# Dr Oliver Mathematics
# Mathematics
# Factor Theorems and Remainder Theorems
# Past Examination Questions

This booklet consists of 24 questions across a variety of examination topics.
The total number of marks available is 170.
(Please read *Synthetic Division* if you have not done so.)

1.
$$\mathrm{f}(x) \equiv x^3 - 2x^2 + ax + b,$$

where $a$ and $b$ are constants.

When $\mathrm{f}(x)$ is divided by $(x-2)$, the remainder is 1.

When $\mathrm{f}(x)$ is divided by $(x+1)$, the remainder is 28.

(a) Find the value of $a$ and the value of $b$. (6)

(b) Show that $(x-3)$ is a factor of $\mathrm{f}(x)$. (2)

2. (a) Use the factor theorem to show that $(x+4)$ is a factor of $2x^3 + x^2 - 25x + 12$. (2)

(b) Factorise $2x^3 + x^2 - 25x + 12$. (4)

3.
$$f(x) \equiv 2x^3 + x^2 - 5x + c,$$

where $c$ is a constant. Given that $\mathrm{f}(1) = 0$,

(a) find the value of $c$, (2)

(b) factorise $\mathrm{f}(x)$ completely, (4)

(c) find the remainder when $\mathrm{f}(x)$ is divided by $(2x-3)$. (2)

4.
$$\mathrm{f}(x) \equiv 2x^3 + 3x^2 - 29x - 60.$$

(a) Find the remainder when $\mathrm{f}(x)$ is divided by $(x+2)$. (2)

(b) Use the factor theorem to show that $(x+3)$ is a factor of $\mathrm{f}(x)$. (2)

(c) Factorise $\mathrm{f}(x)$ completely. (4)

5.
$$\mathrm{f}(x) \equiv x^3 + 4x^2 + x - 6.$$

(a) Use the factor theorem to show that $(x+2)$ is a factor of $\mathrm{f}(x)$. (2)

(b) Factorise $f(x)$ completely. (4)

(c) Write down all the solutions to the equation (1)

$$x^3 + 4x^2 + x - 6 = 0.$$

6.

$$f(x) \equiv 3x^3 - 5x^2 - 16x + 12.$$

(a) Find the remainder when $f(x)$ is divided by $(x - 2)$. (2)

Given that $(x + 2)$ is a factor of $f(x)$,

(b) factorise $f(x)$ completely. (4)

7. Find the remainder when

$$x^3 - 2x^2 - 4x + 8$$

is divided by

(a) (i) $x - 3$, (3)

(ii) $x + 2$.

(b) Hence, or otherwise, find all the solutions to the equation (4)

$$x^3 - 2x^2 - 4x + 8 = 0.$$

8.

$$f(x) \equiv 2x^3 - 3x^2 - 39x + 20.$$

(a) Use the factor theorem to show that $(x + 4)$ is a factor of $f(x)$. (2)

(b) Factorise $f(x)$ completely. (4)

9.

$$f(x) \equiv x^4 + 5x^3 + ax + b,$$

where $a$ and $b$ are constants.

The remainder when $f(x)$ divided by $(x - 2)$ is equal to the remainder when $f(x)$ is divided by $(x + 1)$.

(a) Find the value of $a$. (5)

Given that $(x + 3)$ is a factor of $f(x)$,

(b) find the value of $b$. (3)

10.

$$f(x) \equiv (3x - 2)(x - k) - 8,$$

where $k$ is a constant.

(a) Write down the value of $f(k)$. (1)

When $f(x)$ is divided by $(x - 2)$ the remainder is 4.

(b) Find the value of $k$. (2)

(c) Factorise $f(x)$ completely. (3)

11.
$$f(x) \equiv 2x^3 + ax^2 + bx - 6,$$

where $a$ and $b$ are constants.

When $f(x)$ is divided by $(2x - 1)$, the remainder is $-5$.

When $f(x)$ is divided by $(x + 2)$, there is no remainder.

(a) Find the value of $a$ and the value of $b$. (6)

(b) Factorise $f(x)$ completely. (3)

12.
$$f(x) \equiv 3x^3 - 5x^2 - 58x + 40.$$

(a) Find the remainder when $f(x)$ is divided by $(x - 3)$. (2)

Given that $(x - 5)$ is a factor of $f(x)$,

(b) find all the solutions of $f(x) = 0$. (5)

13.
$$f(x) \equiv x^4 + x^3 + 2x^2 + ax + b,$$

where $a$ and $b$ are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$. (2)

When $f(x)$ is divided by $(x + 2)$, the remainder is $-8$.

(b) Find the value of $a$ and the value of $b$. (5)

14.
$$f(x) \equiv 2x^3 - 7x^2 - 5x + 4.$$

(a) Find the remainder when $f(x)$ is divided by $(x - 1)$. (2)

(b) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$. (2)

(c) Factorise $f(x)$ completely. (4)

15.
$$\mathrm{f}(x) \equiv x^3 + ax^2 + bx + 3,$$
where $a$ and $b$ are constants.

Given that when $\mathrm{f}(x)$ is divided by $(x + 2)$ the remainder is 7,

(a) show that $2a - b = 6$. (2)

Given also that when $\mathrm{f}(x)$ is divided by $(x - 1)$ the remainder is 4,

(b) find the value of $a$ and the value of $b$. (4)

16.
$$\mathrm{f}(x) \equiv 2x^3 - 7x^2 - 10x + 24.$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $\mathrm{f}(x)$. (2)

(b) Factorise $\mathrm{f}(x)$ completely. (4)

17.
$$\mathrm{f}(x) \equiv ax^3 + bx^2 - 4x - 3,$$
where $a$ and $b$ are constants.

Given that $(x - 1)$ is a factor of $\mathrm{f}(x)$,

(a) show that $a + b = 7$. (2)

Given also that, when $\mathrm{f}(x)$ is divided by $(x + 2)$, the remainder is 9,

(b) find the value of $a$ and the value of $b$, showing each step in your working. (4)

18.
$$\mathrm{f}(x) \equiv 2x^3 - 5x^2 + ax + 18,$$
where $a$ is a constant.

Given that $(x - 3)$ is a factor of $\mathrm{f}(x)$,

(a) show that $a = -9$, (2)

(b) factorise $\mathrm{f}(x)$ completely. (4)

19.
$$\mathrm{f}(x) \equiv ax^3 - 11x^2 + bx + 4,$$
where $a$ and $b$ are constants.

When $\mathrm{f}(x)$ is divided by $(x - 3)$ the remainder is 55.

When $\mathrm{f}(x)$ is divided by $(x + 1)$ the remainder is $-9$.

(a) Find the value of $a$ and the value of $b$. (5)

Given that $(3x + 2)$ is a factor of $\mathrm{f}(x)$,

(b) factorise $\mathrm{f}(x)$ completely. (4)

20.
$$\mathrm{f}(x) \equiv 2x^3 - 7x^2 + 4x + 4.$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $\mathrm{f}(x)$. (2)

(b) Factorise $\mathrm{f}(x)$ completely. (4)

21.
$$\mathrm{f}(x) \equiv -4x^3 + ax^2 + 9x - 18,$$

where $a$ is a constant.

Given that $(x - 2)$ is a factor of $\mathrm{f}(x)$,

(a) find the value of $a$, (2)

(b) factorise $\mathrm{f}(x)$ completely, (3)

(c) find the remainder when $\mathrm{f}(x)$ is divided by $(2x - 1)$. (2)

22.
$$\mathrm{f}(x) \equiv 6x^3 + 3x^2 + Ax + B,$$

where $A$ and $B$ are constants.

Given that when $\mathrm{f}(x)$ is divided by $(x + 1)$ the remainder is 45,

(a) show that $B - A = 48$. (2)

Given also that $(2x + 1)$ is a factor of $\mathrm{f}(x)$,

(b) find the value of $A$ and the value of $B$. (4)

(c) Factorise $\mathrm{f}(x)$ fully. (3)

23.
$$\mathrm{f}(x) \equiv 6x^3 + 13x^2 - 4.$$

(a) Use the remainder theorem to find the remainder when $\mathrm{f}(x)$ is divided by $(2x + 3)$. (2)

(b) Use the factor theorem to show that $(x + 2)$ is a factor of $\mathrm{f}(x)$. (2)

(c) Factorise $\mathrm{f}(x)$ completely. (4)

24.
$$\mathrm{f}(x) = -6x^3 - 7x^2 + 40x + 21.$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $\mathrm{f}(x)$. (2)

(b) Factorise $\mathrm{f}(x)$ completely. (4)