Dr Oliver Mathematics
GCSE Mathematics
2009 November Paper 4H: Calculator
1 hour 45 minutes

The total number of marks available is 100.
You must write down all the stages in your working.

1. Ali asked 200 students which sport they like best. (3)
They could choose swimming or tennis or athletics.
The two-way table shows some information about their answers.

	Swimming	Tennis	Athletics	Total
Female			19	
Male	36	42		
Total	79		54	200

Complete the two-way table.

Solution

	Swimming	Tennis	Athletics	Total
Female	43	25	19	87
Male	36	42	35	113
Total	79	67	54	200

2. (a) Use your calculator to work out the value of (2)

$$\frac{8.7 \times 12.3}{9.5 - 5.73}.$$

Write down all the digits from your calculator.
Give your answer as a decimal.

Solution

$$\frac{8.7 \times 12.3}{9.5 - 5.73} = \frac{107.01}{3.77}$$
$$= \underline{\underline{28.384\,615\,38 \text{ (FCD)}}}.$$

(b) Write your answer to part (a) correct to 1 significant figure. (1)

Solution

$\underline{\underline{30 \text{ (1 sf)}}}$.

3. (a) $p = 2$. (2)
$q = -4$.
Work out the value of
$$3p + 5q.$$

Solution

$$3 \times 2 + 5 \times (-4) = 6 - 20 = \underline{\underline{-14}}.$$

(b) Factorise $3m - 6$. (1)

Solution

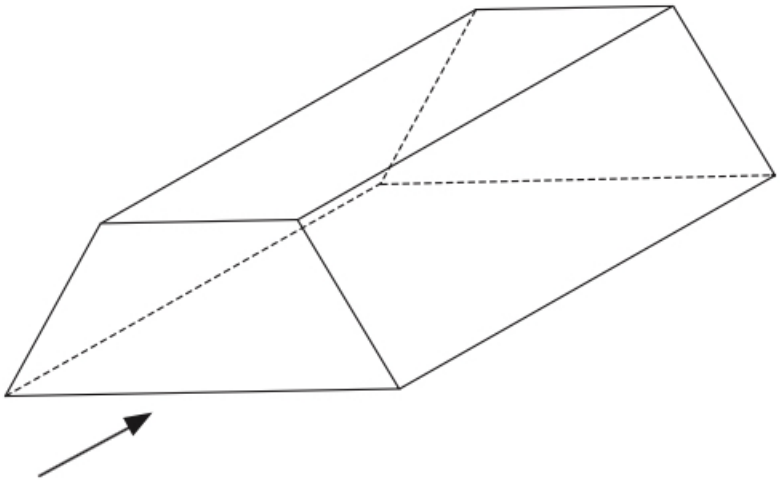
$$3m - 6 = \underline{\underline{3(m - 2)}}.$$

4. Frank did a survey on the areas of pictures in a magazine. (1)
The magazine had 60 pages.
Frank worked out the area of each of the pictures in the first 2 pages.
This may not be a good method to do the survey.
Explain why.

Solution

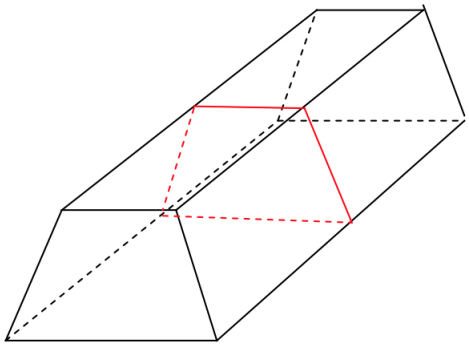
E.g., there is no good reason why the pictures in the first 2 pages will be representative of the whole newspaper, the sample size is too small.

5. The diagram shows a prism.



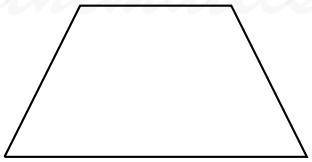
(a) On the diagram, draw in **one** plane of symmetry for the prism. (2)

Solution

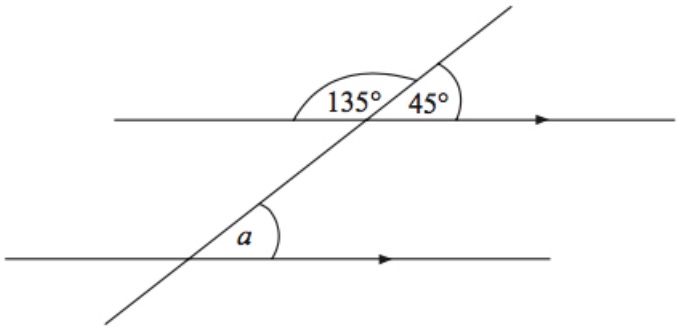


(b) In the space below, sketch the front elevation from the direction marked with an arrow. (2)

Solution



6. Here is a diagram.



(a) Write down the size of the angle marked a. (1)

Solution

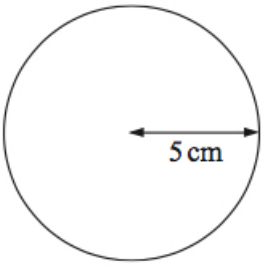
$45°$.

(b) Give a reason for your answer. (1)

Solution

Corresponding angles.

7. A circle has a radius of 5 cm. (2)

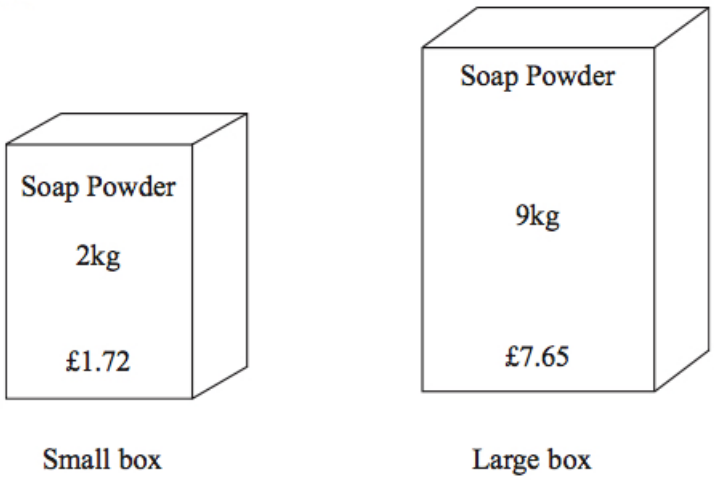


Work out the area of the circle.
Give your answer correct to 3 significant figures.

Solution

$$\pi \times 5^2 = 78.539\,816\,34 \text{ (FCD)}$$
$$= 78.5 \text{ cm}^2 \text{ (3 sf)}.$$

8. Soap powder is sold in two sizes of box. (3)



Small box Large box

A small box contains 2 kg of soap powder and costs £1.72.
A large box contains 9 kg of soap powder and costs £7.65.
Which size of box gives the better value for money?
Explain your answer.
You must show all your working.

Solution

2 kg:

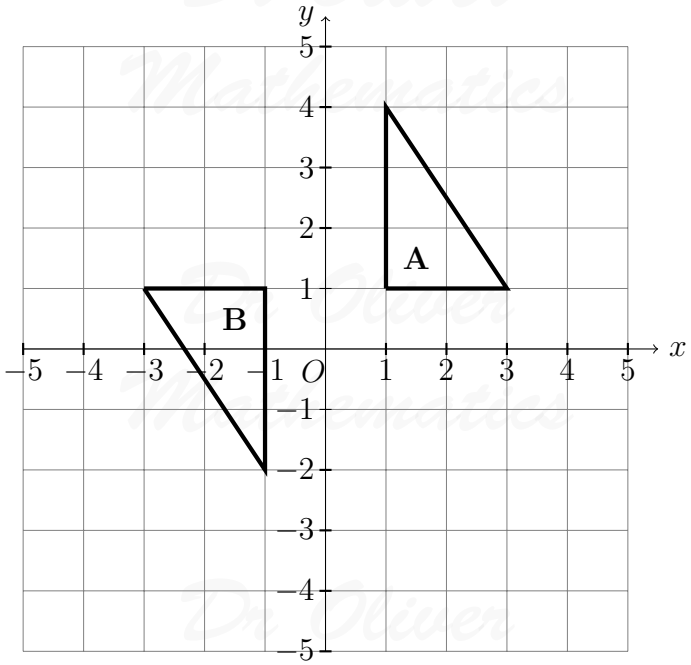
$$\frac{1.72}{2} = 0.86.$$

9 kg:

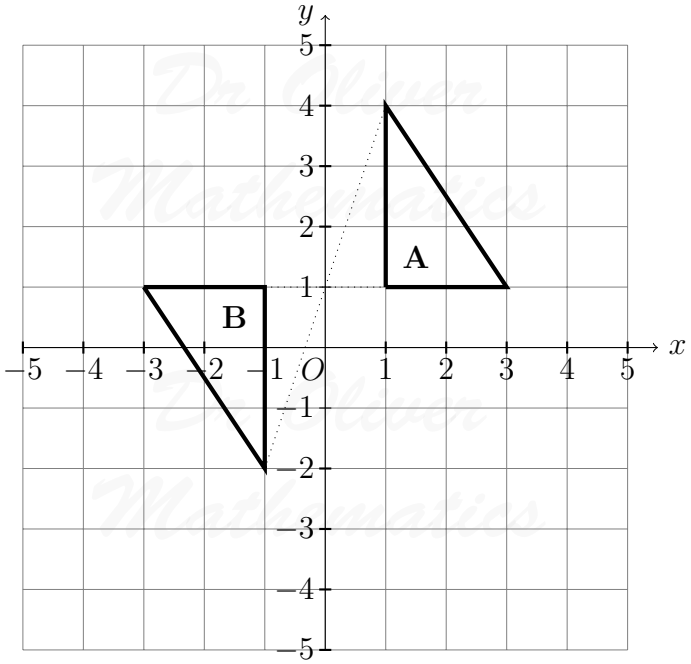
$$\frac{7.65}{9} = 0.85.$$

The 9 kg gives better value for money.

9. Describe fully the single transformation that maps triangle **A** onto triangle **B**. (3)



Solution



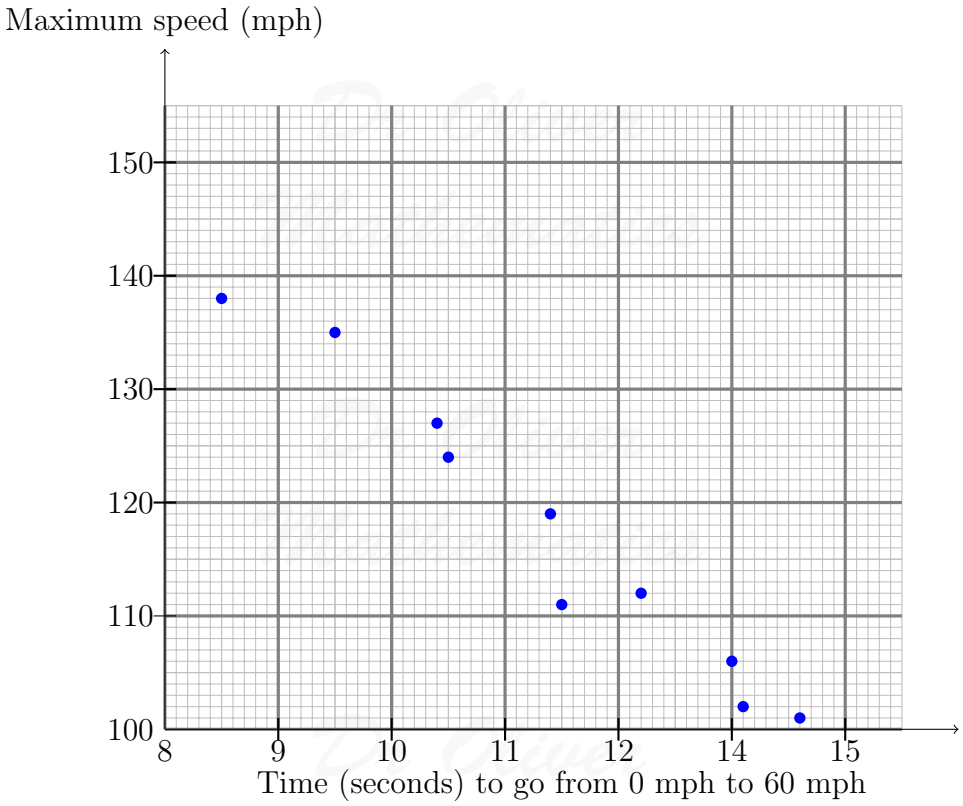
Rotation, through $180°$, about centre $(0, 1)$.

10. A computer costs £360 plus $17\frac{1}{2}\%$ VAT. (3)
Calculate the total cost of the computer.

Solution

$$360 \times 1.175 = \underline{\underline{£423}}.$$

11. The scatter graph shows some information about 10 cars.
It shows the time, in seconds, it takes each car to go from 0 mph to 60 mph.
For each car, it also shows the maximum speed, in mph.



(a) What type of correlation does this scatter graph show? (1)

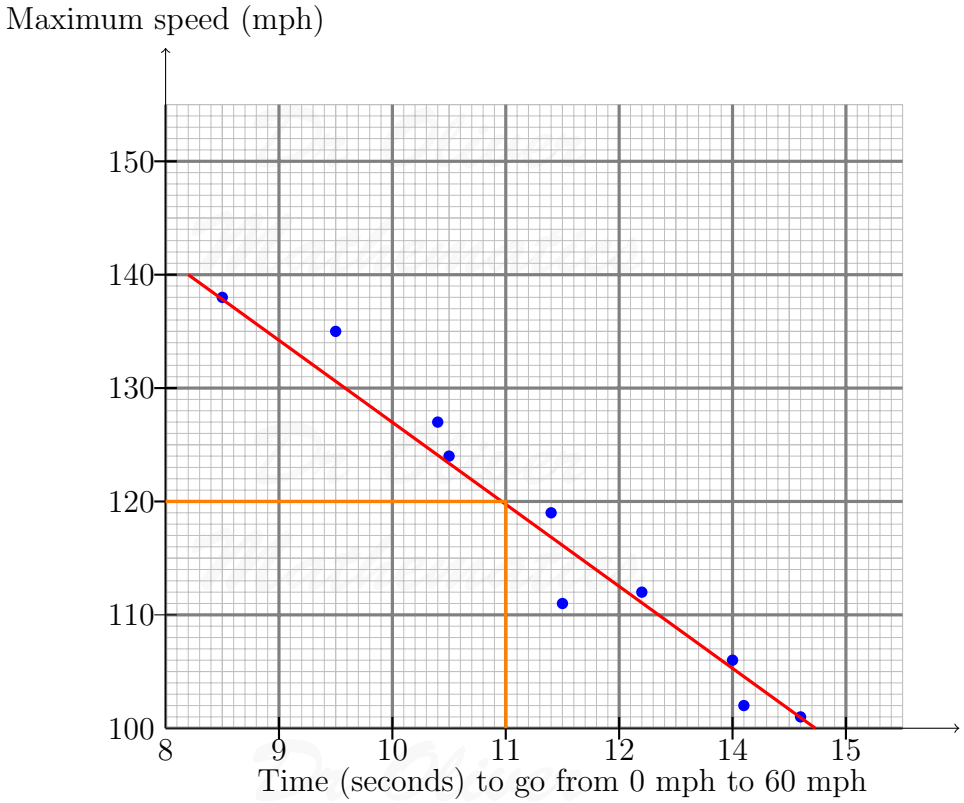
Solution

<u>Negative correlation</u>.

The time a car takes to go from 0 mph to 60 mph is 11 seconds.

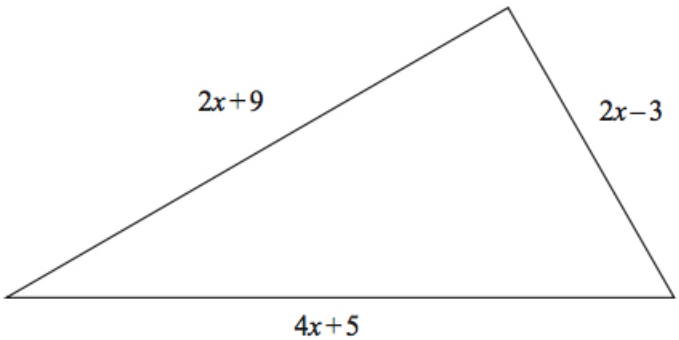
(b) Estimate the maximum speed for this car. (2)

Solution



Correct read-off: approximately 120 mph.

12. In the diagram, all measurements are in centimetres.



The lengths of the sides of the triangle are $(2x + 9)$, $(2x - 3)$, and $(4x + 5)$.

(a) Find an expression, in terms of x, for the perimeter of the triangle. (2)
Give your expression in its simplest form.

Solution

$$(2x + 9) + (2x - 3) + (4x + 5) = \underline{\underline{(8x + 11) \text{ cm}}}.$$

The perimeter of the triangle is 39 cm.

(b) Find the value of x. (2)

Solution

$$8x + 11 = 39 \Rightarrow 8x = 28$$
$$\Rightarrow \underline{\underline{x = 3\tfrac{1}{2} \text{ cm}}}.$$

13. A piece of wood is 180 cm long. (3)
Tom cuts it into three pieces in the ratio 2 : 3 : 4.
Work out the length of the longest piece.

Solution

$2 + 3 + 4 = 9$ and so the length of the longest piece is

$$\tfrac{4}{9} \times 180 = \underline{\underline{80 \text{ cm}}}.$$

14. The equation (4)

$$x^3 + 2x = 60$$

has a solution between 3 and 4.
Use a trial and improvement method to find this solution.
Give your answer correct to 1 decimal place.
You must show all your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 + 2X$; then you press $\boxed{=}$.

Start? and you enter 3; then you press [=].
End? and you enter 4; then you press [=].
Step? and enter 0.05 – 1 decimal place divided by 2; then you press [=].

x	f(x)	Comment
3.7	58.053	too low
3.75	60.234	too high

Clearly,

$$3.7 < x < 3.75$$

and the answer is

$$\underline{\underline{x = 3.7 \text{ (1 dp)}}}.$$

15. (a) Simplify $m^3 \times m^4$. (1)

Solution

$$m^3 \times m^4 = \underline{\underline{m^7}}.$$

(b) Simplify $p^7 \div p^3$. (1)

Solution

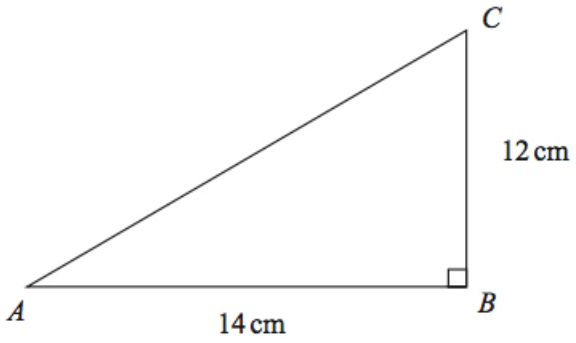
$$p^7 \div p^3 = \underline{\underline{p^4}}.$$

(c) Simplify $4x^2y^3 \times 3xy^2$. (2)

Solution

$$4x^2y^3 \times 3xy^2 = \underline{\underline{12x^3y^5}}.$$

16. ABC is a right-angled triangle. (3)



$AB = 14$ cm.
$BC = 12$ cm.
Calculate the length of AC.
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} AC &= \sqrt{12^2 + 14^2} \\ &= 18.439\,088\,91 \text{ (FCD)} \\ &= \underline{\underline{18.4 \text{ cm (3 sf)}}}. \end{aligned}$$

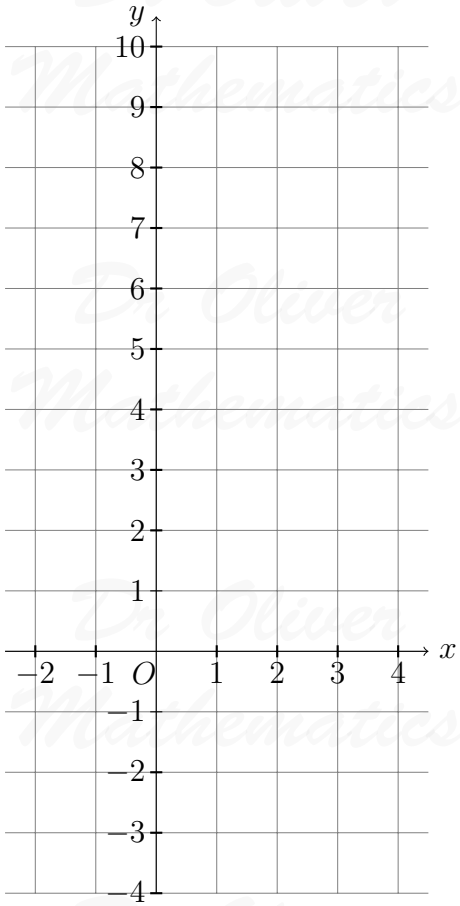
17. (a) Complete the table of values for $y = x^2 - 3x - 1$. (2)

x	-2	-1	0	1	2	3	4
y		3	-1	-3		-1	

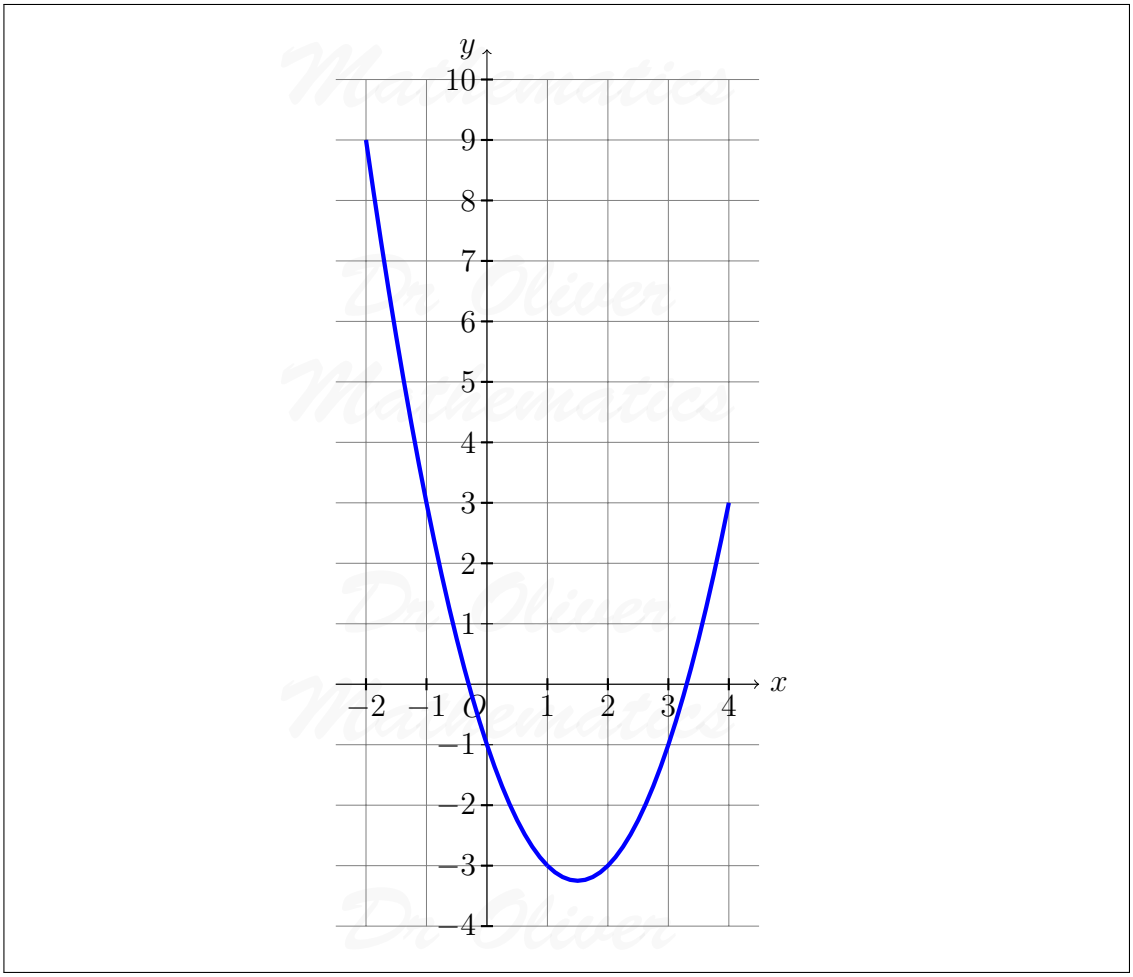
Solution

x	-2	-1	0	1	2	3	4
y	$\underline{9}$	3	-1	-3	$\underline{\underline{-3}}$	-1	$\underline{3}$

(b) On the grid, draw the graph of $y = x^2 - 3x - 1$ for values of x from -2 to 4. (2)



Solution



18. The table shows some information about the heights (h cm) of 100 students.

Height (h cm)	Frequency
$120 \leqslant h < 130$	8
$130 \leqslant h < 140$	16
$140 \leqslant h < 150$	25
$150 \leqslant h < 160$	30
$160 \leqslant h < 170$	21

(a) Find the class interval in which the median lies. (1)

Solution

Height (h cm)	Frequency	Cumulative Frequency
$120 \leqslant h < 130$	8	8
$130 \leqslant h < 140$	16	$16 + 8 = 24$
$140 \leqslant h < 150$	25	$25 + 24 = 49$
$150 \leqslant h < 160$	30	$30 + 49 = 79$
$160 \leqslant h < 170$	21	$21 + 79 = 100$

Now, $\frac{100+1}{2} = 50\frac{1}{2}$th and the median is $\underline{\underline{150 \leqslant h < 160}}$.

(b) Work out an estimate for the mean height of the students. (4)

Solution

Height (h cm)	Frequency	Midpoint	Frequency×Midpoint
$120 \leqslant h < 130$	8	125	$125 \times 8 = 1\,000$
$130 \leqslant h < 140$	16	135	$135 \times 16 = 2\,160$
$140 \leqslant h < 150$	25	145	$145 \times 25 = 3\,625$
$150 \leqslant h < 160$	30	155	$155 \times 8 = 4\,650$
$160 \leqslant h < 170$	21	165	$165 \times 8 = 3\,465$
Total	100		14 900

The mean height is

$$\begin{aligned} \bar{x} &\approx \frac{\sum fh}{\sum f} \\ &= \frac{14\,900}{100} \\ &= \underline{\underline{149 \text{ cm}}}. \end{aligned}$$

19. (a) Expand and simplify (2)

$$(x - 3)(x + 5).$$

Solution

$\times$	x	-3
x	x^2	$-3x$
$+5$	$+5x$	-15

Hence,

$$(x-3)(x+5) = \underline{\underline{x^2+2x-15}}.$$

(b) Solve (3)

$$\frac{29-x}{4} = x+5.$$

Solution

$$\begin{aligned}\frac{29-x}{4} = x+5 &\Rightarrow 29-x = 4(x+5)\\ &\Rightarrow 29-x = 4x+20\\ &\Rightarrow 9 = 5x\\ &\Rightarrow \underline{\underline{x = 1.8}}.\end{aligned}$$

20. The table gives information about the cost of the gas used by a family.

Month	Jan-Mar 2007	Apr-Jun 2007	Jul-Sep 2007	Oct-Dec 2007	Jan-Mar 2008	Apr-Jun 2008	Jul-Sep 2008
Cost of gas (in £)	124	63	24	121	136	71	32

(a) Work out the four-point moving averages for this information. (2)
The first three have been worked out for you: £83, £86, £88.

Solution

$$\frac{121+136+71+32}{4} = \frac{360}{4} = \underline{\underline{£90}}.$$

(b) Use the moving averages to describe the trend. (1)

Solution

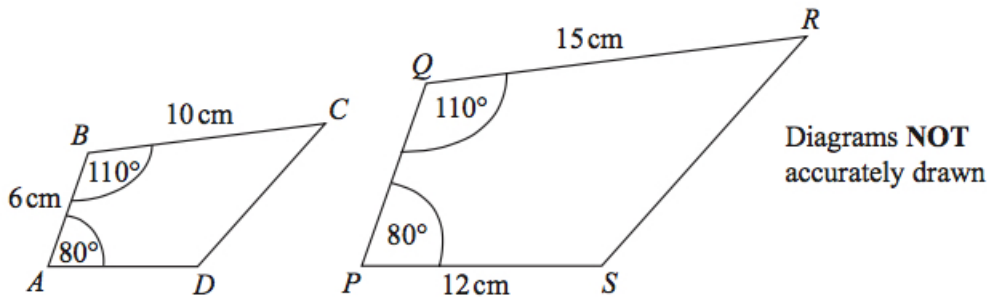
They are increasing.

21. In a sale, normal prices are reduced by 12%. (3)
The sale price of a digital camera is £132.88.
Work out the normal price of the digital camera.

Solution

$$\frac{132.88}{1-0.12} = £151.$$

22. $ABCD$ and $PQRS$ are mathematically similar.



(a) Find the length of PQ. (2)

Solution

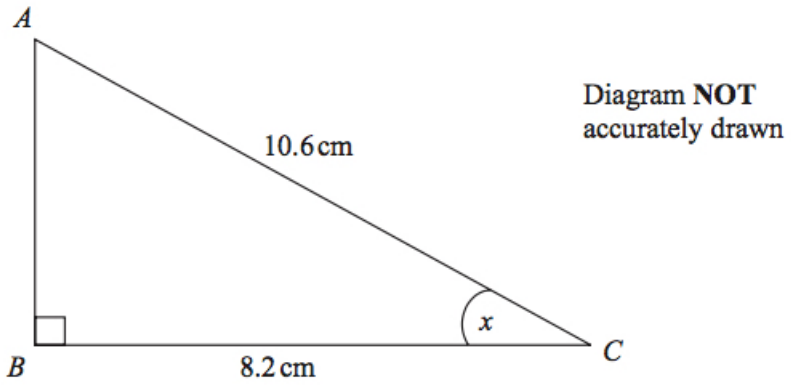
$$PQ = \frac{6 \times 15}{10} = 9 \text{ cm}.$$

(b) Find the length of AD. (2)

Solution

$$AD = \frac{12 \times 10}{15} = 8 \text{ cm}.$$

23. ABC is a right-angled triangle. (3)



$AC = 10.6$ cm.
$BC = 8.2$ cm.
Calculate the size of the angle marked x.
Give your answer correct to 3 significant figures.

Solution

$$\cos = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos x = \frac{8.2}{10.6}$$
$$\Rightarrow x = 39.323\,089\,18 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 39.3^\circ \text{ (3 sf)}}}.$$

24. The table below gives some information about some students in a school. (2)

Year Group	Boys	Girls	Total
Year 12	126	94	220
Year 13	77	85	162
Total	203	179	382

Andrew is going to carry out a survey of these students.
He uses a sample of 50 students, stratified by year group and gender.
Work out the number of Year 13 girls that should be in his sample.

Solution

$$\frac{85}{382} \times 50 = 11.12\ldots$$

and 11 students Year 13 girls should be in his sample.

25. y is directly proportional to x.
When $x = 500$, $y = 10$.

(a) Find a formula for y in terms of x. (3)

Solution

$y \propto x \Rightarrow y = kx$ for some k. Now,

$$10 = 500x \Rightarrow k = \tfrac{1}{50}$$

and so

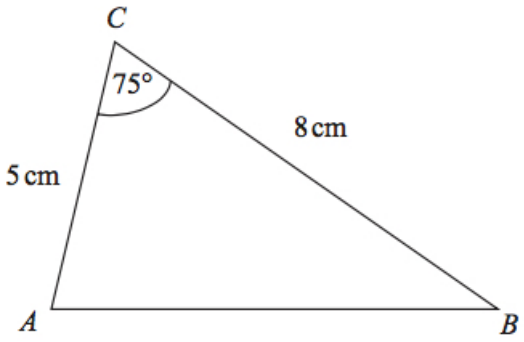
$$y = \tfrac{1}{50}x.$$

(b) Calculate the value of y when $x = 350$. (1)

Solution

$$y = \tfrac{1}{50} \times 350 = 7.$$

26. In triangle ABC, $AC = 5$ cm, $BC = 8$ cm, and angle $ACB = 75°$.



(a) Calculate the area of triangle ABC. (2)
Give your answer correct to 3 significant figures.

Solution

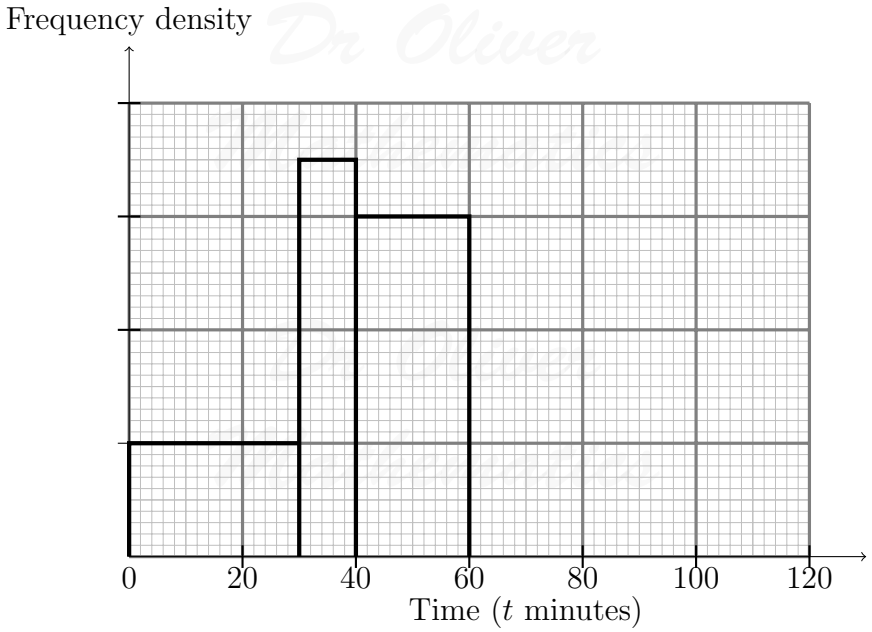
$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 5 \times 8 \times \sin 75^\circ \\ &= 19.318\,516\,53 \text{ (FCD)} \\ &= \underline{\underline{19.3 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

(b) Calculate the length of AB. (3)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}AB &= \sqrt{5^2 + 8^2 - 2 \times 5 \times 8 \times \cos 75^\circ} \\ &= 8.264\,047\,216 \text{ (FCD)} \\ &= \underline{\underline{8.26 \text{ cm (3 sf)}}}.\end{aligned}$$

27. The incomplete histogram and table give some information about the times, in minutes, that cars were parked in a car park.



(a) Use the information in the histogram to complete the frequency table. (2)

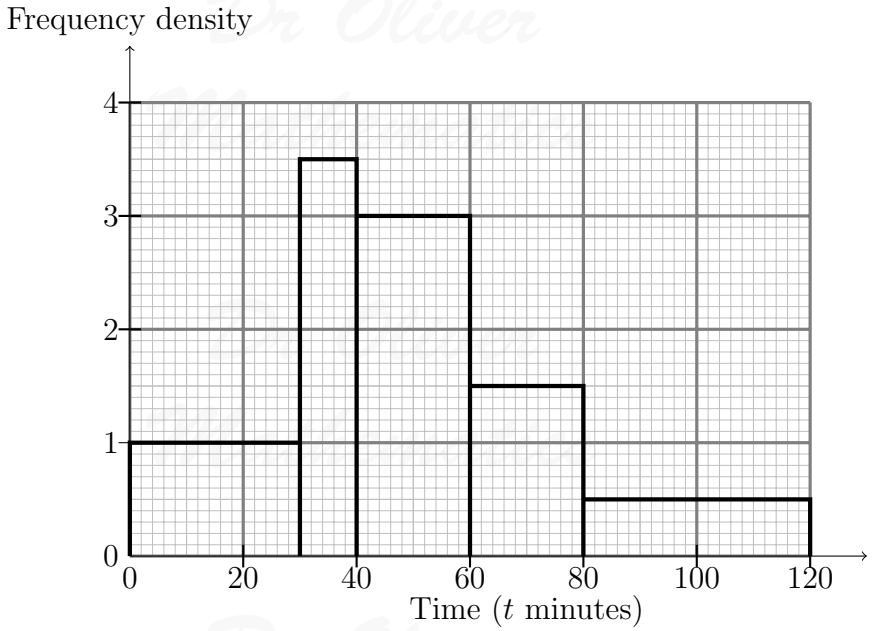
Time (t minutes)	Frequency
$0 < t \leqslant 30$	
$30 < t \leqslant 40$	35
$40 < t \leqslant 60$	
$60 < t \leqslant 80$	30
$80 < t \leqslant 120$	20

Solution

Time (t minutes)	Frequency	Width	Frequency density
$0 < t \leqslant 30$	$\underline{\underline{30}}$	30	$\frac{30}{30} = 1$
$30 < t \leqslant 40$	35	10	$\frac{35}{10} = 3.5$
$40 < t \leqslant 60$	$\underline{\underline{60}}$	20	$\frac{60}{20} = 3$
$60 < t \leqslant 80$	30	20	$\frac{30}{20} = 1.5$
$80 < t \leqslant 120$	20	40	$\frac{20}{40} = 0.5$

(b) Use the information in the table to complete the histogram. (2)

Solution



28. (5)

$$v = \sqrt{\frac{a}{b}}.$$

$a = 6.43$, correct to 2 decimal places.
$b = 5.514$, correct to 3 decimal places.
By considering bounds, work out the value of v to a suitable degree of accuracy.
You must show all your working and give a reason for your final answer.

Solution

$$6.425 \leqslant a < 6.435$$

and

$$5.513\,5 \leqslant b < 5.514\,5.$$

Now, the lower bound is

$$\sqrt{\frac{6.425}{5.514\,5}} = 1.079\,402\,689 \text{ (FCD)}$$

and the upper bound is

$$\sqrt{\frac{6.435}{5.513\,5}} = 1.080\,340\,323 \text{ (FCD)}.$$

Degree of accuracy	Lower bound	Upper bound
1 sf	1	1
2 sf	1.1	1.1
3 sf	1.08	1.08
4 sf	1.079	1.080

The value is 1.08 (3 sf) because the lower and upper bounds agree but not to 4 significant figures.

29. Solve (5)

$$\frac{4}{x+3} + \frac{3}{2x-1} = 1.$$

Solution

$$\frac{4}{x+3} + \frac{3}{2x-1} = 1$$

$$\Rightarrow \quad 4(2x-1) + 3(x+3) = (2x-1)(x+3)$$

$\times$	$2x$	-1
x	$2x^2$	$-x$
$+3$	$+6x$	-3

$$\Rightarrow \quad 8x - 4 + 3x + 9 = 2x^2 + 5x - 3$$

$$\Rightarrow \quad 2x^2 - 6x - 8 = 0$$

$$\Rightarrow \quad x^2 - 3x - 4 = 0$$

$$\left.\begin{array}{ll} \text{add to:} & -3 \\ \text{multiply to:} & -4 \end{array}\right\} -4,\ +1$$

$$\Rightarrow \quad (x+1)(x-4) = 0$$

$$\Rightarrow \quad x + 1 = 0 \text{ or } x - 4 = 0$$

$$\Rightarrow \quad \underline{\underline{x = -1 \text{ or } x = 4}}.$$