Dr Oliver Mathematics
Advanced Subsidiary Paper 22: Mechanics
June 2022: Calculator
1 hour 15 minutes

The total number of marks available is 30.
You must write down all the stages in your working.
Inexact answers should be given to three significant figures unless otherwise stated.
(It goes with Paper 21: Statistics)

1. The point A is 1.8 m vertically above horizontal ground.

   At time $t = 0$, a small stone is projected vertically upwards with speed U ms^{-1} from the point A.

   At time $t = T$ seconds, the stone hits the ground.

   The speed of the stone as it hits the ground is 10 ms^{-1}.

   In an initial model of the motion of the stone as it moves from A to where it hits the ground,

   - the stone is modelled as a particle moving freely under gravity and
   - the acceleration due to gravity is modelled as having magnitude 10 ms^{-2}.

   Using the model,

   (a) find the value of U, (3)

   (b) find the value of T. (2)

   (c) Suggest one refinement, apart from including air resistance, that would make the model more realistic. (1)

   In reality the stone will not move freely under gravity and will be subject to air resistance.

   (d) Explain how this would affect your answer to part (a). (1)

2. A train travels along a straight horizontal track from station P to station Q.

   In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of 25 ms^{-1}.

   The train then travels at this constant speed of 25 ms^{-1} before finally moving with constant deceleration until it comes to rest at Q.

The time spent decelerating is four times the time spent accelerating.

The journey from P to Q takes 700 s.

Using the model,

(a) sketch a speed-time graph for the motion of the train between the two stations P and Q. (1)

The distance between the two stations is 15 km.

Using the model,

(b) show that the time spent accelerating by the train is 40 s, (3)

(c) find the acceleration, in ms^{-2}, of the train, (1)

(d) find the speed of the train 572 s after leaving P. (2)

(e) State one limitation of the model which could affect your answers to parts (b) and (c). (1)

3. A fixed point O lies on a straight line.

A particle P moves along the straight line.

At time t seconds, $t \geqslant 0$, the distance, s metres, of P from O is given by

$$s = \tfrac{1}{3}t^3 - \tfrac{5}{2}t^2 + 6t.$$

(a) Find the acceleration of P at each of the times when P is at instantaneous rest. (6)

(b) Find the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$. (3)

4. In Figure 1, a vertical rope PQ has its end Q attached to the top of a small lift cage.

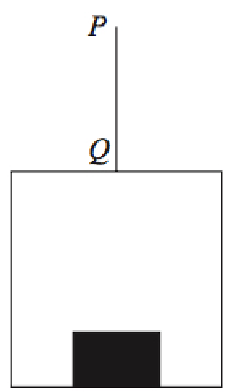

Figure 1: a vertical rope PQ

The lift cage has mass 40 kg and carries a block of mass 10 kg, as shown in the figure.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration 0.2 ms^{-2}.

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

(a) find the tension in the rope PQ, (3)

(b) find the magnitude of the force exerted on the block by the lift cage. (3)