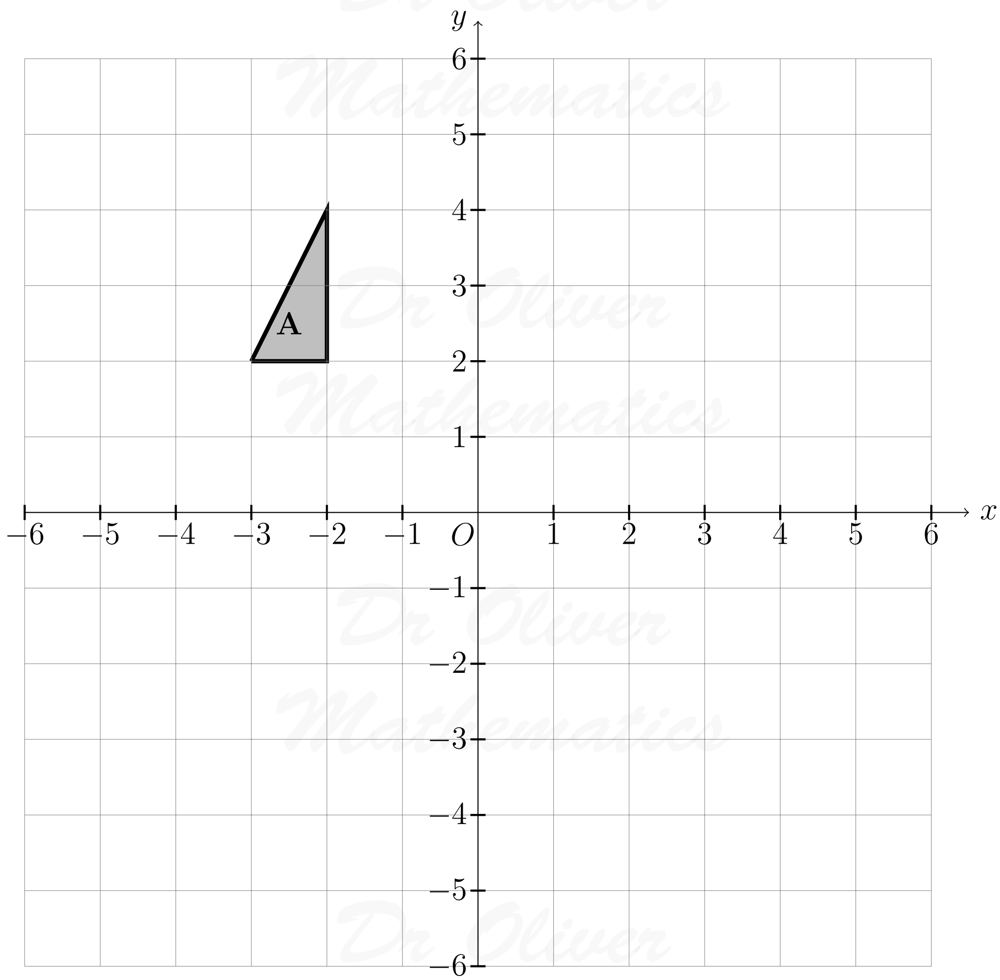
Insert this into (1):

$$\begin{aligned} 5(3) - 2y = 23 &\Rightarrow 15 - 2y = 23 \\ &\Rightarrow -2y = 8 \\ &\Rightarrow \underline{\underline{y = -4}}. \end{aligned}$$

[Check: $2(3) - 3(-4) = 6 + 12 = 18$ ✓]

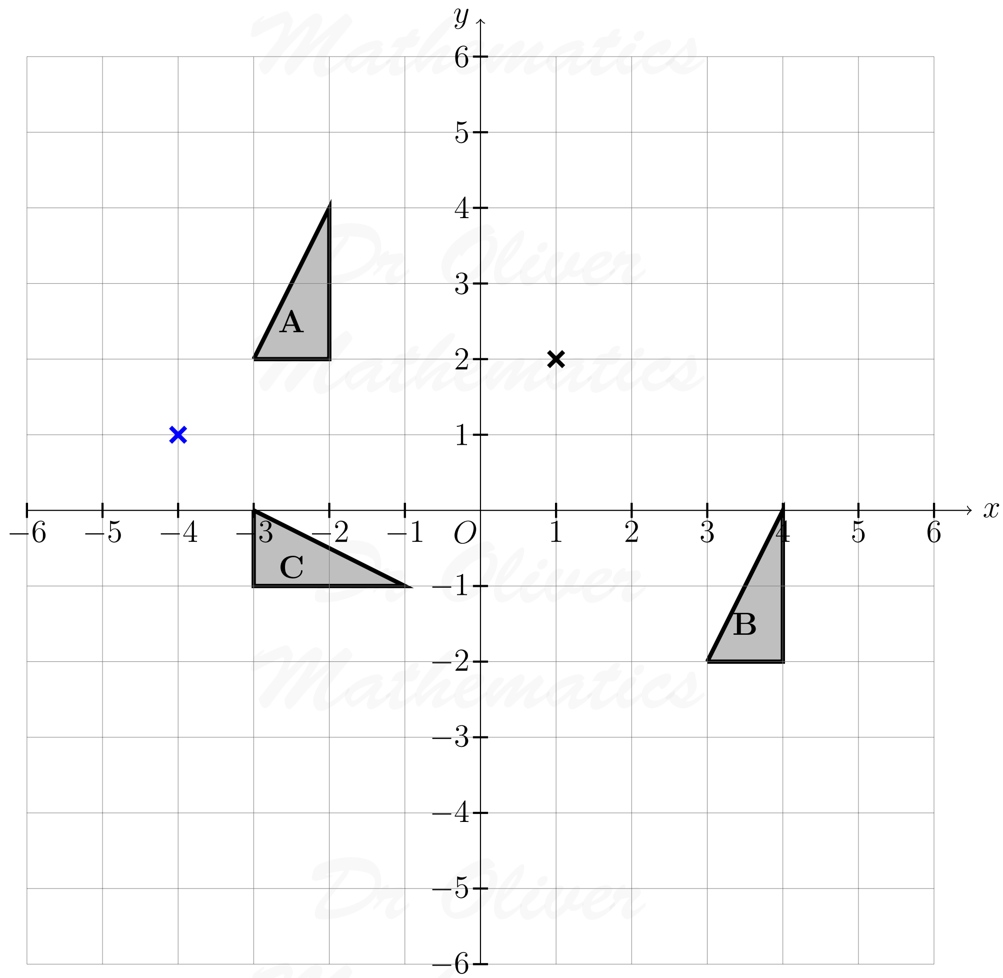
11. Triangle **A** is translated by the vector $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$ to give triangle **B**. (3)

Triangle **B** is rotated 90° clockwise about the point $(1, 2)$ to give triangle **C**.



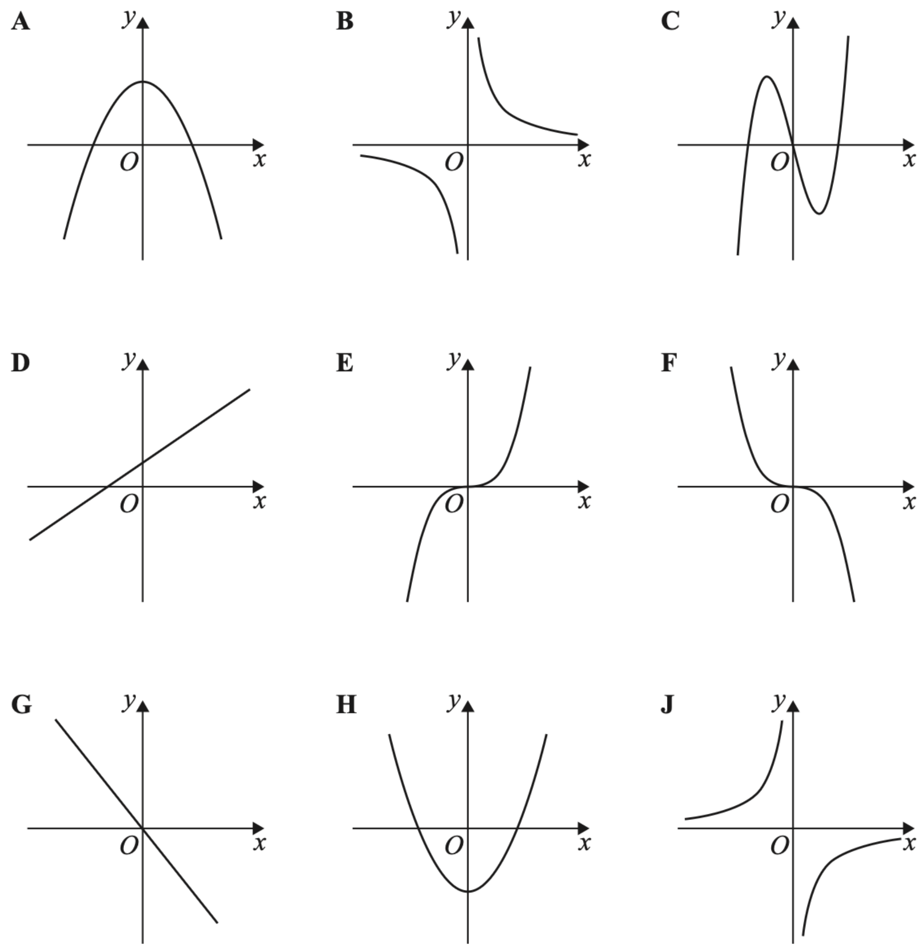
Describe fully the single transformation that maps triangle **A** onto triangle **C**.

Solution



Rotation, about $(-4, 1)$, $90°$ clockwise.

12. Here are some graphs.



Write down the letter of the graph that could have the equation:

(a) $y = x^2 - 4$, (1)

Solution

H.

(b) $y = -x^3$, (1)

Solution

F.

(c) $y = -\dfrac{5}{x}$. (1)

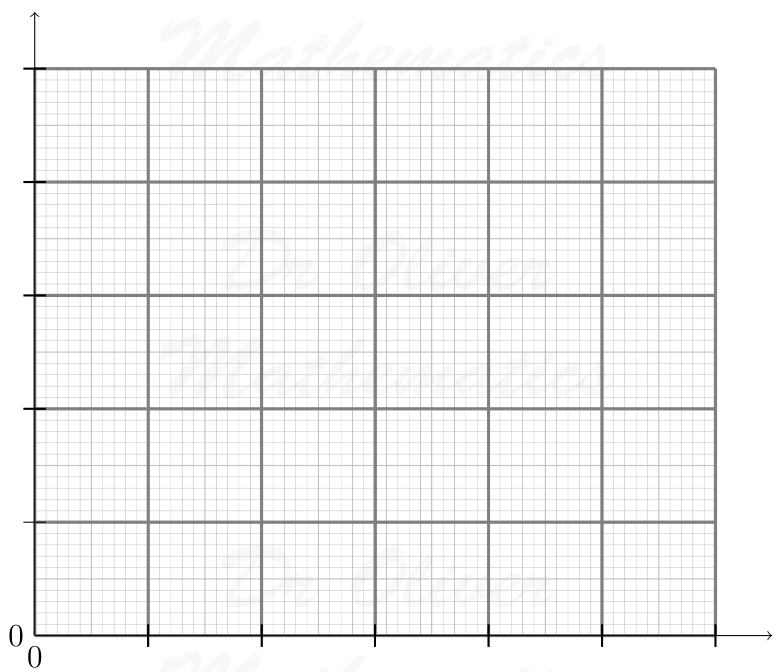
Solution

J.

13. The table gives information about the amount of time that each of 150 people were in a shop.

Time (t minutes)	Frequency
$0 < t \leqslant 10$	20
$10 < t \leqslant 30$	70
$30 < t \leqslant 35$	22
$35 < t \leqslant 50$	30
$50 < t \leqslant 60$	8

(a) On the grid, draw a histogram for this information. (3)

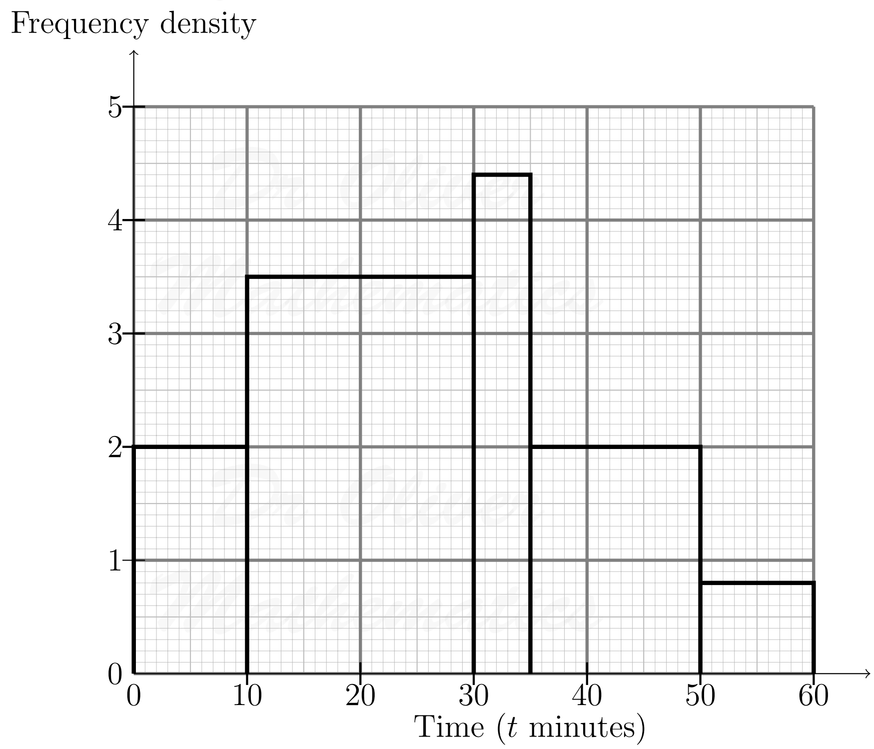


Solution

We first make up a table:

Time (t minutes)	Frequency	Width	Frequency Density
$0 < t \leqslant 10$	20	10	$\frac{20}{10} = 2$
$10 < t \leqslant 30$	70	20	$\frac{70}{20} = 3.5$
$30 < t \leqslant 35$	22	5	$\frac{22}{5} = 4.4$
$35 < t \leqslant 50$	30	15	$\frac{30}{15} = 2$
$50 < t \leqslant 60$	8	10	$\frac{8}{10} = 0.8$

Then, we complete the histogram:



(b) Work out an estimate for the fraction of these 150 people who were in the shop for between 20 minutes and 40 minutes. (2)

Solution
Well,

$$\begin{aligned} \text{20 minutes and 40 minutes} &= (\tfrac{1}{2} \times 70) + 22 + (\tfrac{1}{3} \times 30) \\ &= 35 + 22 + 10 \\ &= 67, \end{aligned}$$

so the fraction is $\underline{\underline{\frac{67}{150}}}$.

14. Expand and simplify (3)

$$(3x - 1)(2x + 3)(x - 5).$$

Solution

Well,

$\times$	$3x$	-1
$2x$	$6x^2$	$-2x$
$+3$	$+9x$	-3

so

$$(3x - 1)(2x + 3) = 6x^2 + 7x - 3.$$

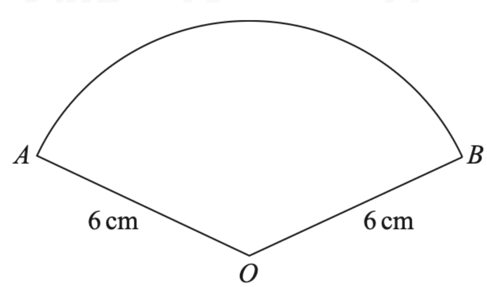
Now,

$\times$	$6x^2$	$+7x$	-3
x	$6x^3$	$+7x^2$	$-3x$
-5	$-30x^2$	$-35x$	$+15$

so

$$(3x - 1)(2x + 3)(x - 5) = \underline{\underline{6x^3 - 23x^2 - 38x + 15}}.$$

15. OAB is a sector of a circle with centre O and radius 6 cm. (4)



The length of the arc AB is 5π cm.

Work out, in terms of π, the area of the sector.
Give your answer in its simplest form.

Solution

Let θ° be the angle in the sector. Then

$$\begin{aligned}\frac{\theta}{360} = \frac{5\pi}{2\pi \times 6} &\Rightarrow \frac{\theta}{360} = \tfrac{5}{12} \\ &\Rightarrow \theta = \tfrac{5}{12} \times 360 \\ &\Rightarrow \theta = 5 \times 30 \\ &\Rightarrow \theta = 150.\end{aligned}$$

Finally,

$$\begin{aligned}\text{area} &= \pi \times 6^2 \times \tfrac{150}{360} \\ &= 36\pi \times \tfrac{15}{36} \\ &= \underline{\underline{15\pi}}\end{aligned}$$

16. There are only n orange sweets and 1 white sweet in a bag. (2)

- Saira takes at random a sweet from the bag and eats the sweet.
- She then takes at random another sweet from the bag and eats this sweet.

Show that the probability that Saira eats two orange sweets is

$$\frac{n-1}{n+1}.$$

Solution

Well, we hace $(n+1)$ sweets and, picking one, leaves Saira with n. So,

$$\begin{aligned}\mathrm{P}(OO) &= \frac{n}{n+1} \times \frac{n-1}{n} \\ &= \frac{\cancel{n}^1}{n+1} \times \frac{n-1}{\cancel{n}^1} \\ &= \underline{\underline{\frac{n-1}{n+1}}},\end{aligned}$$

as required.

17. (a) Rationalise the denominator of (1)

$$\frac{1}{\sqrt{7}}.$$

Solution

$$\begin{aligned}\frac{1}{\sqrt{7}} &= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} \\ &= \underline{\underline{\frac{\sqrt{7}}{7}}}.\end{aligned}$$

(b) Simplify fully (2)

$$\sqrt{80} - \sqrt{5}.$$

Solution

Well,

$$\begin{aligned}\sqrt{80} &= \sqrt{16 \times 5} \\ &= \sqrt{16} \times \sqrt{5} \\ &= 4\sqrt{5},\end{aligned}$$

and so

$$\begin{aligned}\sqrt{80} - \sqrt{5} &= 4\sqrt{5} - \sqrt{5} \\ &= \underline{\underline{3\sqrt{5}}}.\end{aligned}$$

18. Show that (3)

$$0.\dot{1}\dot{5} + 0.2\dot{2}\dot{7}$$

can be written in the form $\dfrac{m}{66}$, where m is an integer.

Solution

Let $x = 0.\dot{1}\dot{5}$ and $y = 0.2\dot{2}\dot{7}$. Now,

$$\begin{aligned} x &= 0.\dot{1}\dot{5} \quad (1) \\ 100x &= 15.\dot{1}\dot{5} \quad (2). \end{aligned}$$

Do $(2) - (1)$:

$$99x = 15 \Rightarrow x = \tfrac{15}{99}.$$

Now,

$$\begin{aligned} 10y &= 2.\dot{2}\dot{7} \quad (3) \\ 1\,000y &= 227.\dot{2}\dot{7} \quad (4). \end{aligned}$$

Do $(4) - (3)$:

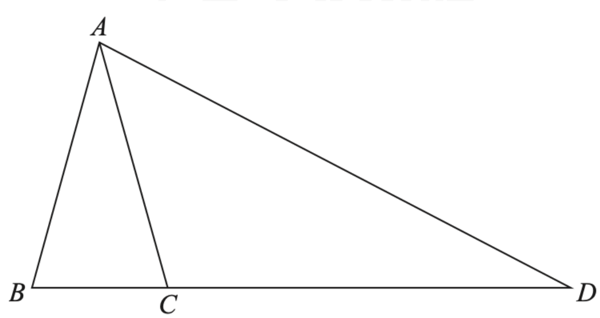
$$990y = 225 \Rightarrow y = \tfrac{225}{990}.$$

Finally,

$$\begin{aligned} 0.\dot{1}\dot{5} + 0.2\dot{2}\dot{7} &= \tfrac{15}{99} + \tfrac{225}{990} \\ &= \tfrac{150}{990} + \tfrac{225}{990} \\ &= \tfrac{150+225}{990} \\ &= \tfrac{375}{990} \\ &= \tfrac{25\times 15}{66\times 15} \\ &= \underline{\underline{\tfrac{25}{66}}}; \end{aligned}$$

hence, $\underline{\underline{m = 25}}$.

19. ABC and DAB are similar isosceles triangles. (3)



- $AB = AC$.

- $AD = BD$.
- $BC : CD = 4 : 21$.

Find the ratio

$$AB : AD.$$

Solution

Well, the question tell us that

$$\begin{aligned} BD &= BC + CD \\ &= 4 + 21 \\ &= 25. \end{aligned}$$

So

$$BC = \tfrac{4}{25}x \text{ and } CD = \tfrac{21}{25}x.$$

Similar triangles:

$$\begin{aligned} \frac{BC}{AB} = \frac{AC}{BC} &\Rightarrow \frac{\frac{4}{25}}{AB} = \frac{AB}{x} \\ &\Rightarrow AB^2 = \tfrac{4}{25} \\ &\Rightarrow AB = \tfrac{2}{5}. \end{aligned}$$

Finally,

$$\begin{aligned} AB : AD &= \tfrac{2}{5} : 1 \\ &= \underline{\underline{2 : 5}}. \end{aligned}$$

20. (3)

$$2^x = \frac{2^n}{\sqrt[3]{2}} \text{ and } 2^y = \left(\sqrt{2}\right)^5.$$

Given that

$$x + y = 8,$$

work out the value of n.

Solution

Well,

$$\begin{aligned} 2^x &= \frac{2^n}{\sqrt[3]{2}} \\ &= \frac{2^n}{2^{\frac{1}{3}}} \\ &= 2^{n-\frac{1}{3}}, \end{aligned}$$

and

$$\begin{aligned} 2^y &= \left(\sqrt{2}\right)^5 \\ &= \left(2^{\frac{1}{2}}\right)^5 \\ &= 2^{\frac{5}{2}}. \end{aligned}$$

Now,

$$\begin{aligned} 2^x \times 2^y &= 2^{x+y} \\ &= 2^{n-\frac{1}{3}+\frac{5}{2}} \\ &= 2^{n-\frac{2}{6}+\frac{15}{6}} \\ &= 2^{n+\frac{13}{6}}. \end{aligned}$$

Finally,

$$\begin{aligned} x+y=8 &\Rightarrow n+\tfrac{13}{6}=8 \\ &\Rightarrow n = 8-\tfrac{13}{6} \\ &\Rightarrow n = \tfrac{48}{6}-\tfrac{13}{6} \\ &\Rightarrow \underline{\underline{n = \tfrac{35}{6}}}. \end{aligned}$$

21. A solid cuboid has a volume of 300 cm^3. (5)
The cuboid has a total surface area of 370 cm^2.

The length of the cuboid is 20 cm.

The width of the cuboid is greater than the height of the cuboid.

Work out the height of the cuboid.

You must show all your working.

Solution

Let w be the width of the cuboid and let h be the height of the cuboid. Then

$$20hw = 300 \Rightarrow w = \frac{15}{h} \quad (1)$$

and

$$\begin{aligned} 2(20w + 20h + hw) = 370 &\Rightarrow 20w + 20h + hw = 185 \\ &\Rightarrow 20w + hw = 185 - 20h \\ &\Rightarrow (20 + h)w = 185 - 20h \\ &\Rightarrow w = \frac{185 - 20h}{20 + h} \quad (2). \end{aligned}$$

Do (1) = (2) and cross-multiply:

$$\begin{aligned} \frac{15}{h} = \frac{185 - 20h}{20 + h} &\Rightarrow 15(20 + h) = h(185 - 20h) \\ &\Rightarrow 15(20 + h) = 5h(37 - 4h) \end{aligned}$$

divide each side by 5:

$$\begin{aligned} &\Rightarrow 3(20 + h) = h(37 - 4h) \\ &\Rightarrow 60 + 3h = 37h - 4h^2 \\ &\Rightarrow 4h^2 - 34h + 60 = 0 \\ &\Rightarrow 2(2h^2 - 17h + 30) = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & -17 \\ \text{multiply to:} & (+2) \times (+30) = +60 \end{array}\right\} -5, -12$$

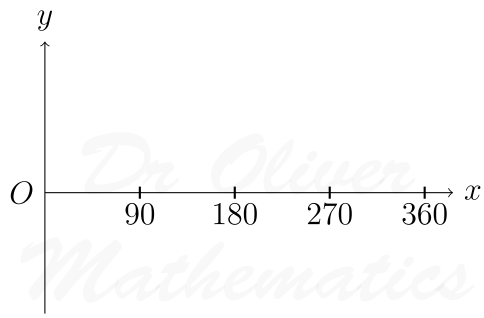
e.g.,

$$\begin{aligned} &\Rightarrow 10[2h^2 - 5h - 12 + 30] = 0 \\ &\Rightarrow 10[h(2h - 5) - 6(2h - 5)] = 0 \\ &\Rightarrow 10(h - 6)(2h - 5) = 0 \\ &\Rightarrow h - 6 = 0 \text{ or } 2h - 5 = 0 \\ &\Rightarrow h = 6 \text{ or } h = 2.5. \end{aligned}$$

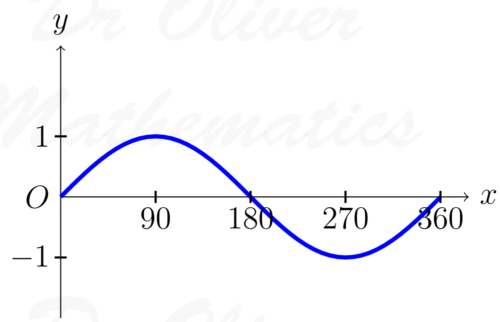
Now, the question says that "The width of the cuboid is greater than the height of the cuboid": since w and h are inversely proportional, $\underline{\underline{h = 2.5}}$.

22. (a) Sketch the graph of (2)

$$y = \sin x^\circ, \text{ for } 0 \leqslant x \leqslant 360.$$



Solution



(b) Solve the equation (2)

$$2\sin x^\circ = -1, \text{ for } 0 \leqslant x \leqslant 360.$$

Solution

Well,

$$\begin{aligned} 2\sin x^\circ = -1 &\Rightarrow \sin x^\circ = -\tfrac{1}{2} \\ &\Rightarrow \underline{\underline{x = 210,\ 330}}. \end{aligned}$$

23. **C** is a circle with centre $(0, 0)$. (5)
L is a straight line.

The circle **C** and the line **L** intersect at the points P and Q.
The coordinates of P are $(5, 10)$.

The x-coordinate of Q is -2.

L has a positive gradient and crosses the y-axis at the point $(0, k)$.

Find the value of k.

Solution

Well,

$$\begin{aligned} x^2 + y^2 &= 5^2 + 10^2 \\ &= 25 + 100 \\ &= 125, \end{aligned}$$

so **C** is

$$x^2 + y^2 = 125.$$

Now,

$$\begin{aligned} x = -2 &\Rightarrow (-2)^2 + y^2 = 125 \\ &\Rightarrow 4 + y^2 = 125 \\ &\Rightarrow y^2 = 121 \\ &\Rightarrow y = \pm 11. \end{aligned}$$

Next, the question says that **L** has a positive gradient so $y = -11$.
Then, $Q(-2, -11)$.

Gradient:

$$\begin{aligned} m &= \frac{-11 - 10}{-2 - 5} \\ &= \frac{-21}{-7} \\ &= 3, \end{aligned}$$

and the equation of the line PQ is

$$y - 10 = 3(x - 5).$$

Finally,

$$\begin{aligned} x = 0 &\Rightarrow y - 10 = 3(0 - 5) \\ &\Rightarrow y - 10 = -15 \\ &\Rightarrow y = -5; \end{aligned}$$

hence, $\underline{\underline{k = -5}}$.