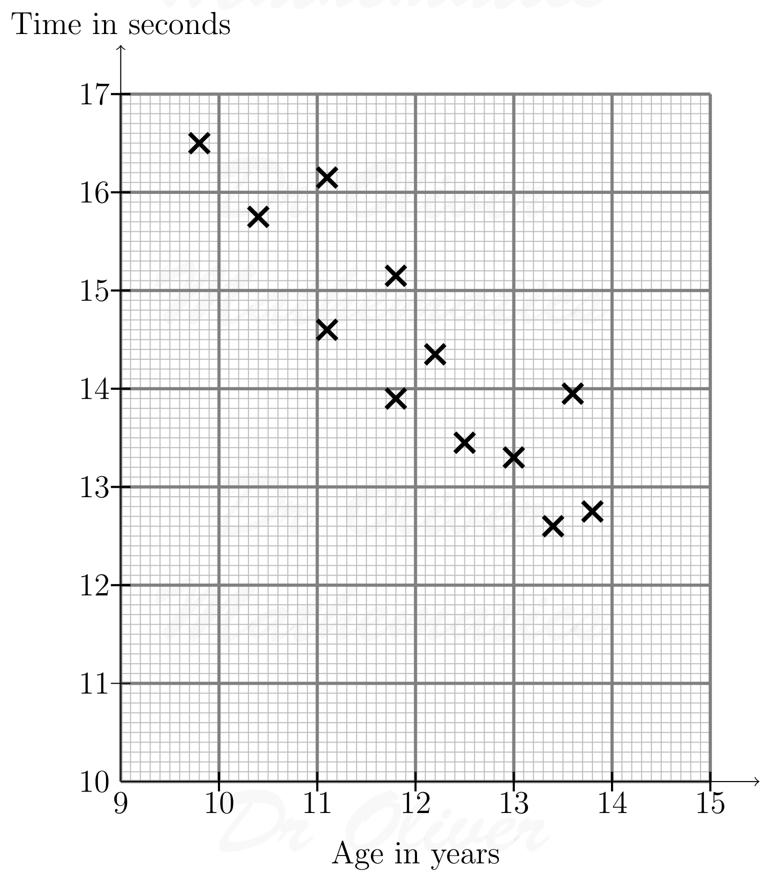
Dr Oliver Mathematics
GCSE Mathematics
2018 Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. The scatter diagram shows information about 12 girls.
   It shows the age of each girl and the best time she takes to run 100 metres.



(a) Write down the type of correlation. (1)

> **Solution**
> Negative correlation, i.e., as the age increases, the time decreases.

Kristina is 11 years old.
Her best time to run 100 metres is 12 seconds.
The point representing this information would be an outlier on the scatter diagram.

(b) Explain why. (1)

Solution
It is significantly below the line of best fit for the scatter diagram.

Debbie is 15 years old.
Debbie says, "The scatter diagram shows I should take less than 12 seconds to run 100 metres"

(c) Comment on what Debbie says. (1)

Solution
There is no data for 15 year olds (except via extrapolation).

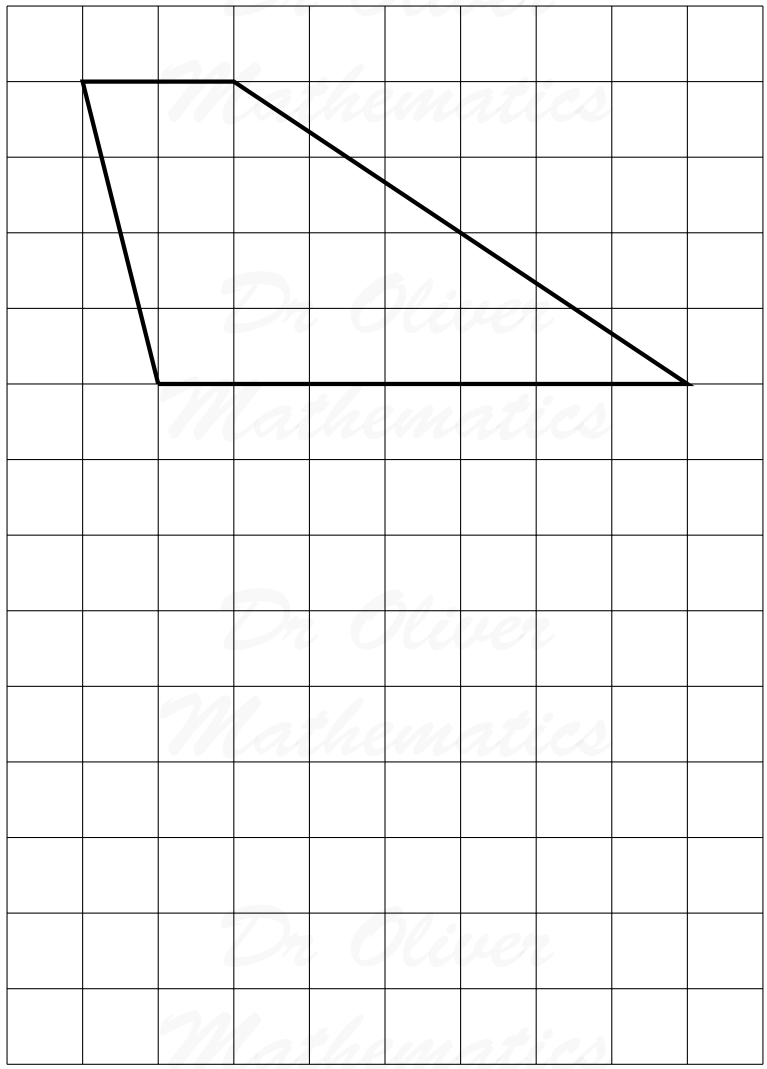
2. Expand and simplify (2)

$$5(p+3)-2(1-2p).$$

Solution

$$\begin{aligned}5(p+3)-2(1-2p) &= 5p+15-2+4p\\ &= \underline{\underline{9p+13}}.\end{aligned}$$

3. Here is a trapezium drawn on a centimetre grid. (2)



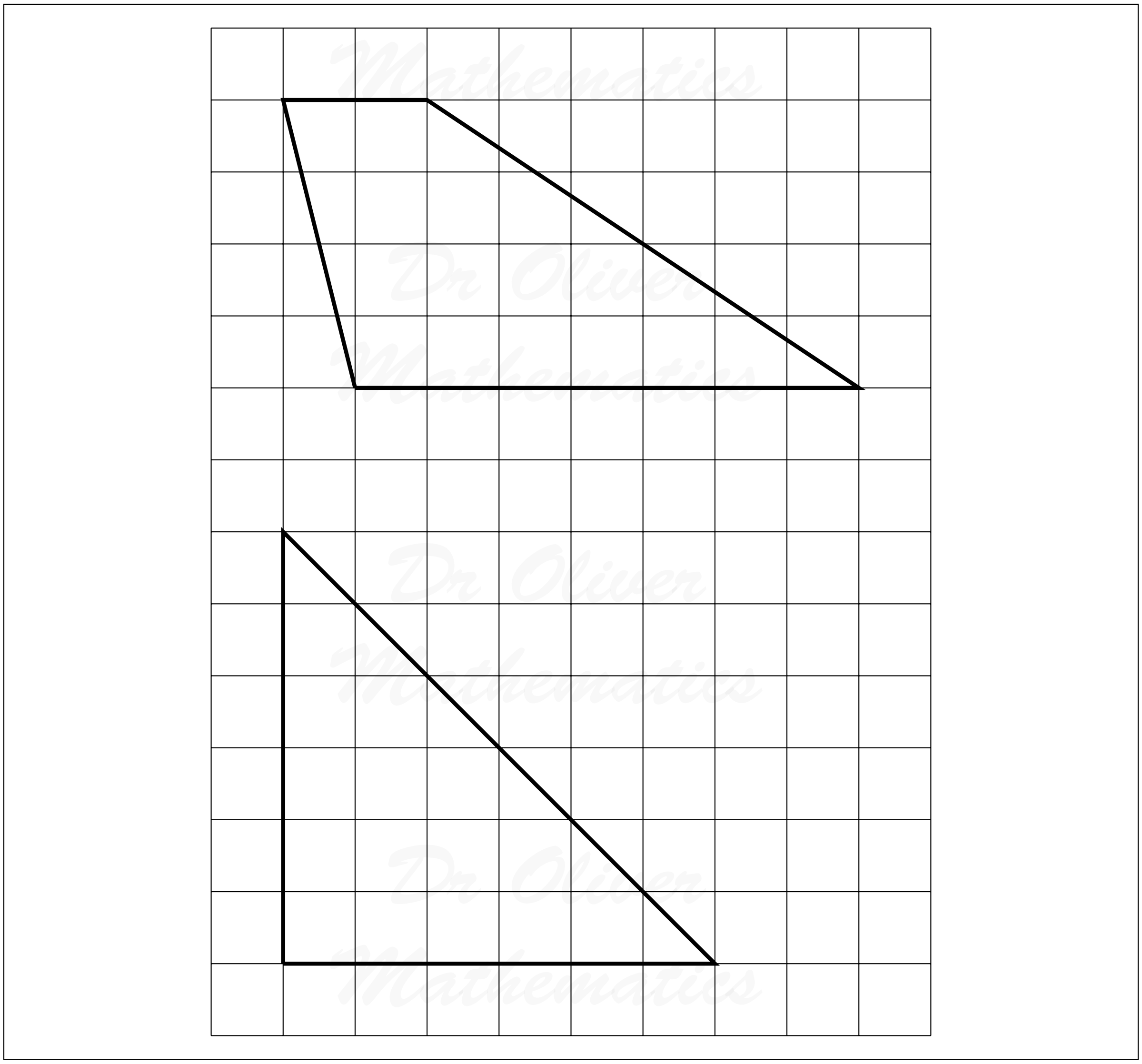
On the grid, draw a triangle equal in area to this trapezium.

Solution

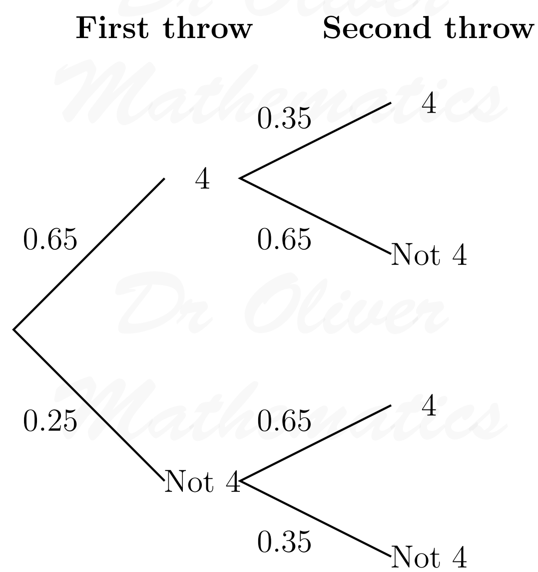
The triangle must have an area of

$$\begin{aligned}(\tfrac{1}{2} \times 4 \times 1) + 4 + (\tfrac{1}{2} \times 6 \times 4) &= 2 + 4 + 12 \\ &= 18 \text{ units}^2.\end{aligned}$$

E.g., a base of 6 and a height of 6, a base of 4 and a height of 9.



4. When a biased 6-sided dice is thrown once, the probability that it will land on 4 is 0.65. (2)
The biased dice is thrown twice.
Amir draws this probability tree diagram.
The diagram is **not** correct.

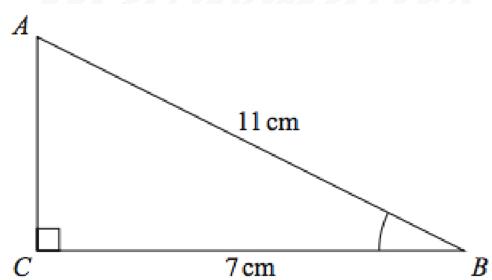


Write down **two** things that are wrong with the probability tree diagram.

Solution

First reason: On the first throw, $0.65 + 0.25 = 0.9$, not 1.
Second reason: On the second throw, the numbers are reversed: it should be 0.65 on the 4 and 0.35 on the Not 4.

5. ABC is a right-angled triangle.



(a) Work out the size of angle ABC. (2)
Give your answer correct to 1 decimal place.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos ABC = \frac{7}{11}$$
$$\Rightarrow \angle ABC = 50.478\,803\,64 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle ABC = 50.5^\circ \text{ (1 dp)}}}.$$

The length of the side AB is reduced by 1 cm.
The length of the side BC is still 7 cm.
Angle ACB is still 90°.

(b) Will the value of $\cos ABC$ increase or decrease? (1)
You must give a reason for your answer.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos ABC = \frac{7}{10};$$

hence, the answer has <u>increased</u> since $\frac{7}{10} > \frac{7}{11}$.

6. There are some counters in a bag.
The counters are red or white or blue or yellow.
Bob is going to take at random a counter from the bag.
The table shows each of the probabilities that the counter will be blue or will be yellow.

Colour	Red	White	Blue	Yellow
Probability			0.45	0.25

There are 18 blue counters in the bag.
The probability that the counter Bob takes will be red is twice the probability that the counter will be white.

(a) Work out the number of red counters in the bag. (4)

Solution
Let r be the number of red counters in the bag. Then $\frac{1}{2}r$ is the number of white counters in the bag. So,

$$r + \tfrac{1}{2}r + 0.45 + 0.25 = 1 \Rightarrow \tfrac{3}{2}r = 0.3$$
$$\Rightarrow r = 0.2$$

and the number of red counters in the bag is

$$\frac{0.2}{0.45} \times 18 = \underline{\underline{8}}.$$

A marble is going to be taken at random from a box of marbles.
The probability that the marble will be silver is 0.5.

There must be an even number of marbles in the box.

(b) Explain why. (1)

Solution

$$0.5 \times \text{odd number} = n\tfrac{1}{2}$$

for some n and this is not an integer. So, we need an even number: $2m$, for example, and this is $\underline{\underline{\text{even}}}$.

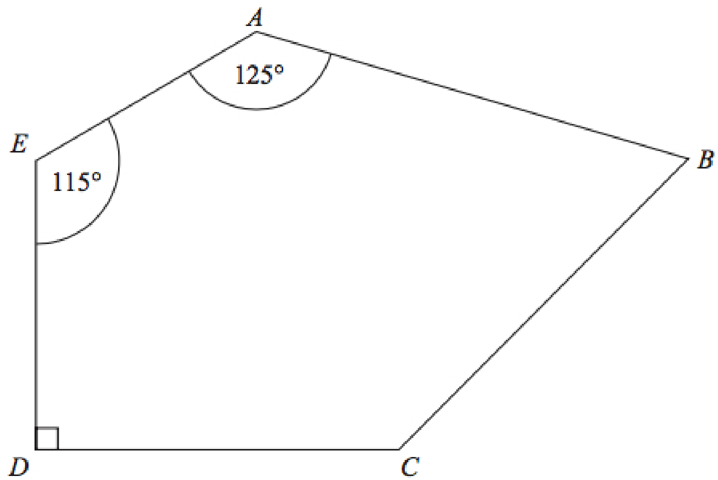
7. Solve (3)

$$\frac{5-x}{2} = 2x - 7.$$

Solution

$$\begin{aligned}\frac{5-x}{2} = 2x - 7 &\Rightarrow 5 - x = 2(2x - 7)\\ &\Rightarrow 5 - x = 4x - 14\\ &\Rightarrow 19 = 5x\\ &\Rightarrow \underline{\underline{x = 3\tfrac{4}{5}}}.\end{aligned}$$

8. $ABCDE$ is a pentagon. (5)



Angle $BCD = 2 \times \angle ABC$.

Work out the size of angle BCD.
You must show all your working.

Solution

The sum of all of the angles in a pentagon is

$$(5-2) \times 180 = 540°.$$

Now,

$$\begin{aligned} 125 + 115 + 90 + \angle ABC + 2\angle ABC = 540 &\Rightarrow 3\angle ABC = 210 \\ &\Rightarrow \angle ABC = 70 \\ &\Rightarrow \underline{\underline{\angle BCD = 140°}}. \end{aligned}$$

9.

$$T = \sqrt{\frac{w}{d^3}}.$$

$w = 5.6 \times 10^{-5}$.
$d = 1.4 \times 10^{-4}$.

(a) Work out the value of T. (2)
Give your answer in standard form correct to 3 significant figures.

Solution

$$T = \sqrt{\frac{5.6 \times 10^{-5}}{(1.4 \times 10^{-4})^3}}$$
$$= 4\,517.539\,515 \text{ (FCD)}$$
$$= \underline{\underline{4.52 \times 10^3 \text{ (3 sf)}}}.$$

w is increased by 10%.
d is increased by 5%.
Lottie says, "The value of T will increase because both w and d are increased."

(b) Lottie is wrong. (2)
Explain why.

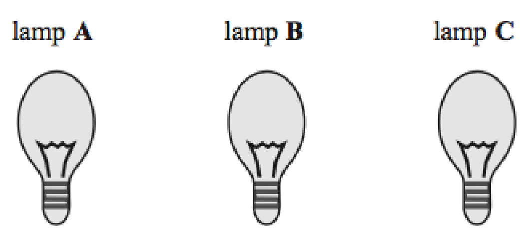
Solution

w increases from 5.6×10^{-5} to 6.16×10^{-5} and d increases from 1.4×10^{-4} to $1.47. \times 10^{-4}$. Now,

$$T = \sqrt{\frac{6.16 \times 10^{-5}}{(1.47 \times 10^{-4})^3}}$$
$$= 4\,403.665\,816 \text{ (FCD)}$$

and so T will $\underline{\underline{\text{decrease}}}$.

10. Here are three lamps. (3)



Lamp **A** flashes every 20 seconds.
Lamp **B** flashes every 45 seconds.
Lamp **C** flashes every 120 seconds.
The three lamps start flashing at the same time.
How many times in one hour will the three lamps flash at the same time?

Solution

$$\begin{array}{r|l} & 20 \\ \hline 2 & 10 \\ \hline 2 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So

$$20 = 2 \times 2 \times 5 = 2^2 \times 5.$$

$$\begin{array}{r|l} & 45 \\ \hline 3 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So

$$45 = 3 \times 3 \times 5 = 3^2 \times 5.$$

$$\begin{array}{r|l} & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

So

$$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5.$$

Hence,

$$\text{LCF}(20, 45, 120) = 2^3 \times 3^2 \times 5 = 360.$$

Finally, the number of times the three lamps flash at the same time is

$$\begin{aligned} \frac{1 \text{ hour}}{360 \text{ seconds}} &= \frac{60 \text{ minutes}}{360 \text{ seconds}} \\ &= \frac{3\,600 \text{ seconds}}{360 \text{ seconds}} \\ &= \underline{\underline{10}}. \end{aligned}$$

11. In 2003, Jerry bought a house. (3)
In 2007, Jerry sold the house to Mia.
He made a profit of 20%.
In 2012, Mia sold the house for £162 000.
She made a loss of 10%.
Work out how much Jerry paid for the house in 2003.

Solution

Well,

$$\text{new price for Jerry} = \text{old price for Jerry} \times 1.2$$

and

$$\text{new price for Mia} = \text{old price for Mia} \times 0.9.$$

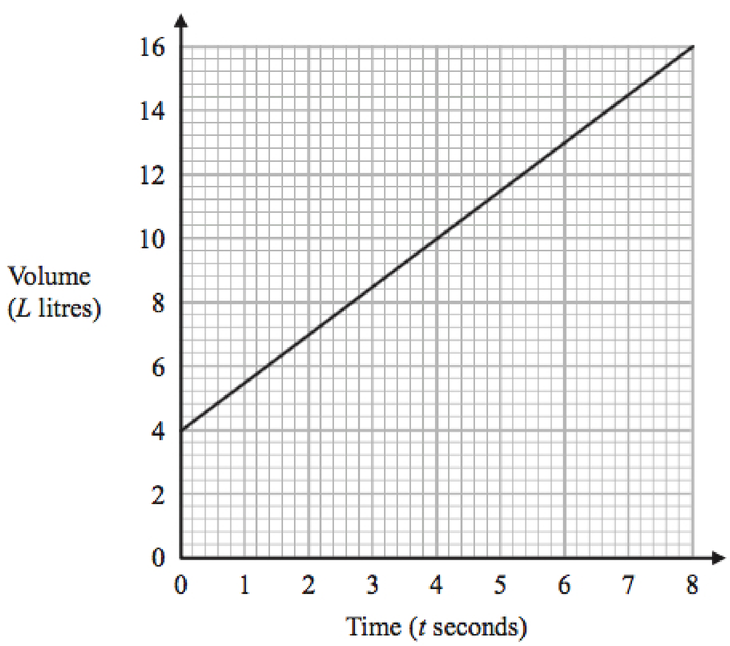
But the new price for Mia was £162 000 so

$$\text{old price for Mia} = \frac{162\,000}{0.9} = £180\,000$$

so that means

$$\text{old price for Jerry} = \frac{180\,000}{1.2} = \underline{\underline{£150\,000}}.$$

12. The graph shows the volume of liquid (L litres) in a container at time t seconds.



(a) Find the gradient of the graph. (2)

Solution

E.g., (0, 4) and (8, 16):

$$\text{gradient} = \frac{16-4}{8-0} = \frac{12}{8} = \underline{\underline{\tfrac{3}{2}}}.$$

(b) Explain what this gradient represents. (1)

Solution

For every 1 second, there is an additional $\frac{3}{2}$ litres.

The graph intersects the volume axis at $L = 4$.

(c) Explain what this intercept represents. (1)

Solution

When $t = 0$, the volume is 4 litres.

13. Here are two similar solid shapes. (3)

A

B

Surface area of shape **A** : surface area of shape B = 3 : 4.
The volume of shape B is 10 cm^3.

Work out the volume of shape **A**.
Give your answer correct to 3 significant figures.

Solution

The area scale factor (ASF), going from **B** to **A**, is $\frac{3}{4}$.

The length scale factor (LSF), going from **B** to **A**, is $\sqrt{\frac{3}{4}}$.

The volume scale factor (VSF), going from **B** to **A**, is $(\sqrt{\frac{3}{4}})^3$.

Hence, the volume of shape **A** is

$$
\begin{aligned}
10 \times \left(\sqrt{\tfrac{3}{4}}\right)^3 &= 6.495\,190\,528 \text{ (FCD)} \\
&= \underline{\underline{6.50 \text{ cm}^3 \text{ (3 sf)}}}.
\end{aligned}
$$

14. There are 16 hockey teams in a league. (2)
Each team played two matches against each of the other teams.

Work out the total number of matches played.

Solution

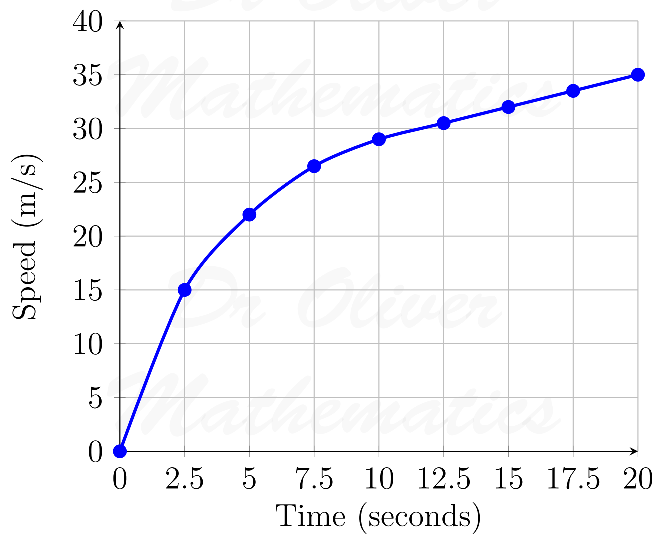
16 divided by 2 is 8 and a team plays

$$(16 - 1) \times 2 = 30 \text{ matches.}$$

Hence, the total number of matches played is

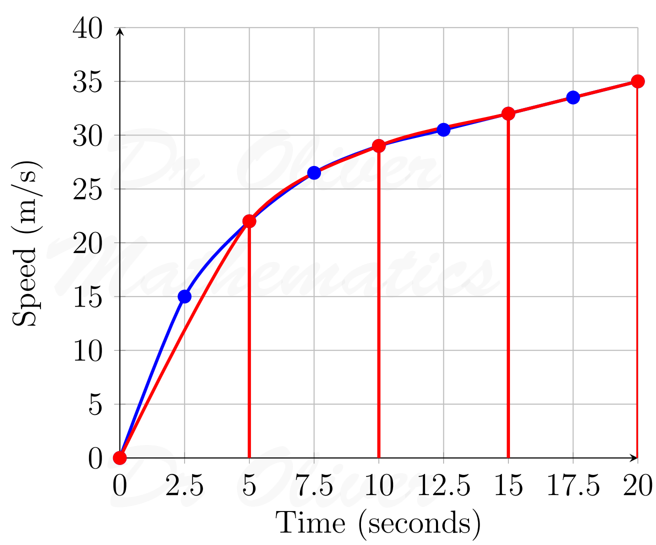
$$8 \times 30 = \underline{\underline{240 \text{ matches played}}}.$$

15. The graph shows the speed of a car, in metres per second, during the first 20 seconds of a journey.



(a) Work out an estimate for the distance the car travelled in the first 20 seconds. Use 4 strips of equal width. (3)

Solution



$$\begin{aligned}\text{Area} &\approx \tfrac{1}{2} \times 5 \times [0 + 2(22 + 29 + 32) + 35] \\ &= \underline{\underline{502.5 \text{ m}}}.\end{aligned}$$

(b) Is your answer to part (a) an underestimate or an overestimate of the actual distance (1)

the car travelled in the first 20 seconds?
Give a reason for your answer.

Solution
It is an underestimate as part of the graph is not included.

16. The nth term of a sequence is given by

$$an^2 + bn,$$

where a and b are integers.

The 2nd term of the sequence is -2.
The 4th term of the sequence is 12.

(a) Find the 6th term of the sequence. (4)

Solution

$$4a + 2b = -2 \quad (1)$$
$$16a + 4b = 12 \quad (2).$$

Now,

$$8a + 4b = -4 \quad (3).$$

Next, (2) − (3):

$$\begin{aligned} 8a = 16 \Rightarrow a &= 2 \\ &\Rightarrow 8 + 2b = -2 \\ &\Rightarrow 2b = -10 \\ &\Rightarrow b = -5. \end{aligned}$$

So, nth term of a sequence is given by

$$2n^2 - 5n,$$

and the 6th term of the sequence is

$$(2 \times 6^2) - (5 \times 6) = \underline{\underline{42}}.$$

Here are the first five terms of a different quadratic sequence:

$$0 \quad 2 \quad 6 \quad 12 \quad 20.$$

(b) Find an expression, in terms of n, for the nth term of this sequence. (2)

Solution

Let the

$$n\text{th term} = an^2 + bn + c.$$

We only need the second line of differences (why?):

$$\begin{array}{ccccc} 0 & & 2 & & 6 \\ & 2 & & 4 & \\ & & 2 & & \end{array}$$

$$\begin{array}{ccccc} a+b+c & & 4a+2b+c & & 9a+3b+c \\ & 3a+b & & 5a+b & \\ & & 2a & & \end{array}$$

We compare terms:

$$2a = 2 \Rightarrow a = 1,$$

$$\begin{aligned} 3a + b = 2 &\Rightarrow 3 \times 1 + b = 2 \\ &\Rightarrow b = -1, \end{aligned}$$

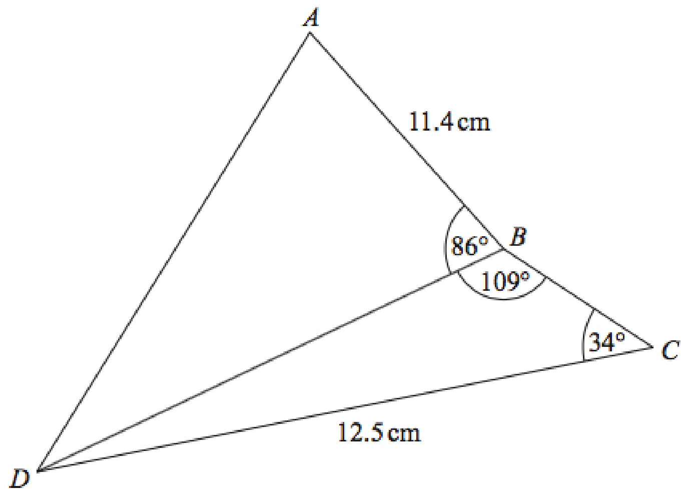
and

$$\begin{aligned} a + b + c = 0 &\Rightarrow 1 - 1 + c = 0 \\ &\Rightarrow c = 0; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{n^2 - n}}.$$

17. Work out the length of AD. (5)



Give your answer correct to 3 significant figures.

Solution

$$\frac{BD}{\sin BCD} = \frac{CD}{\sin CBD} \Rightarrow \frac{BD}{\sin 34^\circ} = \frac{12.5}{\sin 109^\circ}$$
$$\Rightarrow BD = \frac{12.5 \sin 34^\circ}{\sin 109^\circ}$$
$$\Rightarrow BD = 7.392\,674\,744 \text{ (FCD)}.$$

Now,

$$\begin{aligned} AD &= \sqrt{AB^2 + BD^2 - 2 \cdot AB \cdot BD \cdot \cos ABD} \\ &= \sqrt{11.4^2 + 7.392\ldots^2 - 2 \cdot 11.4 \cdot 7.392\ldots \cdot \cos 86^\circ} \\ &= 13.147\,394\,34 \text{ (FCD)} \\ &= \underline{\underline{13.1 \text{ cm (3 sf)}}}. \end{aligned}$$

18. (a) Show that the equation (2)

$$x^3 + x = 7$$

has a solution between 1 and 2.

Solution

Let $f(x) = x^3 + x - 7$. Then

$$f(1) = 1 + 1 - 7 = -7$$
$$f(2) = 8 + 1 - 7 = 2.$$

The function is continuous and there is a change of sign and so the root lies $\underline{\underline{1 < f(x) < 2}}$.

(b) Show that the equation (1)

$$x^3 + x = 7$$

can be rearranged to give

$$x = \sqrt[3]{7 - x}.$$

Solution

$$x^3 + x = 7 \Rightarrow x^3 = 7 - x$$
$$\Rightarrow \underline{\underline{x = \sqrt[3]{7 - x}}},$$

as required.

(c) Starting with $x_0 = 2$, use the iteration formula (3)

$$x_{n+1} = \sqrt[3]{7 - x_n}.$$

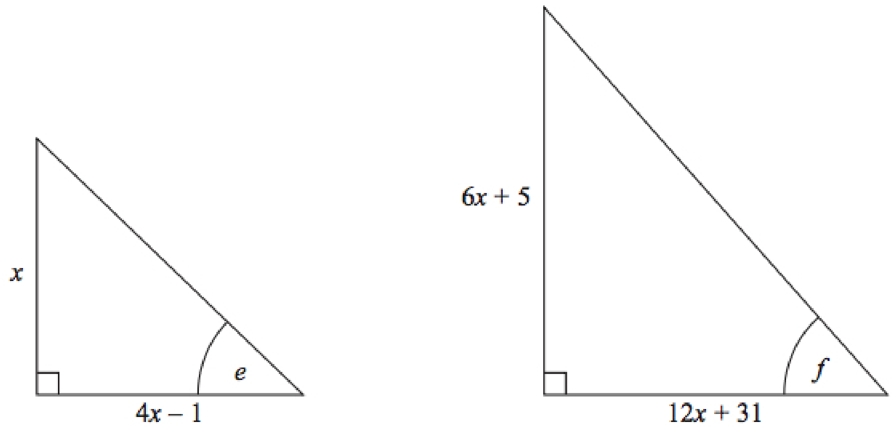
three times to find an estimate for a solution of

$$x^3 + x = 7.$$

Solution

$x_1 = 1.709\,975\,947$ (FCD)
$x_2 = 1.742\,418\,802$ (FCD)
$x_3 = \underline{\underline{1.738\,849\,506\text{ (FCD)}}}$.

19. Here are two right-angled triangles. (5)



Given that

$$\tan e = \tan f,$$

find the value of x.
You must show all your working.

Solution

$$\tan e = \tan f \Rightarrow \frac{x}{4x-1} = \frac{6x+5}{12x+31}$$
$$\Rightarrow x(12x+31) = (4x-1)(6x+5)$$

$\times$	$4x$	-1
$6x$	$24x^2$	$-6x$
$+5$	$+20x$	-5

$$\Rightarrow 12x^2 + 31x = 24x^2 + 14x - 5$$
$$\Rightarrow 12x^2 - 17x - 5 = 0$$

$$\begin{array}{ll} \text{add to:} & -17 \\ \text{multiply to:} & (+12) \times (-5) = -60 \end{array} \Bigg\} -20,\ +3$$

$$\begin{aligned} &\Rightarrow 12x^2 - 20x + 3x - 5 = 0 \\ &\Rightarrow 4x(3x - 5) + (3x - 5) = 0 \\ &\Rightarrow (4x + 1)(3x - 5) = 0 \\ &\Rightarrow 4x + 1 = 0 \text{ or } 3x - 5 = 0 \\ &\Rightarrow x = -\tfrac{1}{4} \text{ or } x = 1\tfrac{2}{3}; \end{aligned}$$

now, $x \neq = -\frac{1}{4}$ (why?) so $\underline{\underline{x = 1\frac{2}{3}}}$.

20. 50 people were asked if they speak French or German or Spanish. (5)

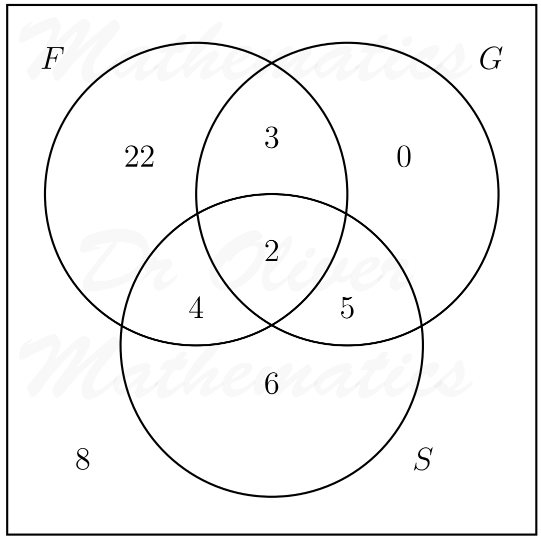
Of these people,
31 speak French;
2 speak French, German, and Spanish;
4 speak French and Spanish but not German;
7 speak German and Spanish;
8 do not speak any of the languages;
all 10 people who speak German speak at least one other language.

Two of the 50 people are chosen at random.

Work out the probability that they both only speak Spanish.

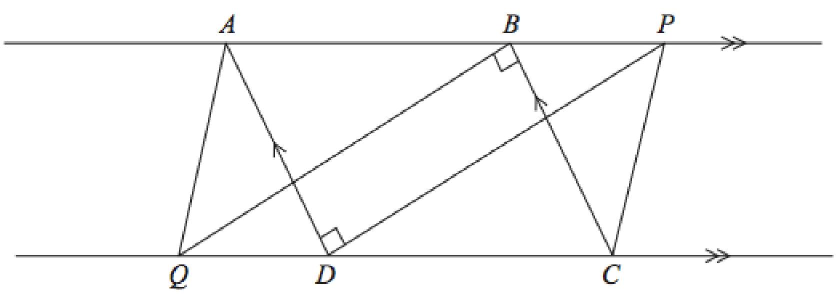
Solution

Let F, G, and S be the persons who speak French, German, and Spanish respectively and we make up a Venn diagram.



$$\text{P(both only speak Spanish)} = \tfrac{6}{50} \times \tfrac{5}{49}$$
$$= \underline{\underline{\tfrac{3}{245}}}.$$

21. $ABCD$ is a parallelogram.



ABP and QDC are straight lines.
Angle ADP = angle $CBQ = 90°$.

(a) Prove that triangle ADP is congruent to triangle CBQ. (3)

Solution
$\angle ADP = \angle CBQ$ (given)
$\angle PAD = \angle DCQ$ (opposite angles of a parallelogram)

AD and BC are the same length (opposite sides of a parallelogram)
So, triangles ADP is congruent to CBQ (AAS).

(b) Explain why AQ is parallel to PC. (2)

Solution

$AP = QC$ (triangle ADP is congruent to triangle CBQ)
So $APCQ$ is a parallelogram (why?)
Finally, AQ is parallel to PC (opposite sides of a parallelogram)