

Dr Oliver Mathematics
GCSE Mathematics
2007 November Paper 6H: Calculator
2 hours

The total number of marks available is 100.

You must write down all the stages in your working.

1. In April 2004, the population of the European Community was 376 million. (3)
In April 2005, the population of the European Community was 451 million.
Work out the percentage increase in population.
Give your answer correct to 1 decimal place.

Solution

$$\frac{451 - 376}{376} \times 100\% = 19.94680851 \text{ (FCD)}$$
$$= \underline{\underline{19.9\% \text{ (1 dp)}}}$$

2. The equation (4)
- $$x^3 - 5x = 60$$

has a solution between 4 and 5.

Use a trial and improvement method to find this solution.

Give your answer correct to 1 decimal place.

You must show **all** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

x	$f(x)$	Comment
4.3	58.007	too low
4.35	60.5562	too high

Clearly,

$$4.3 < x < 4.35$$

and the answer is

$$\underline{\underline{x = 4.3 \text{ (1 dp)}}}$$

3. In the triangle XYZ , $XY = 5.6 \text{ cm}$, $YZ = 10.5 \text{ cm}$, angle $XYZ = 90^\circ$.

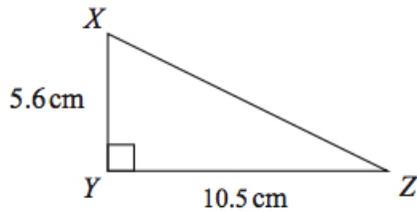


Diagram **NOT**
accurately drawn

- (a) Work out the length of XZ .

(3)

Solution

$$\begin{aligned} XZ &= \sqrt{XY^2 + YZ^2} \\ &= \sqrt{5.6^2 + 10.5^2} \\ &= \underline{\underline{11.9 \text{ cm}}} \end{aligned}$$

4 copies of the triangle are fitted together to make the shape shown in the diagram.

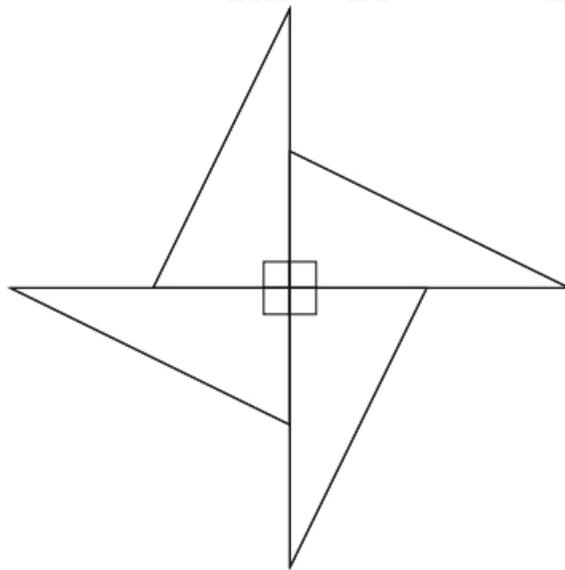


Diagram **NOT**
accurately drawn

(b) Calculate the perimeter of the shape.

(2)

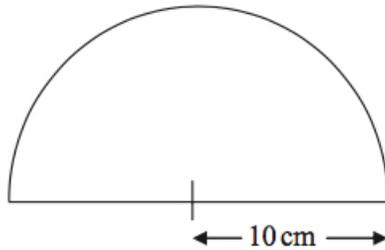
Solution

$$\begin{aligned}\text{Perimeter} &= 4 \times 11.9 + 4(10.5 - 5.6) \\ &= \underline{\underline{67.2 \text{ cm}}}.\end{aligned}$$

4. The diagram shows a semicircle.

(3)

Diagram **NOT**
accurately drawn



The radius of the semicircle is 10 cm.

Calculate the area of the semicircle.

Give your answer correct to 3 significant figures.

State the units of your answer.

Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \pi \times 10^2 \\ &= 157.079\ 632\ 7 \text{ (FCD)} \\ &= \underline{\underline{157 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

5. The table gives information about the times, in minutes, that 106 shoppers spent in a supermarket.

Time (t minutes)	Frequency
$0 < t \leq 10$	20
$10 < t \leq 20$	17
$20 < t \leq 30$	12
$30 < t \leq 40$	32
$40 < t \leq 50$	25

- (a) Find the class interval that contains the median. (1)

Solution

Time (t minutes)	Frequency	Cumulative frequency
$0 < t \leq 10$	20	20
$10 < t \leq 20$	17	$20 + 17 = 37$
$20 < t \leq 30$	12	$37 + 12 = 49$
$30 < t \leq 40$	32	$49 + 32 = 81$
$40 < t \leq 50$	25	$81 + 25 = 106$

The class interval that contains the median is $30 < t \leq 40$.

- (b) Calculate an estimate for the mean time that the shoppers spent in the supermarket. Give your answer correct to 3 significant figures. (4)

Solution

Time (t minutes)	Frequency	Midpoint	Frequency \times Midpoint
$0 < t \leq 10$	20	5	100
$10 < t \leq 20$	17	15	255
$20 < t \leq 30$	12	25	300
$30 < t \leq 40$	32	35	1 120
$40 < t \leq 50$	25	45	1 125
Total	106		2 900

The mean time that the shoppers is approximately

$$\begin{aligned} \frac{2\,900}{106} &= 27.358\,490\,57 \text{ (FCD)} \\ &= \underline{\underline{27.4 \text{ mins (3 sf)}}}. \end{aligned}$$

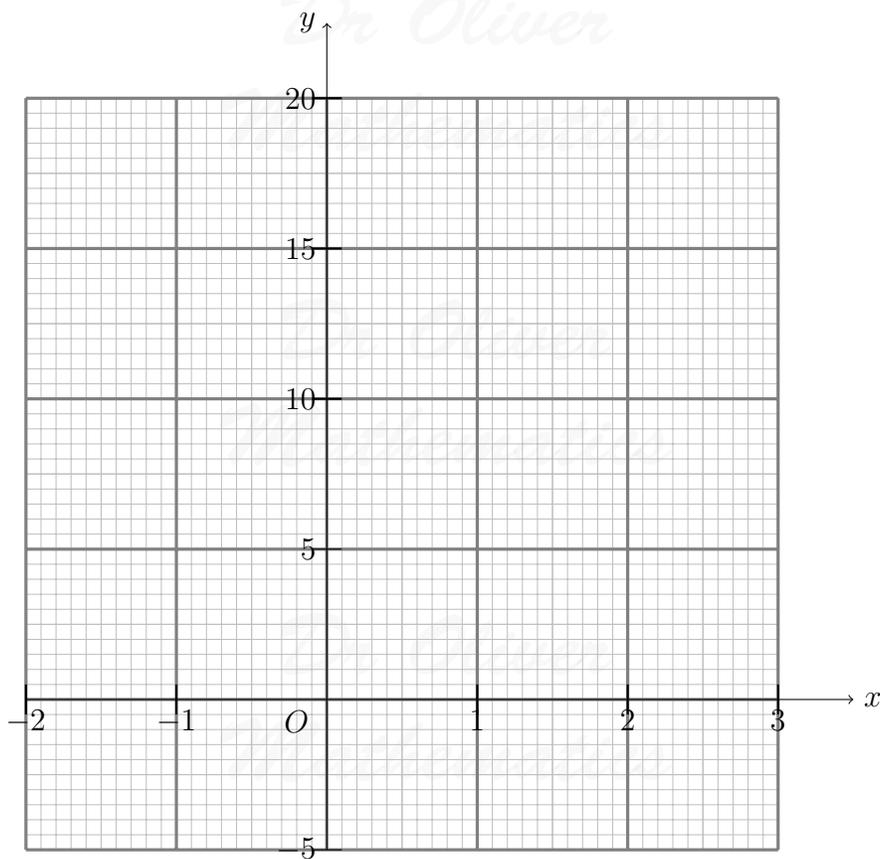
6. (a) Complete the table of values for $y = 2x^2 - 4x$. (2)

x	-2	-1	0	1	2	3
y	16		0		6	

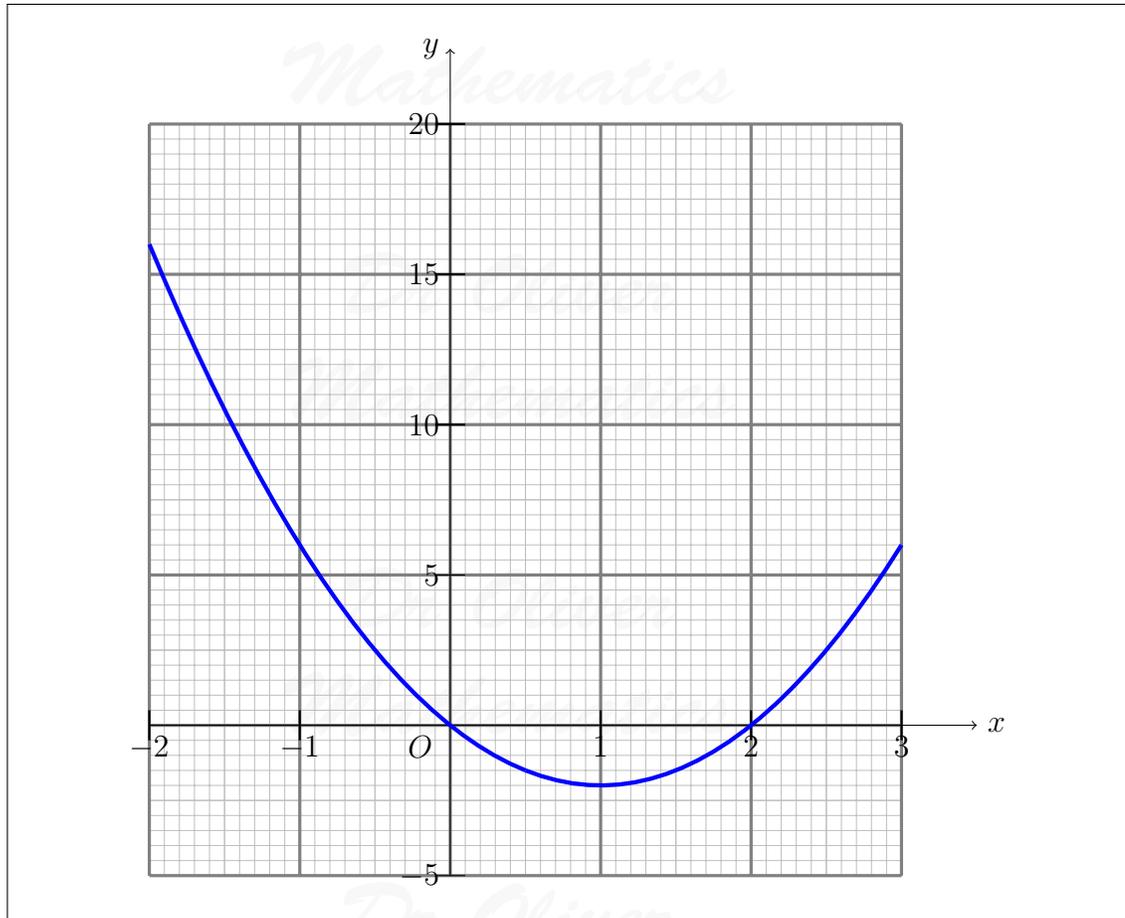
Solution

x	-2	-1	0	1	2	3
y	16	<u>6</u>	0	<u>-2</u>	<u>0</u>	6

- (b) On the grid, draw the graph of $y = 2x^2 - 4x$ for values of x from -2 to 3 . (2)



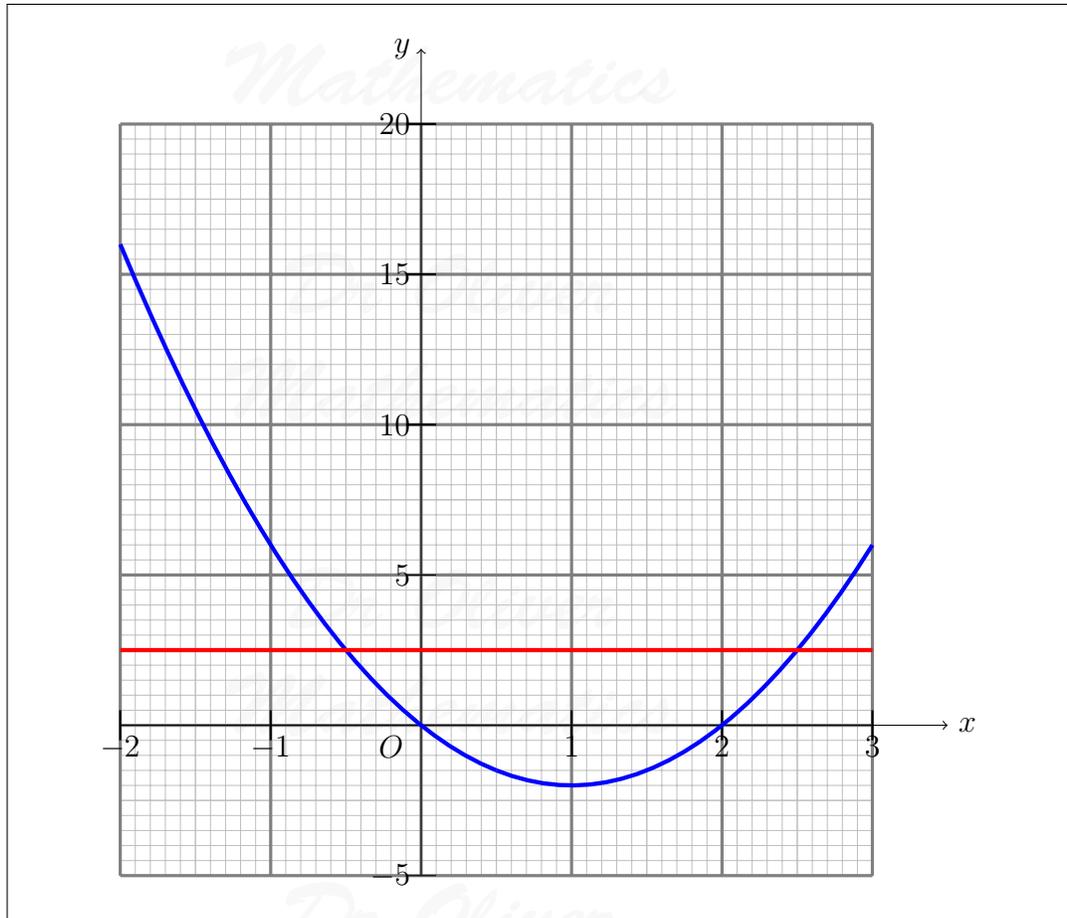
Solution



- (c) (i) On the same axes, draw the straight line $y = 2.5$.

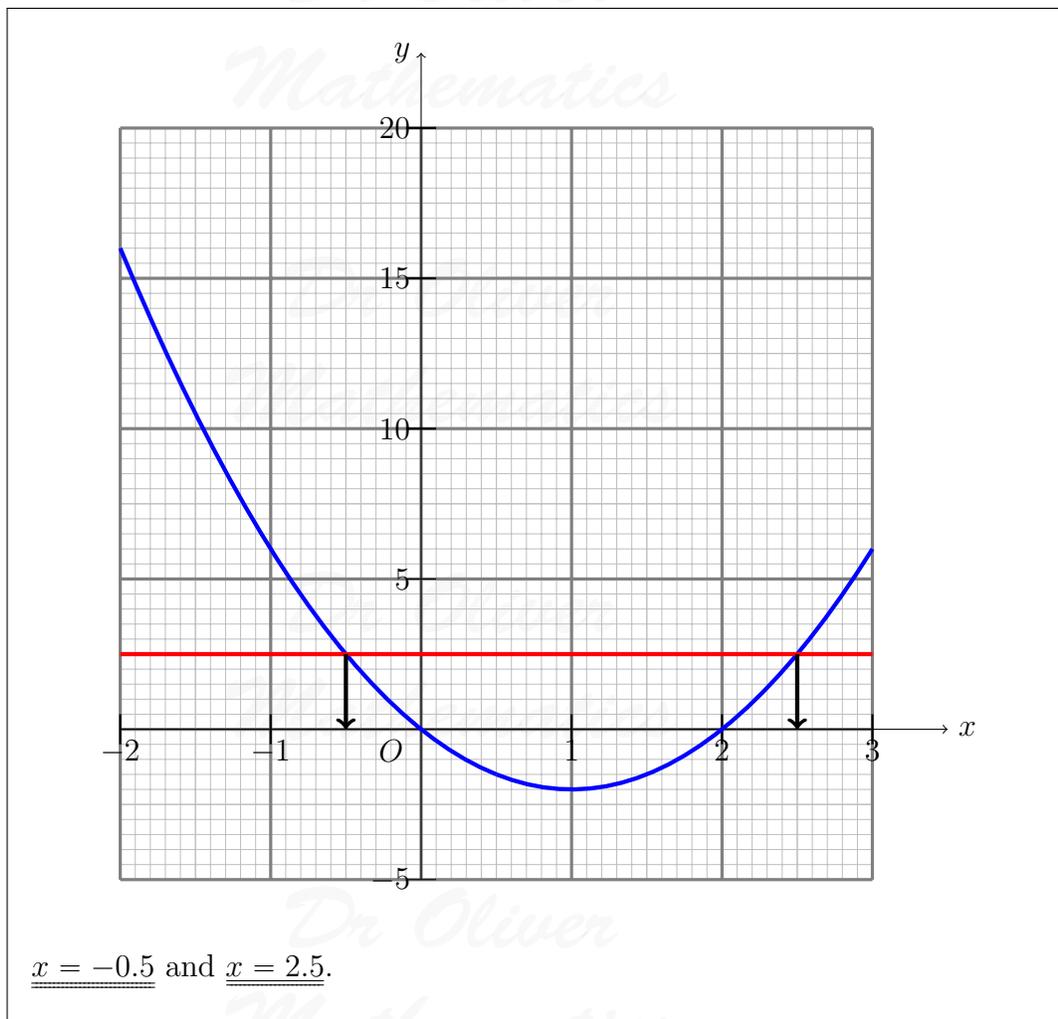
(2)

Solution



- (ii) Write down the values of x for which $2x^2 - 4x = 2.5$.

Solution



7. On July 1st 2004, Jack invested £2 000 at 5% per annum compound interest. (3)
 Work out the value of Jack's investment on July 1st 2006.

Solution

$$2000 \times 1.05^2 = \underline{\underline{\pounds 2\,205}}$$

8. Write 720 as a product of its prime factors. (2)

Solution

$$\begin{array}{r|l}
 & 720 \\
 2 & 360 \\
 2 & 180 \\
 2 & 90 \\
 2 & 45 \\
 3 & 15 \\
 3 & 5 \\
 5 & 1
 \end{array}$$

and so

$$\begin{aligned}
 720 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \\
 &= \underline{\underline{2^4 \times 3^2 \times 5}}.
 \end{aligned}$$

9. (a) Simplify $p^7 \times p^9$.

(1)

Solution

$$p^7 \times p^9 = \underline{\underline{p^{16}}}.$$

- (b) Simplify

(1)

$$\frac{q^{12} \times q^4}{q^6}.$$

Solution

$$\frac{q^{12} \times q^4}{q^6} = \frac{q^{16}}{q^6} = \underline{\underline{q^{10}}}.$$

10. In a sale, normal prices are reduced by 25%.

(3)

The sale price of a saw is £12.75.

Calculate the normal price of the saw.

Solution

$1 - 0.25 = 0.75$ and the normal price of the saw is

$$\frac{12.75}{0.75} = \underline{\underline{£17}}.$$

11. Work out

(3)

$$\frac{2 \times (2.2 \times 10^{12}) \times (1.5 \times 10^{12})}{2.2 \times 10^{12} - 1.5 \times 10^{12}}.$$

Give your answer in standard form correct to 3 significant figures.

Solution

$$\begin{aligned} \frac{2 \times (2.2 \times 10^{12}) \times (1.5 \times 10^{12})}{2.2 \times 10^{12} - 1.5 \times 10^{12}} &= \frac{6.6 \times 10^{24}}{0.7 \times 10^{12}} \\ &= 9.428\,571\,429 \times 10^{12} \text{ (FCD)} \\ &= \underline{\underline{9.43 \times 10^{12} \text{ (3 sf)}}}. \end{aligned}$$

12. Solve

(3)

$$\begin{aligned} 3x + y &= 8 \\ 4x + 2y &= 9. \end{aligned}$$

Solution

$$\begin{aligned} 3x + y &= 8 \quad (1) \\ 4x + 2y &= 9 \quad (2) \end{aligned}$$

$$2 \times (1) - (2):$$

$$\begin{aligned} 6x - 4x &= 16 - 9 \Rightarrow 2x = 7 \\ &\Rightarrow \underline{\underline{x = 3\frac{1}{2}}} \\ &\Rightarrow \underline{\underline{y = -2\frac{1}{2}}}. \end{aligned}$$

13. (a) In the diagram, O is the centre of the circle.

(2)

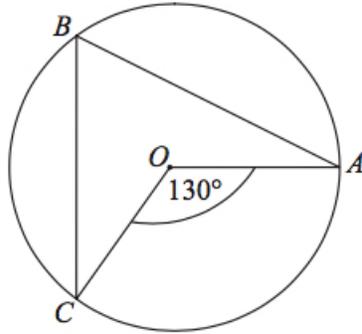


Diagram NOT accurately drawn

A , B , and C are points on the circle.
Angle $COA = 130^\circ$.

- (i) Find the size of angle CBA .

Solution

65° .

- (ii) Give a reason for your answer.

Solution

The angle at the centre is twice the angle at the circumference.

- (b) In the diagram, O is the centre of the circle.

(2)

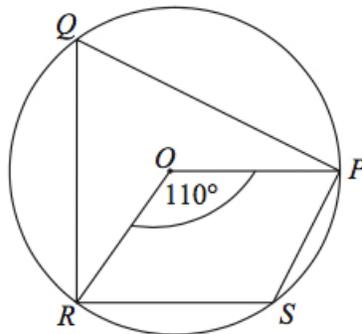


Diagram NOT accurately drawn

P , Q , R , and S are points on the circle.
Angle $ROP = 110^\circ$.
Calculate the size of angle RSP .

Solution

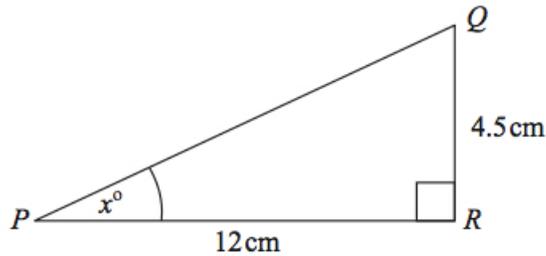
Reflex angle $ROP = 250^\circ$ (angles in a circle).

Angle $RSP = 125^\circ$ (angle at the centre is twice the angle at the circumference).

14. PQR is a right-angled triangle.

(3)

Diagram **NOT**
accurately drawn



$$PR = 12\text{ cm.}$$

$$QR = 4.5\text{ cm.}$$

$$\text{Angle } PRQ = 90^\circ.$$

Work out the value of x .

Give your answer correct to one decimal place.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan x^\circ = \frac{4.5}{12}$$

$$\Rightarrow x^\circ = 20.556\ 045\ 22 \text{ (FCD)}$$

$$\Rightarrow \underline{\underline{x^\circ = 20.6^\circ \text{ (1 dp)}}}$$

- 15.

(3)

$$A = \frac{h(x + 10)}{2}$$

$$A = 27.$$

$$h = 4.$$

Work out the value of x .

Solution

$$\begin{aligned}27 &= \frac{4(x + 10)}{2} \Rightarrow \frac{27}{4} = \frac{x + 10}{2} \\ &\Rightarrow \frac{27}{2} = x + 10 \\ &\Rightarrow \underline{\underline{x = 3\frac{1}{2}}}.\end{aligned}$$

16. The table shows the number of televisions sold in a shop in the first five months of 2006.

Month	Jan	Feb	Mar	Apr	May	June
Number	1 240	1 270	1 330	1 300	1 330	x

- (a) Work out the first 3-month moving average for the information in the table. (2)

Solution

$$\frac{1\,240 + 1\,270 + 1\,330}{3} = \underline{\underline{1\,280}}.$$

The fourth 3-month moving average of the number of televisions sold in 2006 is 1 350.
The number of televisions sold in the shop in June was x .

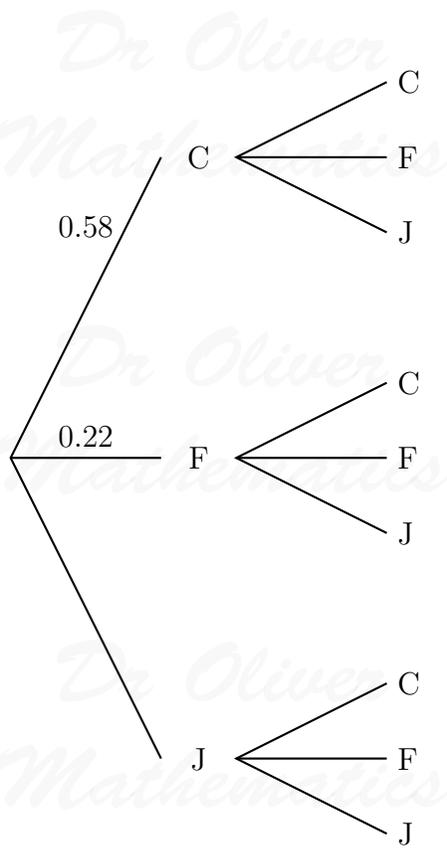
- (b) Work out the value of x . (2)

Solution

$$\begin{aligned}\frac{1\,300 + 1\,330 + x}{3} &= 1\,350 \Rightarrow 2\,630 + x = 4\,050 \\ &\Rightarrow \underline{\underline{x = 1\,420}}.\end{aligned}$$

17. Julie has 100 music CDs. 58 of the CDs are classical.
22 of the CDs are folk.
The rest of the CDs are jazz.
On Saturday, Julie chooses one CD at random from the 100 CDs.
On Sunday, Julie chooses one CD at random from the 100 CDs.

- (a) Complete the probability tree diagram. (2)

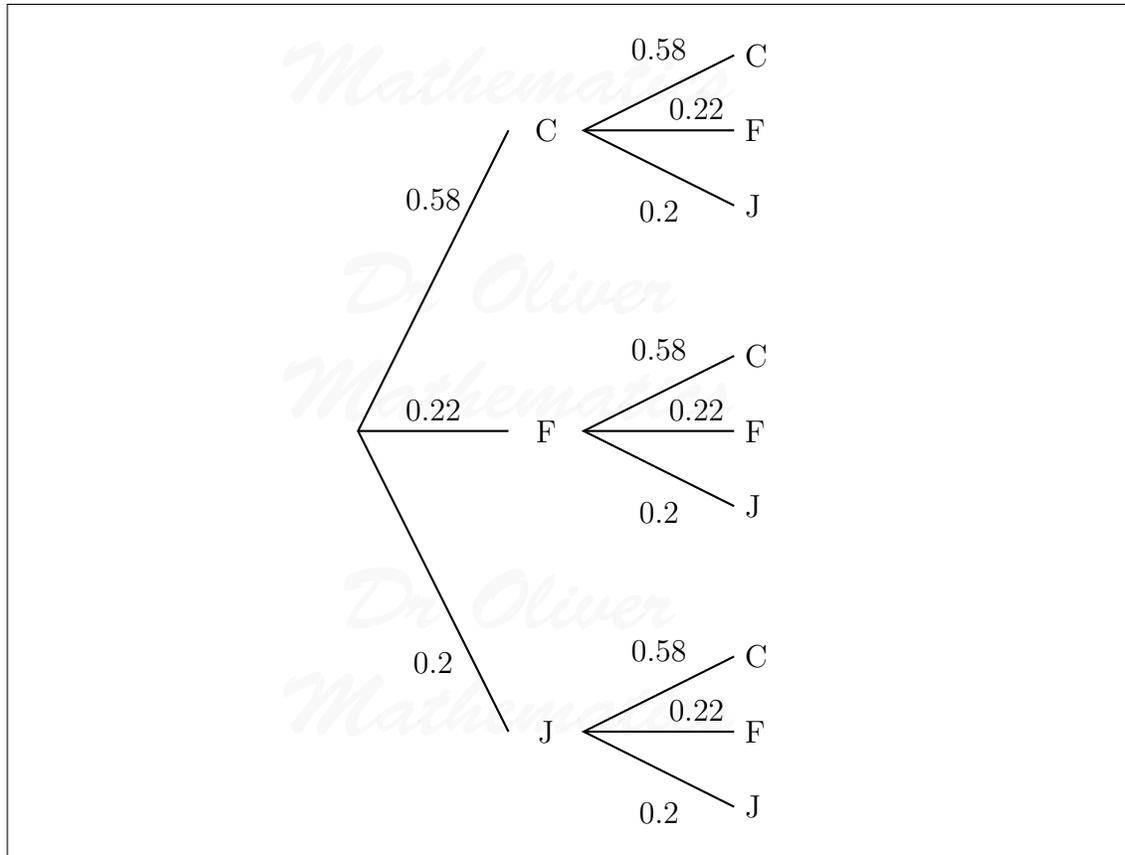


Solution

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- (b) Calculate the probability that Julie will choose a jazz CD on **both** Saturday and Sunday. (2)

Solution

$$0.2 \times 0.2 = \underline{0.04}.$$

- (c) Calculate the probability that Julie will choose at least one jazz CD on Saturday **and** Sunday. (3)

Solution

$$\begin{aligned} P(\text{at least one jazz CD}) &= 1 - P(\text{no jazz CD}) \\ &= 1 - 0.8 \times 0.8 \\ &= 1 - 0.64 \\ &= \underline{0.36}. \end{aligned}$$

18. f is inversely proportional to d .

(3)

When $d = 50$, $f = 256$.

Find the value of f when $d = 80$.

Solution

$$f \propto \frac{1}{d} \Rightarrow f = \frac{k}{d}$$

for some constant k . Now,

$$256 = \frac{k}{50} \Rightarrow k = 12\,800$$

and so

$$f = \frac{12\,800}{d}.$$

Finally

$$f = \frac{12\,800}{80} = \underline{\underline{160}}.$$

19. The diagram shows a 6-sided shape.

All the corners are right angles.

All the measurements are given in centimetres.

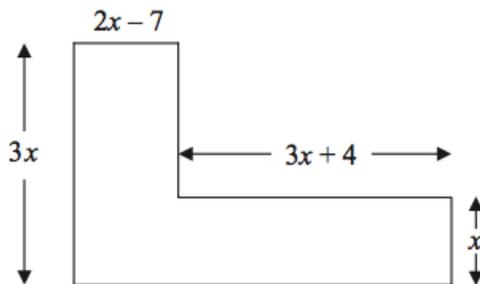


Diagram **NOT**
accurately drawn

The area of the shape is 85 cm^2 .

(a) Show that

(3)

$$9x^2 - 17x - 85 = 0.$$

Solution

$$\begin{aligned}
 3x[(2x - 7) + (3x + 4)] - 2x(3x + 4) &= 85 \Rightarrow 3x(5x - 3) - 2x(3x + 4) = 85 \\
 &\Rightarrow 15x^2 - 9x - 6x^2 - 8x - 85 = 0 \\
 &\Rightarrow \underline{\underline{9x^2 - 17x - 85 = 0}},
 \end{aligned}$$

as required.

(b) (i) Solve

$$9x^2 - 17x - 85 = 0. \quad (4)$$

Give your solutions correct to 3 significant figures.

Solution

$$a = 9, b = -17, c = -85:$$

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{17 \pm \sqrt{(-17)^2 - 4 \times 9 \times (-85)}}{2 \times 9} \\
 &= \frac{17 \pm \sqrt{3349}}{18} \\
 &= -2.270585844 \text{ or } 4.159474732 \text{ (FCD)} \\
 &= \underline{\underline{-2.27 \text{ or } 4.16 \text{ (3 sf)}}}.
 \end{aligned}$$

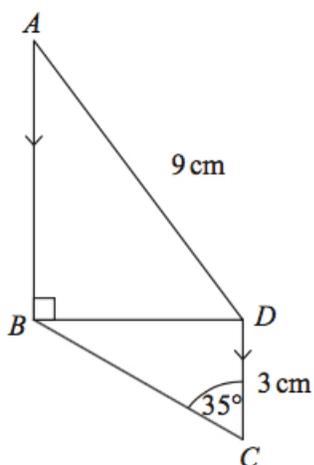
(ii) Hence, work out the length of the shortest side of the 6-sided shape.

Solution

$$2 \times 4.159 \dots - 7 = \underline{\underline{1.32 \text{ (3 sf)}}}.$$

20. AB is parallel to DC .

(4)

Diagram **NOT**
accurately drawn

$$AD = 9 \text{ cm.}$$

$$DC = 3 \text{ cm.}$$

$$\text{Angle } BCD = 35^\circ.$$

$$\text{Angle } ABD = 90^\circ.$$

Calculate the size of angle BAD .

Give your answer correct to one decimal place.

Solution

$$BD = 3 \tan 35^\circ$$

and

$$\sin BAD = \frac{3 \tan 35^\circ}{9} \Rightarrow BAD = 13.49747506 \text{ (FCD)}$$

$$\Rightarrow \underline{\underline{BAD = 13.5^\circ \text{ (1 dp)}}}$$

21. The diagram shows an equilateral triangle.

(3)

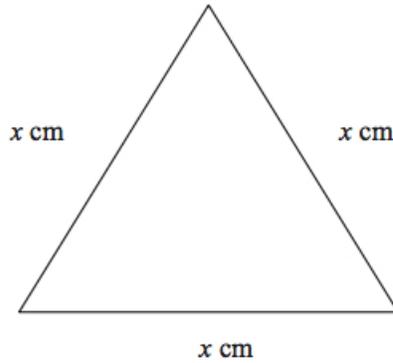


Diagram **NOT**
accurately drawn

The area of the equilateral triangle is 36 cm^2 .
Find the value of x . Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \frac{1}{2}x^2 \sin 60^\circ &= 36 \Rightarrow x^2 \sin 60^\circ = 72 \\ &\Rightarrow x^2 = 83.138\ 438\ 76 \text{ (FCD)} \\ &\Rightarrow x = 9.118\ 028\ 228 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 9.12 \text{ cm (3 sf)}}}. \end{aligned}$$

22. (a) Simplify $(2x^4y^5)^3$. (2)

Solution

$$(2x^4y^5)^3 = \underline{\underline{8x^{12}y^{15}}}.$$

- (b) Rearrange the formula to make t the subject. (4)

Solution

$$y = \frac{2pt}{p-t}$$

$$\begin{aligned}
 y &= \frac{2pt}{p-t} \Rightarrow y(p-t) = 2pt \\
 &\Rightarrow py - ty = 2pt \\
 &\Rightarrow py = 2pt + ty \\
 &\Rightarrow py = t(2p+y) \\
 &\Rightarrow t = \frac{py}{2p+y}.
 \end{aligned}$$

23. The mass M grams of a cube with edges of length L cm and density D grams per cm^3 is given by the formula (3)

$$M = DL^3.$$

$D = 8$ correct to 1 significant figure.

$L = 6.4$ correct to 1 decimal place.

Calculate the upper bound of M .

Give your answer correct to 2 significant figures.

Solution

Now,

$$7.5 \leq D < 8.5$$

and

$$6.35 \leq L < 6.45.$$

Finally, the upper bound of M is

$$\begin{aligned}
 8.5 \times 6.45^3 &= 2280.857063 \text{ (FCD)} \\
 &= \underline{\underline{2300 \text{ g (2 sf)}}}.
 \end{aligned}$$

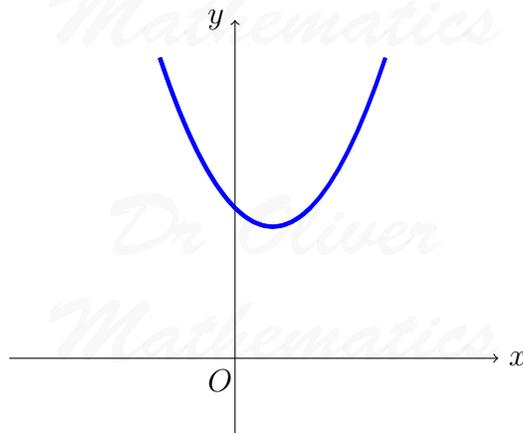
24. $x^2 - 8x + 23 = (x - p)^2 + q$ for all values of x . (3)
- (a) Find the value of p and the value of q .

Solution

$$\begin{aligned}
 x^2 - 8x + 23 &= (x^2 - 8x + 16) + 7 \\
 &= (x - 4)^2 + 7;
 \end{aligned}$$

hence, $p = 4$ and $q = 7$.

Here is a sketch of the curve with equation $x^2 - 8x + 23$.



B is the minimum point on the curve.

(b) Find the coordinates of B .

(1)

Solution

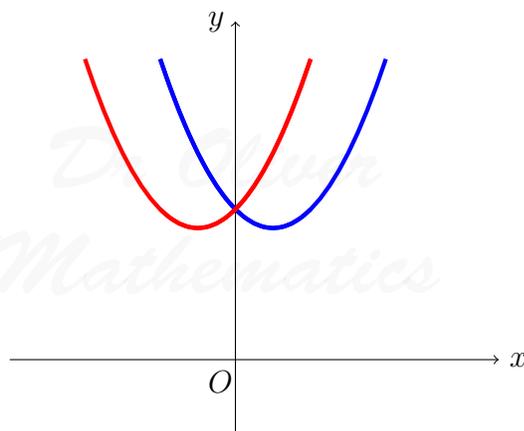
(4, 7).

The equation of the curve can be written in the form written in the form $y = f(x)$ where $f(x) = x^2 - 8x + 23$.

(c) On the diagram below, draw a sketch of the curve $y = f(-x)$.

(1)

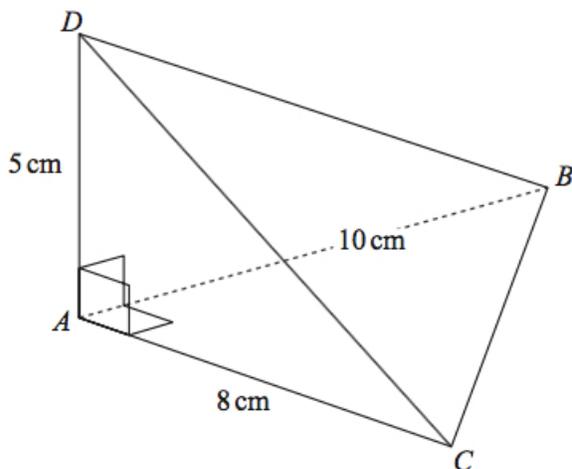
Solution



25. The diagram shows a tetrahedron.

(6)

Diagram NOT
accurately drawn



AD is perpendicular to both AB and AC .

$AB = 10$ cm.

$AC = 8$ cm.

$AD = 5$ cm.

Angle $BAC = 90^\circ$.

Calculate the size of angle BDC .

Give your answer correct to 1 decimal place.

Solution

$$BC = \sqrt{8^2 + 10^2}$$

$$= \sqrt{164},$$

$$CD = \sqrt{5^2 + 8^2}$$

$$= \sqrt{89} \text{ and,}$$

$$BD = \sqrt{5^2 + 10^2}$$

$$= \sqrt{125}.$$

Finally,

$$\begin{aligned}\cos BDC &= \frac{BD^2 + CD^2 - BC^2}{2 \cdot BD \cdot CD} \Rightarrow \cos BDC = \frac{89 + 125 - 164}{2 \cdot \sqrt{89} \cdot \sqrt{125}} \\ &\Rightarrow \cos BDC = \frac{50}{2 \cdot \sqrt{89} \cdot \sqrt{125}} \\ &\Rightarrow BDC = 76.28911407 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{BDC = 76.3^\circ \text{ (1 dp)}}}\end{aligned}$$

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