Dr Oliver Mathematics
Mathematics: Higher
2025 Paper 1: Non-Calculator
1 hour 15 minutes

The total number of marks available is 55.
You must write down all the stages in your working.

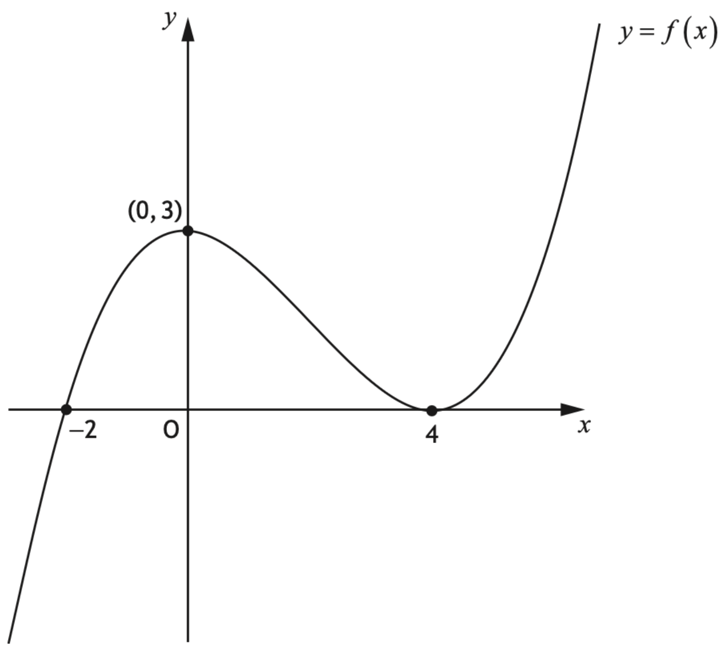
1. A curve has equation (4)
$$y = x^3 - 2x^2 + 5.$$
Find the equation of the tangent to this curve at the point where $x = 2$.

2. Find the equation of the perpendicular bisector of the line joining $A(1, 4)$ and $B(9, 10)$. (4)

3. Find (4)
$$\int \left(\frac{12}{x^2} + x^{\frac{1}{2}} \right) \mathrm{d}x, \; x > 0.$$

4. Evaluate (3)
$$3 \log_3 2 + \log_3 \tfrac{1}{24}.$$

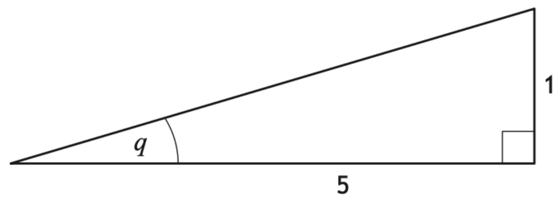
5. The diagram shows the graph of $y = \mathrm{f}(x)$, with stationary points at $(0, 3)$ and $(4, 0)$. (2)



Sketch the graph of

$$y = \mathrm{f}(-x) + 3.$$

6. The diagram shows a right-angled triangle with angle q.

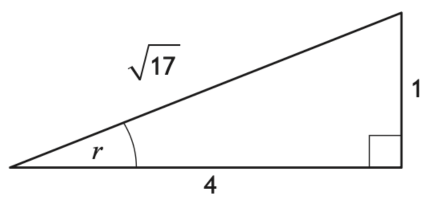


(a) Determine the value of

(i) $\sin 2q$, (3)

(ii) $\cos 2q$. (1)

A second right-angled triangle has angle r as shown.



(b) Find the value of $\sin(2q - r)$. (3)

7. (a) Show that $(x + 3)$ is a factor of (2)

$$5x^3 + 16x^2 - x - 12.$$

(b) Hence, or otherwise, solve (3)

$$5x^3 + 16x^2 - x - 12 = 0.$$

8. Given that (3)

$$\log_a 75 = 2 + \log_a 3,\ a > 0,$$

find the value of a.

9. Find the coordinates of the points of intersection of the line with equation (4)

$$y = x + 1$$

and the circle with equation

$$x^2 + y^2 - 2x + 6y - 15 = 0.$$

10. The vectors $\mathbf{u}$ and $\mathbf{v}$ are such that (5)

- $\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$,
- $\mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ k \end{pmatrix}$, and
- the angle between $\mathbf{u}$ and $\mathbf{v}$ is $45°$.

Find the value of k, where $k > 0$.

11. The equation (4)

$$9x^2 + 3kx + k = 0$$

has two real and distinct roots.

Determine the range of values for k.
Justify your answer.

12. Given that (4)

- $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6\cos x + 8\sin 2x$ and
- $y = 4$ when $x = \frac{1}{6}\pi$,

express y in terms of x.

13. A function, f, is defined on the set of real numbers.

The derivative of f is

$$\mathrm{f}'(x) = (x + 5)(2 - x).$$

(a) Find the x-coordinates of the stationary points on the curve with equation $y = \mathrm{f}(x)$ and determine their nature. (3)

It is known that

- f is a cubic function.
- $\mathrm{f}(0) < 0$.
- The equation $\mathrm{f}(x) = 0$ has exactly one solution. The solution lies between -10 and 10.

(b) Draw a sketch of a possible graph of $y = \mathrm{f}(x)$. (3)