Dr Oliver Mathematics
AQA GCSE Mathematics
2019 November Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Circle the calculation that decreases 250 by 15%: (1)

$$250 \div 1.15 \quad 250 \times 0.15 \quad 250 \times 0.85 \quad 250 \div 0.85$$

Solution

$$250 \div 1.15 \quad 250 \times 0.15 \quad \underline{\underline{250 \times 0.85}} \quad 250 \div 0.85$$

2. Solve (1)

$$3x = 2x.$$

Circle your answer.

$$x = -1 \quad x = 0 \quad x = \tfrac{2}{3} \quad x = \tfrac{3}{2}$$

Solution

Well,

$$3x = 2x \Rightarrow x = 0$$

so

$$x = -1 \quad \underline{\underline{x = 0}} \quad x = \tfrac{2}{3} \quad x = \tfrac{3}{2}$$

3. $A(2, 13)$ and $B(10, 1)$. (1)
Circle the midpoint of AB.

$$(4, 6) \quad (5, 6.5) \quad (6, 7) \quad (8, 12)$$

Solution

Well, the midpoint is

$$\left(\frac{2+10}{2}, \frac{13+1}{2}\right) = (6, 7)$$

so

$$(4, 6) \quad (5, 6.5) \quad \underline{\underline{(6, 7)}} \quad (8, 12)$$

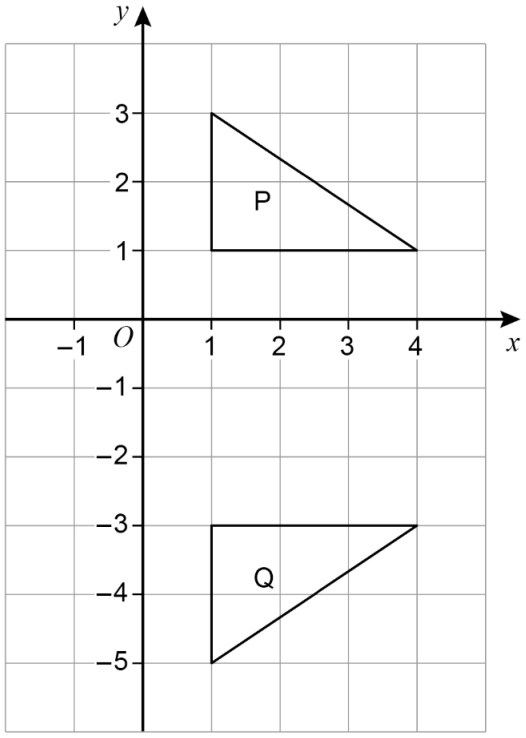
4. Circle the expression equivalent to $(2x)^4$. (1)

$$2x^4 \quad 6x^4 \quad 8x^4 \quad 16x^4$$

Solution

$$2x^4 \quad 6x^4 \quad 8x^4 \quad \underline{\underline{16x^4}}$$

5. (a) Here are two triangles, P and Q. (1)



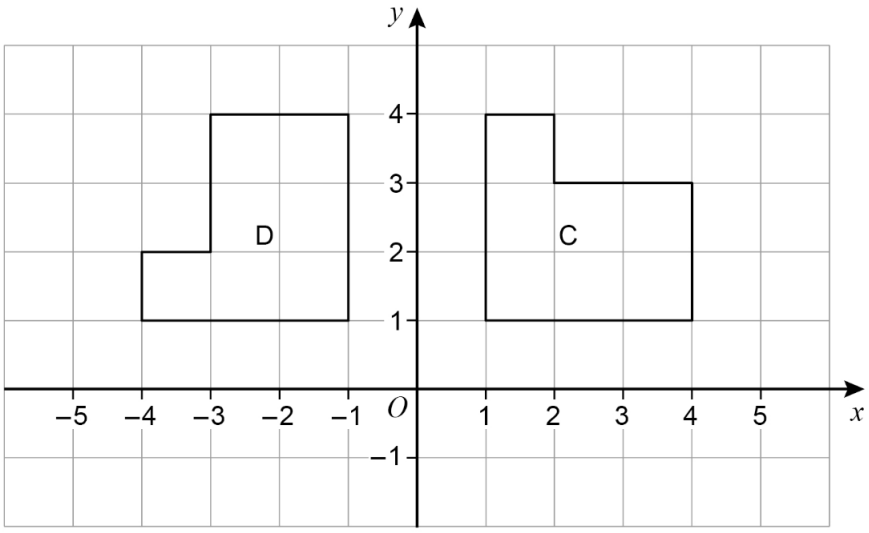
Here is a statement.

A transformation that maps P to Q is a reflection in the line $x = -1$.

Make **one** criticism of the statement.

Solution
E.g., reflection in the line $y = -1$.

(b) Here are two triangles, C and D. (1)



Here is a statement.

A transformation that maps C to D is a rotation through 90° anticlockwise.

Make **one** criticism of the statement.

Solution
E.g., they've forgotten to give the centre of the transformation.

6. (a) A geometric progression starts (1)

4 16

Work out the next term.

Solution

Well,

$$\frac{16}{4} = 4$$

and

$$16 \times 4 = \underline{\underline{64}}.$$

(b) A Fibonacci-type sequence starts (2)

$$3 \quad -8$$

The sequence is continued by adding the previous two terms.

Work out the next $\underline{\underline{\text{two}}}$ terms.

Solution

$$\begin{aligned} \text{3rd term}: &\quad 3 + (-8) = \underline{\underline{-5}} \\ \text{4th term}: &\quad -8 + (-5) = \underline{\underline{-13}}. \end{aligned}$$

7. Given that (2)

$$a \times 60 = b,$$

work out the value of

$$\frac{4b}{a}.$$

Solution

Well,

$$a \times 60 = b \Rightarrow 60 = \frac{b}{a}$$

and

$$\begin{aligned} \frac{4b}{a} &= 4 \times \frac{b}{a} \\ &= 4 \times 60 \\ &= \underline{\underline{240}}. \end{aligned}$$

8. Write (3)

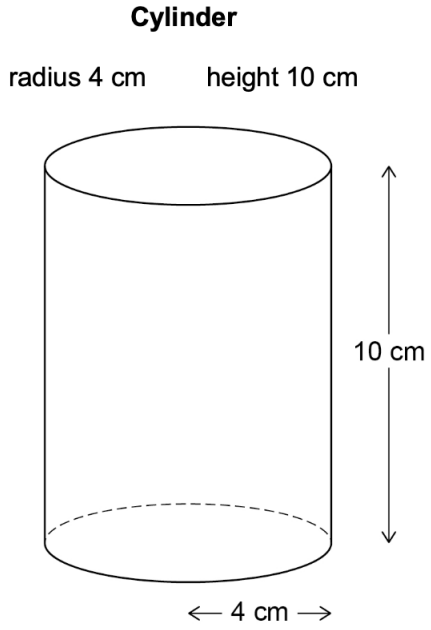
$$27 \times (3^2)^7$$

as a single power of 3.

Solution

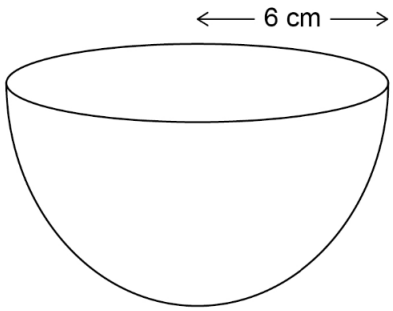
$$27 \times (3^2)^7 = 3^3 \times 3^{14}$$
$$= \underline{\underline{3^{17}}}.$$

9. Here are two solids. (4)



Hemisphere

radius 6 cm



volume of a hemisphere $= \frac{2}{3}\pi r^3$ where r is the radius

Which solid has the greater volume?
You **must** show your working.

Solution

Well,

$$\begin{aligned} \text{volume}_{\text{cylinder}} &= \pi \times 4^2 \times 10 \\ &= 160\pi \end{aligned}$$

and

$$\begin{aligned} \text{volume}_{\text{hemisphere}} &= \tfrac{2}{3} \times \pi \times 6^3 \\ &= \tfrac{2}{3}\pi \times 216 \\ &= 2\pi \times 72 \\ &= 144\pi; \end{aligned}$$

so, the cylinder has more volume.

10. Saj makes Rose Pink paint and Cherry Pink paint. (4)
He mixes red paint with white paint as shown.

Rose Pink	Cherry Pink
red : white = 1 : 2	red : white = 4 : 3

He makes 60 litres of Rose Pink paint.
To this Rose Pink paint he adds 80 litres of red paint and 28 litres of white paint.

Has he now made Cherry Pink paint?
You **must** show your working.

Solution

Well,

$$(\tfrac{1}{1+2}) \times 60 = 20$$

so, for the 60 litres of Rose Pink paint, he uses 20 red and 40 white.

He adds

$$40 + 28 = 68 \text{ white paint}$$

and

$$20 + 80 = 100 \text{ red paint.}$$

What is the ratio red : white? Well,

$$\begin{aligned} \text{red : white} &= 100 : 68 \\ &= \tfrac{100}{25} : \tfrac{68}{25} \\ &= 4 : 2\tfrac{18}{72}; \end{aligned}$$

hence, he has <u>not made</u> Cherry Pink paint.

In fact,

$$40 + 32 = 72 \text{ white paint}$$

and

$$20 + 76 = 96 \text{ red paint}$$

and

$$\begin{aligned} \text{red : white} &= 96 : 72 \\ &= \tfrac{96}{24} : \tfrac{72}{24} \\ &= 4 : 3 \end{aligned}$$

and he <u>made</u> Cherry Pink paint.

11. (a) Work out (2)

$$\frac{2 \times 10^{14}}{8 \times 10^{9}}.$$

Give your answer in standard form.

Solution

$$\begin{aligned}\frac{2 \times 10^{14}}{8 \times 10^{9}} &= \frac{2}{8} \times \frac{10^{14}}{10^{9}} \\ &= 0.25 \times 10^{5} \\ &= \underline{\underline{2.5 \times 10^{4}}}.\end{aligned}$$

(b) (2)

$$6\,200.07 = 6.2 \times 10^{c} + 7 \times 10^{d}.$$

Work out the values of c and d.

Solution

Well,

$$\underline{\underline{c = 3}} \text{ and } \underline{\underline{d = -2}}.$$

12. (1)

$$V = \frac{k}{H}, \text{ where } k \text{ is a constant.}$$

Which **two** statements are correct?
Tick **two** boxes.

- ☐ V is directly proportional to H
- ☐ V is inversely proportional to H
- ☐ V is directly proportional to $\frac{1}{H}$
- ☐ V is inversely proportional to $\frac{1}{H}$

Solution

$\underline{\underline{V \text{ is inversely proportional to } H}}$ and $\underline{\underline{V \text{ is directly proportional to } \frac{1}{H}}}$.

13. The nth term of a sequence is (3)

$$\frac{n(n-4)}{\sqrt{n+3}}.$$

Work out the sum of the 1st and 6th terms.

Solution

$$\begin{aligned}
\text{1st term}: \quad & \frac{1(1-4)}{\sqrt{1+3}} \\
&= \frac{-3)}{\sqrt{4}} \\
&= \frac{-3)}{2} \\
&= -1\tfrac{1}{2}; \\
\text{6th term}: \quad & \frac{6(6-4)}{\sqrt{6+3}} \\
&= \frac{12}{\sqrt{9}} \\
&= \frac{12}{3} \\
&= 4;
\end{aligned}$$

hence,

$$-1\tfrac{1}{2} + 4 = \underline{\underline{-2\tfrac{1}{2}}}.$$

14. (1)

$$8\,300 = 100 \times 83.$$

Circle the number that is closest in value to

$$\sqrt{8\,300}$$

19 90 830 900

Solution

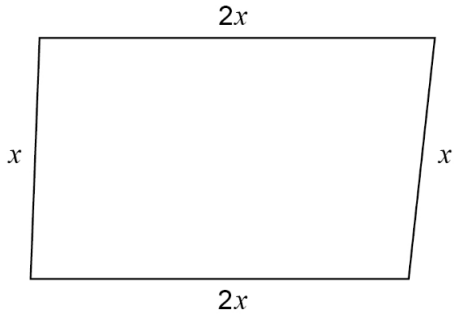
Well,

$$\begin{aligned}\sqrt{8\,300} &= \sqrt{100 \times 83} \\ &= \sqrt{100} \times \sqrt{83} \\ &\approx \sqrt{100} \times \sqrt{81} \\ &= 10 \times 9 \\ &= 90\end{aligned}$$

so

19 $\underline{\underline{90}}$ 830 900

15. Here is a **sketch** of a quadrilateral. (3)
All lengths are in centimetres.



Not drawn accurately

Tick **one** box for each statement.

	True	May be true	Not true
The quadrilateral is a rectangle			
The quadrilateral is a parallelogram			
The quadrilateral is a rhombus			
The quadrilateral is a kite			

Solution

	True	May be true	Not true
The quadrilateral is a rectangle		Yes	
The quadrilateral is a parallelogram	Yes		
The quadrilateral is a rhombus			Yes
The quadrilateral is a kite			Yes

16. In a box there are some buttons. (4)

45 are large and the rest are small.
Some are yellow and the rest are green.

- The number of small is $\frac{5}{3}$ of the number of large.
- The number of green is 300% of the number of yellow.
- There are 12 small yellow buttons.

How many large green buttons are there?
You may use the two-way table to help you.

	Large	Small	Total
Yellow		12	
Green			
Total	45		

Solution

Well,

$$45 \times \tfrac{5}{3} = 15 \times 5 = 75$$

so that means

$$75 - 12 = 63$$

of the small buttons are green. Hence, there are

$$45 + 75 = 120 \text{ buttons.}$$

Now, the number of green is 300% of the number of yellow so let the number of green and yellow be $3x$ and x respectively:

	Large	Small	Total
Yellow		12	x
Green		63	$3x$
Total	45	75	120

Next,

$$\tfrac{1}{4} \times 120 = 30 \text{ and } \tfrac{3}{4} \times 120 = 90$$

and that leaves us

	Large	Small	Total
Yellow		12	30
Green		63	90
Total	45	75	$4x$

We fill in the large buttons:

	Large	Small	Total
Yellow	18	12	30
Green	$\underline{\underline{27}}$	63	90
Total	45	75	$4x$

Hence, there are $\underline{\underline{27}}$ large green buttons.

17. (1)

$$\mathbf{a} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}.$$

Work out

$$\mathbf{a} - 3\mathbf{b}.$$

Circle your answer.

$$\begin{pmatrix} -6 \\ 17 \end{pmatrix} \quad \begin{pmatrix} -6 \\ -13 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 17 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -13 \end{pmatrix}$$

Solution

$$\begin{aligned}\mathbf{a} - 3\mathbf{b} &= \begin{pmatrix} -3 \\ 2 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -5 \end{pmatrix} \\ &= \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -15 \end{pmatrix} \\ &= \begin{pmatrix} -6 \\ 17 \end{pmatrix}\end{aligned}$$

so

$$\underline{\underline{\begin{pmatrix} -6 \\ 17 \end{pmatrix}}} \quad \begin{pmatrix} -6 \\ -13 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 17 \end{pmatrix} \quad \begin{pmatrix} 0 \\ -13 \end{pmatrix}$$

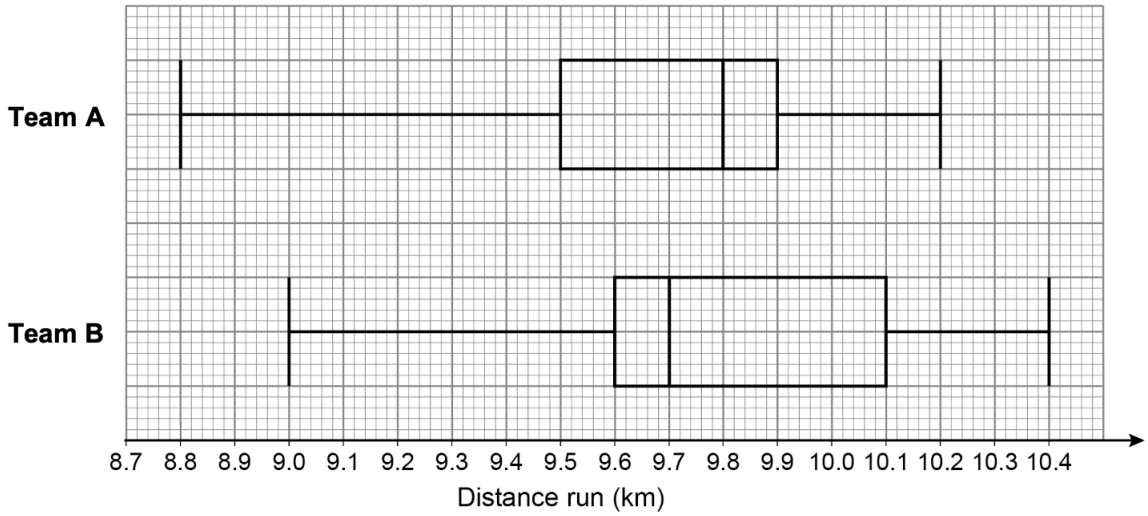
18. Solve (4)

$$\frac{x+15}{3} = 2(x+10).$$

Solution

$$\begin{aligned}\frac{x+15}{3} = 2(x+10) &\Rightarrow x + 15 = 6(x+10) \\ &\Rightarrow x + 15 = 6x + 60 \\ &\Rightarrow -45 = 5x \\ &\Rightarrow \underline{\underline{x = -9}}.\end{aligned}$$

19. The box plots represent the distances run by the players in a football match.



(a) On average, which team's players ran further? (1)
Tick a box.

☐ Team A ☐ Team B

Give a reason for your answer.

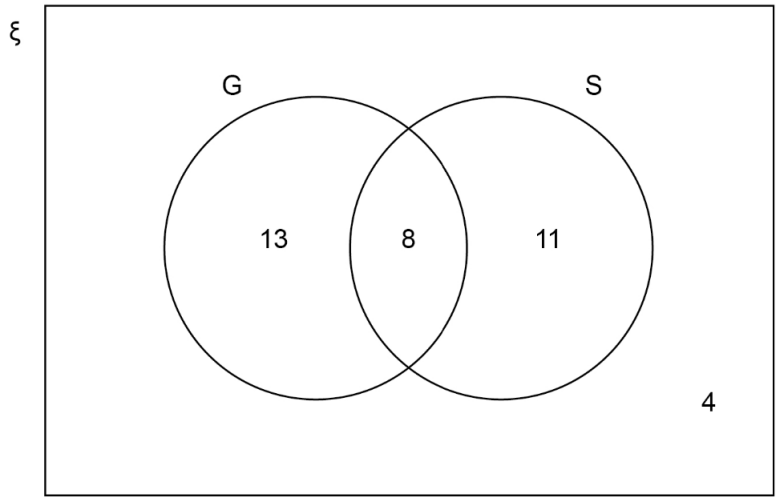
Solution
Team A as the median is higher.

(b) The players in Team A ran more consistent distances. (1)
How do the box plots show this?

Solution
The IQR is smaller.

20. The Venn diagram shows information about some houses.

- G = houses with a garage.
- S = houses with a shed.



A house is chosen at random.

(a) The house has a garage. (1)
What is the probability that it has a shed?

Solution

$$\begin{aligned} \mathrm{P}(S|G) &= \frac{8}{13+8} \\ &= \underline{\underline{\tfrac{8}{21}}}. \end{aligned}$$

(b) The house does **not** have a garage. (1)
What is the probability that it does **not** have a shed?

Solution

$$\begin{aligned} \mathrm{P}(S'|G') &= \frac{4}{11+4} \\ &= \underline{\underline{\tfrac{4}{15}}}. \end{aligned}$$

(c) Show that (2)

$$\mathrm{P}[(G \cap S)'] > P(G \cup S').$$

Solution

Well, there are

$$13 + 8 + 11 + 4 = 36 \text{ houses.}$$

Now,

$$\begin{aligned} \mathrm{P}[(G \cap S)'] &= \frac{36 - 8}{36} \\ &= \tfrac{28}{36} \end{aligned}$$

and

$$\begin{aligned} \mathrm{P}(G \cup S') &= \frac{13 + 8 + 4}{36} \\ &= \tfrac{25}{36}; \end{aligned}$$

hence,

$$\underline{\underline{\mathrm{P}[(G \cap S)'] > P(G \cup S')}},$$

as required.

21. Work out (1)

$$0.70\dot{4}\,\dot{8} - 0.001$$

Circle your answer.

$$0.703\dot{4}\,\dot{8} \quad 0.703\,\dot{8} \quad 0.703\,\dot{8}\dot{4} \quad 0.703\,\dot{8}\dot{4}$$

Solution

Well,

$$\begin{aligned} 0.70\dot{4}\,\dot{8} - 0.001 &= 0.704\,848\,\ldots - 0.001 \\ &= 0.703\,848\,\ldots \\ &= 0.703\,\dot{8}\dot{4} \end{aligned}$$

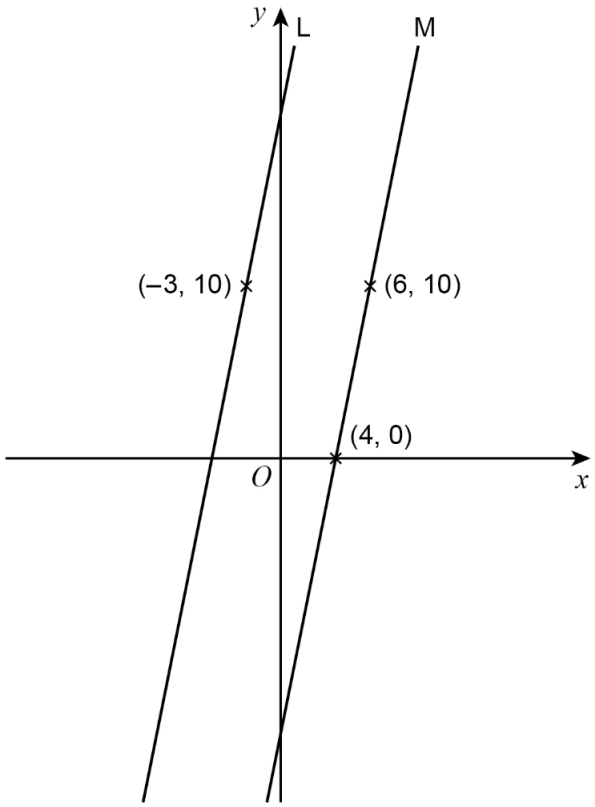
so

$$0.703\dot{4}\,\dot{8} \quad 0.703\,\dot{8} \quad 0.703\,\dot{8}\dot{4} \quad \underline{\underline{0.703\,\dot{8}\dot{4}}}$$

22. (3)

- $(-3, 10)$ is a point on line L.
- $(4, 0)$ and $(6, 10)$ are points on line M.

- L and M are parallel.



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Work out the equation of line L.
Give your answer in the form $y = mx + c$.

Solution

Well,

$$\begin{aligned} m_M &= \frac{10-0}{6-4} \\ &= \frac{10}{2} \\ &= 5. \end{aligned}$$

L and M have same gradient and the equation of L is

$$\begin{aligned} y - 10 = 5[x-(-3)] &\Rightarrow y - 10 = 5(x+3) \\ &\Rightarrow y - 10 = 5x + 15 \\ &\Rightarrow \underline{\underline{y = 5x + 25}}; \end{aligned}$$

hence, $\underline{\underline{m = 5}}$ and $\underline{\underline{c = 25}}$.

23. (a) Factorise (2)

$$5x^2 + 6x - 8.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & +6 \\ \text{multiply to:} & (+5) \times (-8) = -40 \end{array}\right\} +10,\ -4$$

E.g.,

$$\begin{aligned} 5x^2 + 6x - 8 &= 5x^2 + 10x - 4x - 8 \\ &= 5x(x+2) - 4(x+2) \\ &= \underline{\underline{(5x-4)(x+2)}}. \end{aligned}$$

(b) Simplify fully (3)

$$\frac{x^2 + 9x + 14}{x^2 - 4}.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & +9 \\ \text{multiply to:} & +14 \end{array}\right\} +2,\ +7$$

and the difference of two squares:

$$\begin{aligned} \frac{x^2 + 9x + 14}{x^2 - 4} &= \frac{(x+2)(x+7)}{(x+2)(x-2)} \\ &= \underline{\underline{\frac{x+7}{x-2}}}. \end{aligned}$$

24. Work out (4)

$$\sqrt{18} - \frac{28}{\sqrt{50}}.$$

Give your answer in the form

$$\frac{\sqrt{a}}{b},$$

where a and b are integers.

Solution

$$\begin{aligned}
\sqrt{18} - \frac{28}{\sqrt{50}} &= \sqrt{9 \times 2} - \left(\frac{28}{\sqrt{50}} \times \frac{\sqrt{50}}{\sqrt{50}}\right) \\
&= \sqrt{9} \times \sqrt{2} - \frac{28\sqrt{50}}{50} \\
&= 3\sqrt{2} - \frac{14\sqrt{25 \times 2}}{25} \\
&= 3\sqrt{2} - \frac{14 \times 5\sqrt{2}}{25} \\
&= 3\sqrt{2} - \frac{14\sqrt{2}}{5} \\
&= \frac{15\sqrt{2}}{5} - \frac{14\sqrt{2}}{5} \\
&= \underline{\underline{\frac{\sqrt{2}}{5}}};
\end{aligned}$$

hence, $\underline{\underline{a = 2}}$ and $\underline{\underline{b = 5}}$.

25. A bag contains 8 balls.
3 are red and 5 are blue.
2 balls are taken from the bag at random without replacement.

(a) Write down the probability that there is **at least** 1 red ball still in the bag. (1)

Solution

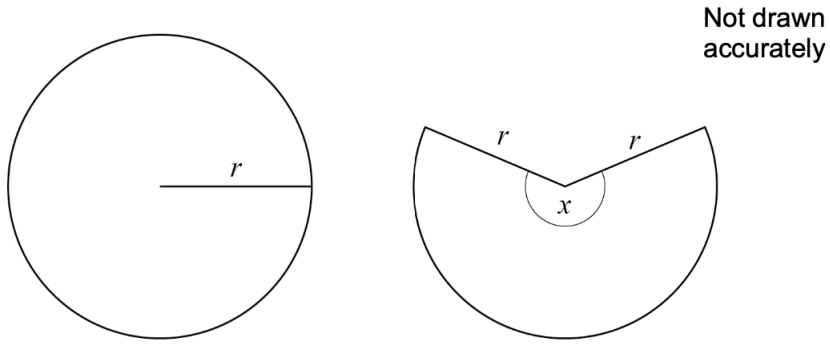
$$\text{P(at least 1 red ball)} = \underline{\underline{1}}.$$

(b) Work out the probability that there are **at least** 2 red balls still in the bag. (3)

Solution

$$\begin{aligned}\text{P(at least 2 red balls)} &= 1 - \text{P}(RR)\\ &= 1 - (\tfrac{3}{8} \times \tfrac{2}{7})\\ &= 1 - (\tfrac{3}{4} \times \tfrac{1}{7})\\ &= 1 - \tfrac{3}{28}\\ &= \underline{\underline{\tfrac{25}{28}}}.\end{aligned}$$

26. Here are a circle and a sector of the circle. (4)
They each have radius r.



circumference of circle = perimeter of sector

Work out the size of angle x.
Give your answer in terms of π.

Solution

Well,

$$\text{circumference}_{\text{circle}} = 2 \times \pi \times r$$

and

$$\text{perimeter}_{\text{sector}} = 2r + \frac{x}{360} \times 2 \times \pi \times r.$$

Now, given that two are equal,

$$2 \times \pi \times r = 2r + \frac{x}{360} \times 2 \times \pi \times r \Rightarrow 2\pi r = 2r + \frac{\pi r x}{180}$$
$$\Rightarrow 2\pi r - 2r = \frac{\pi r x}{180}$$
$$\Rightarrow \frac{180(2\pi - 2)}{\pi} = x$$
$$\Rightarrow \underline{\underline{x = \frac{360(\pi - 1)}{\pi}}}.$$

27. A curve has the equation

$$y = x^2 - 6x + 17.$$

The turning point of the curve is at $(a, 8)$.

(a) By completing the square, or otherwise, work out the value of a. (2)

Solution

$$\begin{aligned} y &= x^2 - 6x + 17 \\ &= (x^2 - 6x + 9) + 8 \\ &= (x - 3)^2 + 8; \end{aligned}$$

hence, $\underline{\underline{a = 3}}$.

The turning point of the curve

$$y = x^2 + 4x + b$$

also has y-coordinate 8.

(b) Work out the value of b. (2)

Solution

$$\begin{aligned} y &= x^2 + 4x + b \\ &= (x^2 + 4x + 4) + (-4 + b) \\ &= (x + 2)^2 + (-4 + b); \end{aligned}$$

hence,

$$8 = -4 + b \Rightarrow \underline{\underline{b = 12}}.$$

28. Work out the value of (2)

$$100^{-\frac{1}{2}}.$$

Solution

$$\begin{aligned} 100^{-\frac{1}{2}} &= \frac{1}{100^{\frac{1}{2}}} \\ &= \frac{1}{\sqrt{100}} \\ &= \underline{\underline{\frac{1}{10}}}. \end{aligned}$$

29. Show that the value of (4)

$$5\sin 30^\circ \times \cos 30^\circ \times 8\tan 30^\circ$$

is an integer.

Solution

$$\begin{aligned} 5\sin 30^\circ \times \cos 30^\circ \times 8\tan 30^\circ &= 5(\tfrac{1}{2}) \times \tfrac{\sqrt{3}}{2} \times 8(\tfrac{\sqrt{3}}{3}) \\ &= \tfrac{5}{2} \times \tfrac{\sqrt{3}}{2} \times \tfrac{8\sqrt{3}}{3} \\ &= \tfrac{5}{2} \times \tfrac{24}{6} \\ &= \tfrac{5}{2} \times 4 \\ &= \underline{\underline{10}}. \end{aligned}$$