Dr Oliver Mathematics
Algebra: Part 1

1. Solve
$$3^{2x^2-7x+3} = 4^{x^2-x-6}.$$

Solution

Well, we need to factorise.

$$\left.\begin{array}{ll}\text{add to:} & \quad\quad -7 \\ \text{multiply to:} & (+2)\times(+3) = +6\end{array}\right\} -6,\ -1$$

$$\begin{aligned}2x^2 - 7x + 3 &= 2x^2 - 6x - x + 1 \\ &= 2x(x-3) - (x-3) \\ &= (2x-1)(x-3).\end{aligned}$$

$$\left.\begin{array}{ll}\text{add to:} & -1 \\ \text{multiply to:} & -6\end{array}\right\} -3,\ +2$$

$$x^2 - x - 6 = (x-3)(x+2)$$

Interesting, isn't it, that they both have $(x-3)$ as a common factor. Now,

$$\begin{aligned}3^{2x^2-7x+3} = 4^{x^2-x-6} &\Rightarrow 3^{(2x-1)(x-3)} = 4^{)x-3)(x+2)} \\ &\Rightarrow \left[3^{(2x-1)}\right]^{(x-3)} = \left[4^{(x+2)}\right]^{(x-3)} \\ &\Rightarrow \ln\left[3^{(2x-1)}\right]^{(x-3)} = \ln\left[4^{(x+2)}\right]^{(x-3)} \\ &\Rightarrow (x-3)\ln\left[3^{(2x-1)}\right] = (x-3)\ln\left[4^{(x+2)}\right] \\ &\Rightarrow (x-3)\ln\left[3^{(2x-1)}\right] - (x-3)\ln\left[4^{(x+2)}\right] = 0 \\ &\Rightarrow (x-3)\left(\ln\left[3^{(2x-1)}\right] - \ln\left[4^{(x+2)}\right]\right) = 0 \\ &\Rightarrow (x-3)\left[(2x-1)\ln 3 - (x+2)\ln 4\right] = 0 \\ &\Rightarrow x-3 = 0 \text{ or } (2x-1)\ln 3 - (x+2)\ln 4 = 0.\end{aligned}$$

$\underline{x-3=0}$:

$$x - 3 = 0 \Rightarrow \underline{\underline{x = 3}}.$$

$\underline{(2x-1)\ln 3-(x+2)\ln 4=0}$:

$$\begin{aligned}(2x-1)\ln 3-(x+2)\ln 4=0 &\Rightarrow (2x\ln 3-\ln 3)-(x\ln 4+2\ln 4)=0\\ &\Rightarrow 2x\ln 3-x\ln 4=\ln 3+2\ln 4\\ &\Rightarrow x(2\ln 3-\ln 4)=\ln 3+2\ln 4\\ &\Rightarrow \underline{\underline{x=\frac{\ln 3+2\ln 4}{2\ln 3-\ln 4}}}.\end{aligned}$$