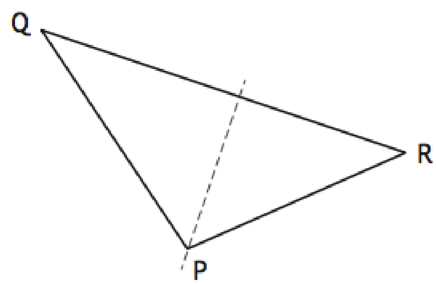
Dr Oliver Mathematics
Mathematics: Higher
2023 Paper 2: Calculator
1 hour 30 minutes

The total number of marks available is 65.
You must write down all the stages in your working.

1. Triangle PQR has vertices $P(5, -1)$, $Q(-2, 8)$, and $R(13, 3)$.



   (a) Find the equation of the altitude from P. (3)

   (b) Calculate the angle that the side PR makes with the positive direction of the x-axis. (2)

2. Find the equation of the tangent to the curve with equation (4)

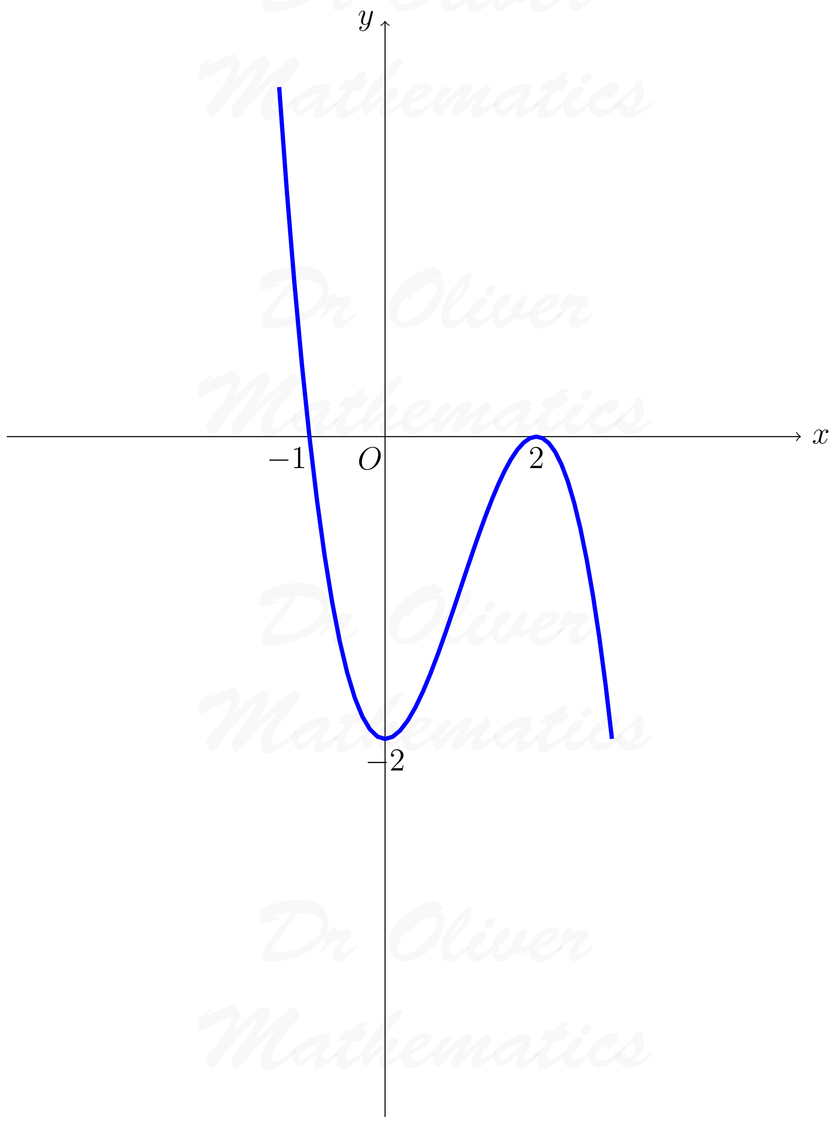
$$y = 2x^5 - 3x$$

   at the point where $x = 1$.

3. Find (2)

$$\int 7\cos(4x + \tfrac{1}{3}\pi)\,\mathrm{d}x.$$

4. The diagram shows the cubic graph of $y = \mathrm{f}(x)$, with stationary points at $(2, 0)$ and $(0, -2)$. (2)



Sketch the graph of $y = 2\,\mathrm{f}(-x)$.

5. A function, f, is defined by (3)

$$\mathrm{f}(x) = (3 - 2x)^4, \text{ where } x \in \mathbb{R}.$$

Calculate the rate of change of f when $x = 4$.

6. A function $\mathrm{f}(x)$ is defined by (3)

$$\mathrm{f}(x) = \frac{2}{x} + 3,\ x > 0.$$

Find the inverse function, $\mathrm{f}^{-1}(x)$.

7. Solve the equation (5)

$$\sin x^\circ + 2 = 3\cos 2x^\circ$$

for $0 \leqslant x < 360$.

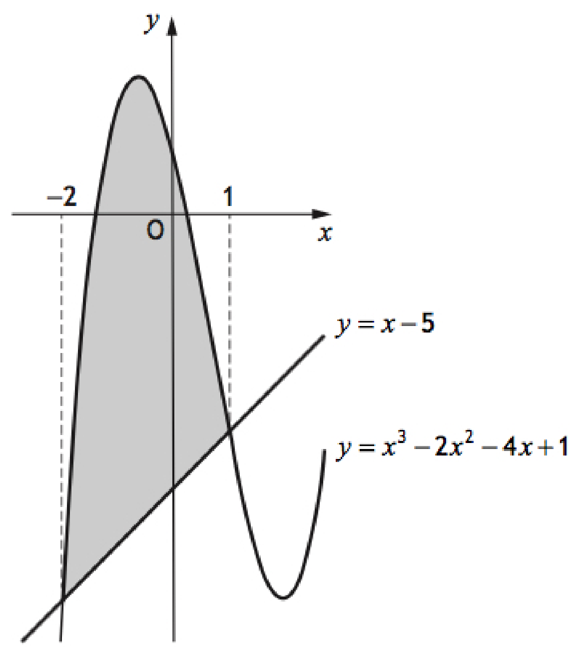
8. The diagram shows part of the curve with equation (5)

$$y = x^3 - 2x^2 - 4x + 1$$

and the line with equation

$$y = x - 5.$$

The curve and the line intersect at the points where $x = -2$ and $x = 1$.



Calculate the shaded area.

9. (a) Express (4)

$$7\cos x^\circ - 3\sin x^\circ$$

in the form

$$k\sin(x + a)^\circ,$$

where $k > 0$ and $0 < a < 360$.

(b) Hence, or otherwise, find:

(i) the maximum value of $14\cos x^\circ - 6\sin x^\circ$, (1)

(ii) the value of x for which it occurs where $0 \leqslant x < 360$. (2)

10. Determine the range of values of x for which the function (4)

$$\mathrm{f}(x) = 2x^3 + 9x^2 - 24x + 6$$

is strictly decreasing.

11. Circle C_1 has equation

$$(x-4)^2 + (y+2)^2 = 37.$$

Circle C_2 has equation

$$x^2 + y^2 + 2x - 6y - 7 = 0.$$

(a) Calculate the distance between the centres of C_1 and C_2. (3)

(b) Hence, show that C_1 and C_2 intersect at two distinct points. (3)

12. A curve, for which (4)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 8x^3 + 3,$$

passes through the point $(-1, 3)$.

Express y in terms of x.

13. A patient is given a dose of medicine.

The concentration of the medicine in the patient's blood is modelled by where

$$C_t = 11\mathrm{e}^{-0.005\,3t},$$

where

- t is the time, in minutes, since the dose of medicine was given aand
- C_t is the concentration of the medicine, in mg/l, at time t.

(a) Calculate the concentration of the medicine 30 minutes after the dose was given (1)

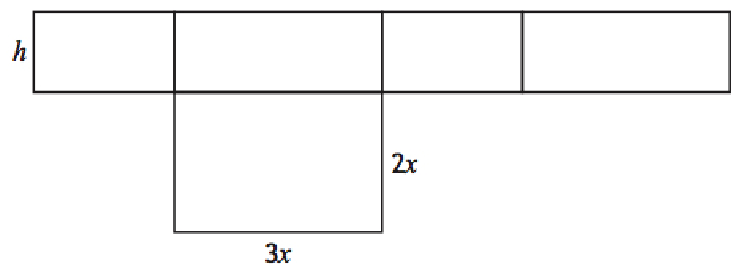
The dose of medicine becomes ineffective when its concentration falls to 0.66 mg/l.

(b) Calculate the time taken for this dose of the medicine to become ineffective. (3)

14. A net of an open box is shown.

The box is a cuboid with height h centimetres.

The base is a rectangle measuring $3x$ centimetres by $2x$ centimetres.



(a) (i) Express the area of the net, A cm^2, in terms of h and x. (1)

(ii) Given that $A = 7\,200$ cm^2, show that the volume of the box, V cm^3, is given by (2)

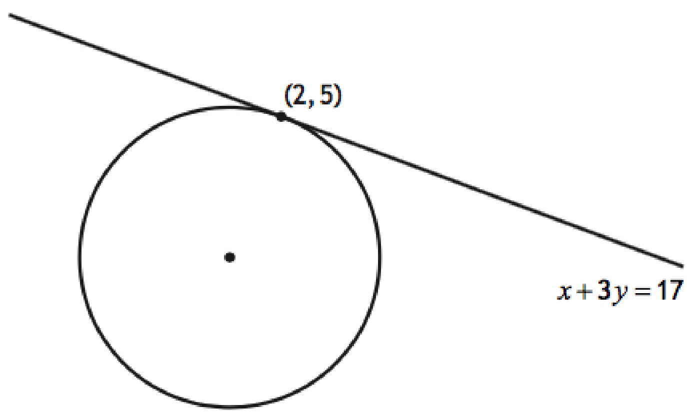
$$V = 4\,320x - \tfrac{18}{5}x^3.$$

(b) Determine the value of x that maximises the volume of the box. (4)

15. The line (4)

$$x + 3y = 17$$

is a tangent to a circle at the point $(2, 5)$.



The centre of the circle lies on the y-axis.

Find the coordinates of the centre of the circle.