

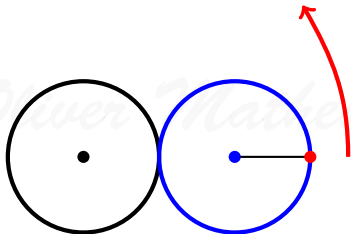
Dr Oliver Mathematics

A Polar Curve

April 3, 2018

Generating a curve

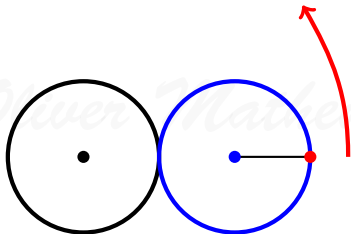
We will roll a blue circle around the outside of a black circle that has the same radius. As it does so the red dot, which is on the circumference of the blue circle, will rotate around the centre of the blue circle and it will trace out a path.



For every time the blue circle goes around the outside of the black circle how many times will the red do go around the centre of the blue circle?

Generating a curve

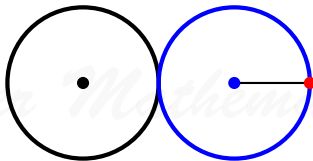
We will roll a blue circle around the outside of a black circle that has the same radius. As it does so the red dot, which is on the circumference of the blue circle, will rotate around the centre of the blue circle and it will trace out a path.



For every time the blue circle goes around the outside of the black circle how many times will the red dot go around the centre of the blue circle? It will make two revolutions.

Generating a curve

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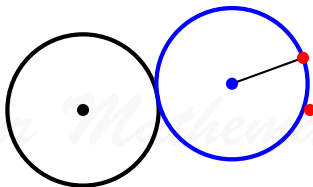


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Generating a curve

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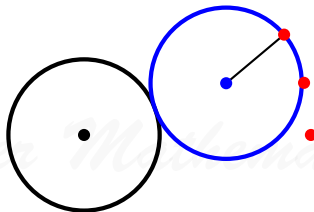


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Generating a curve

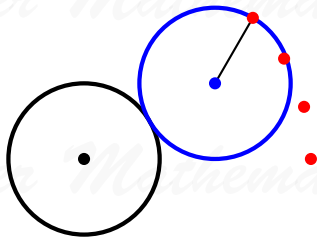
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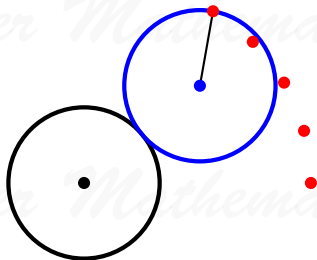
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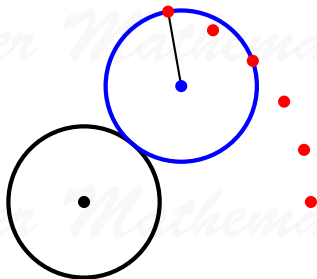
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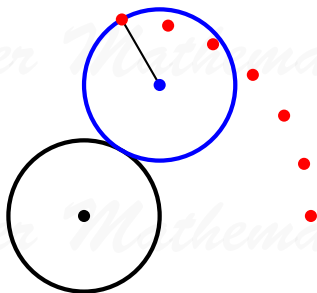
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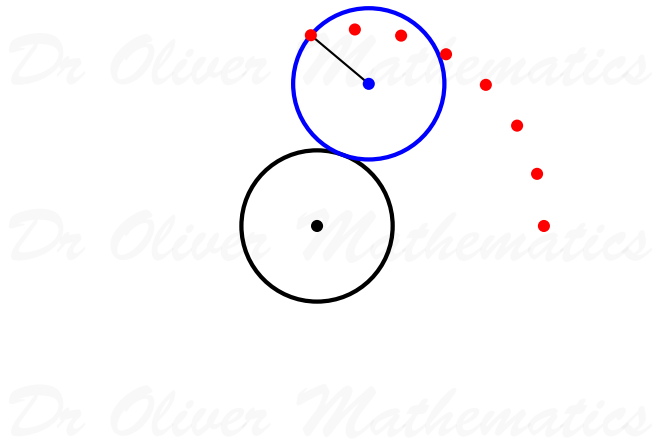
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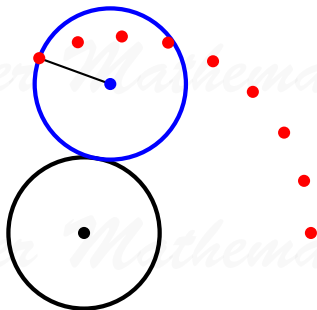
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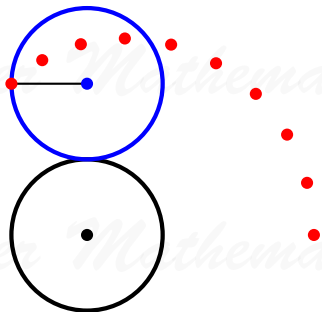
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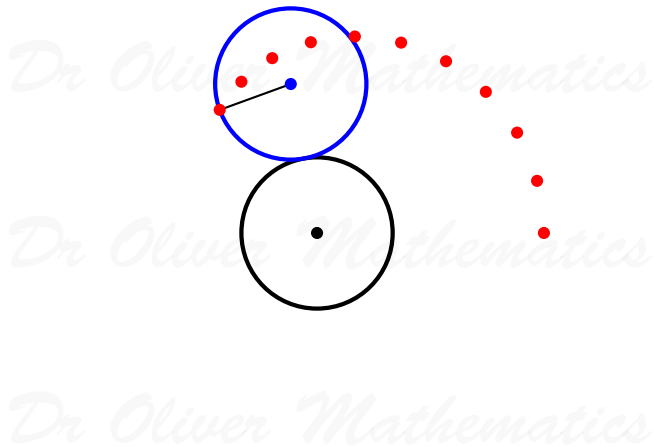
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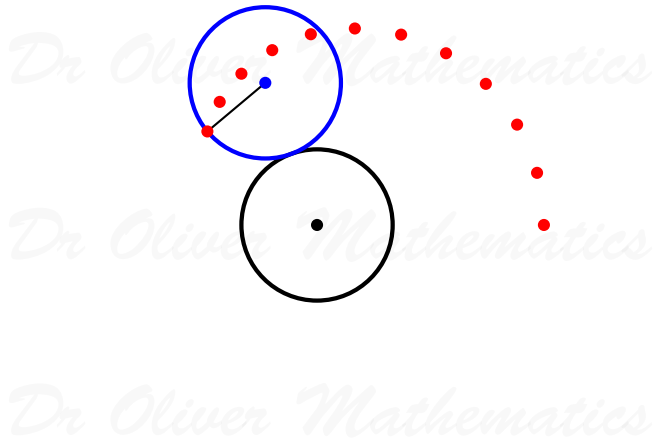
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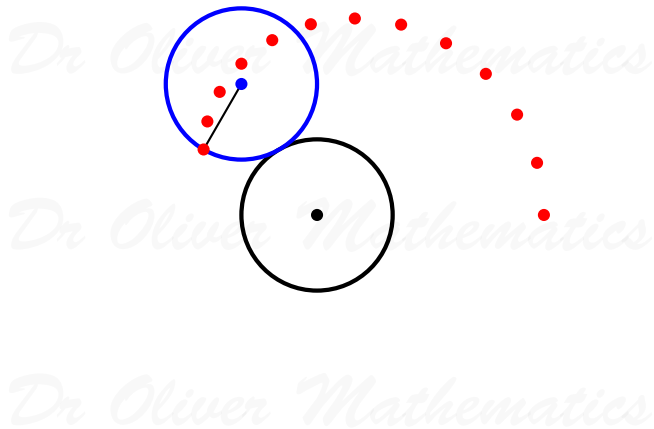
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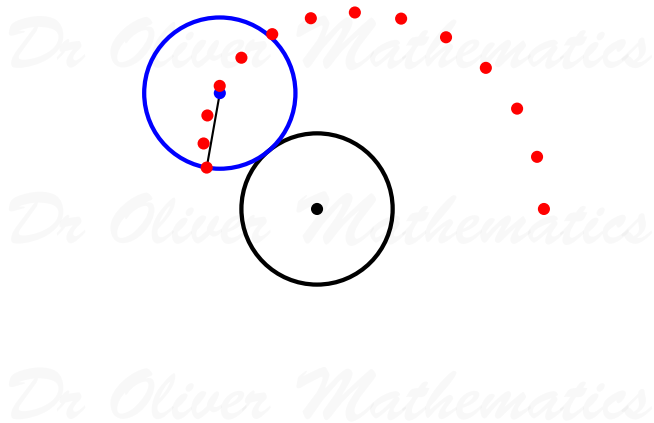
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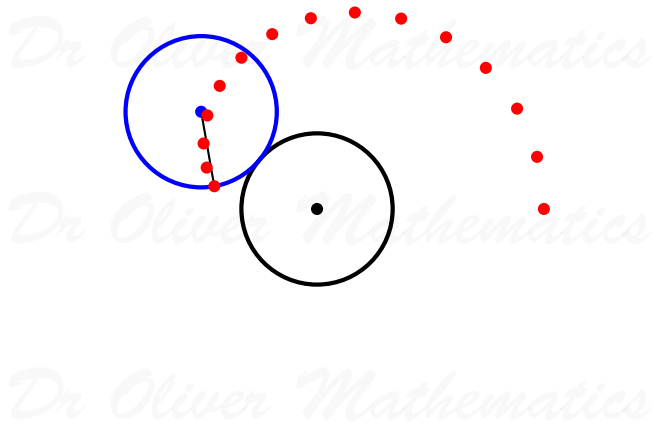
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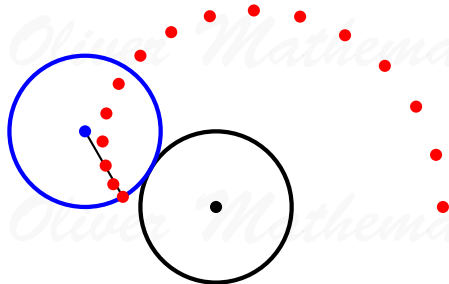
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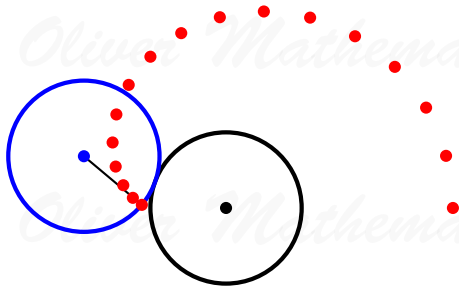
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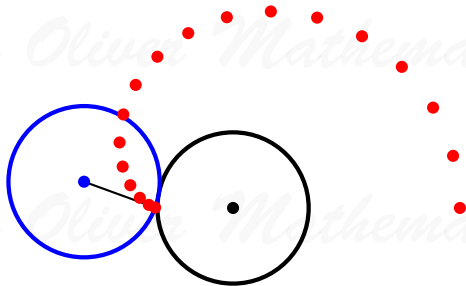
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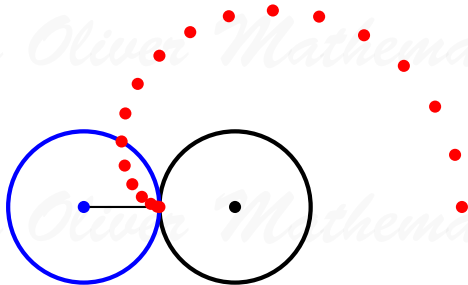
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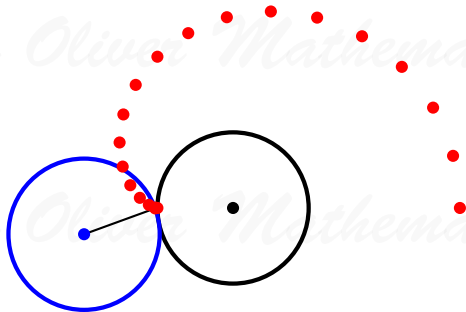
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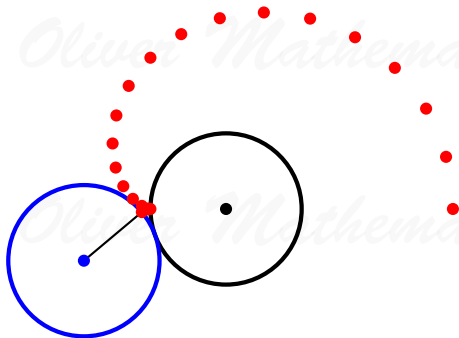
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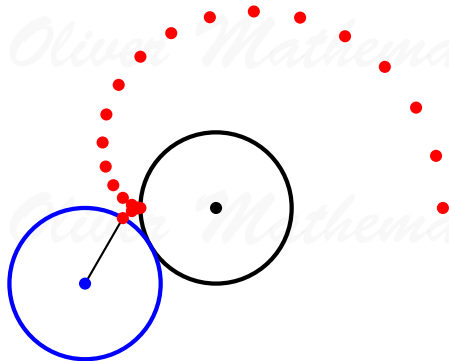
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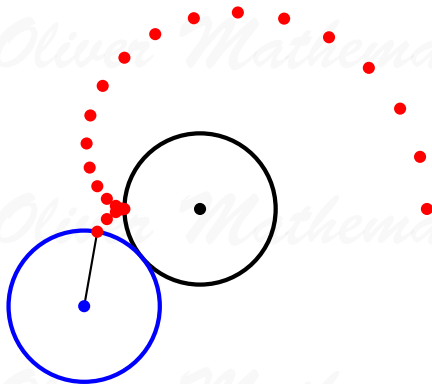
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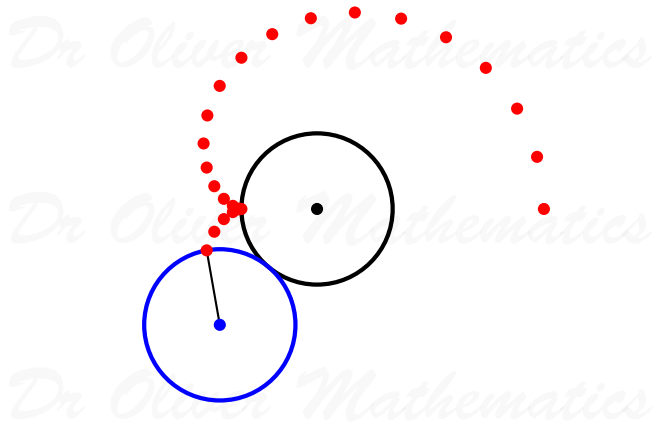
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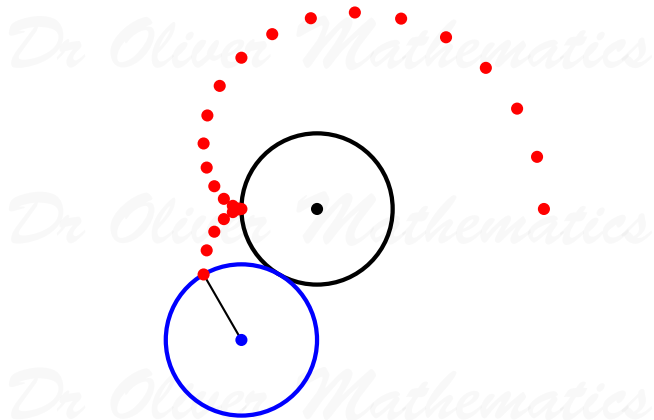
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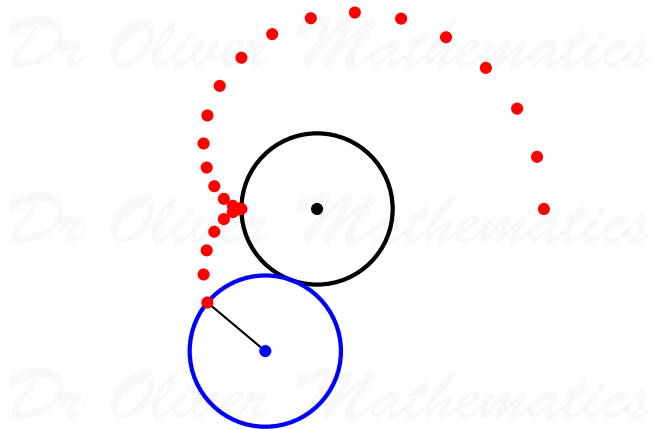
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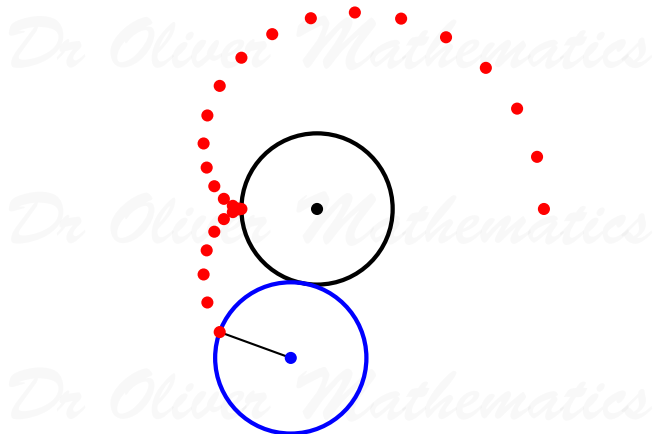
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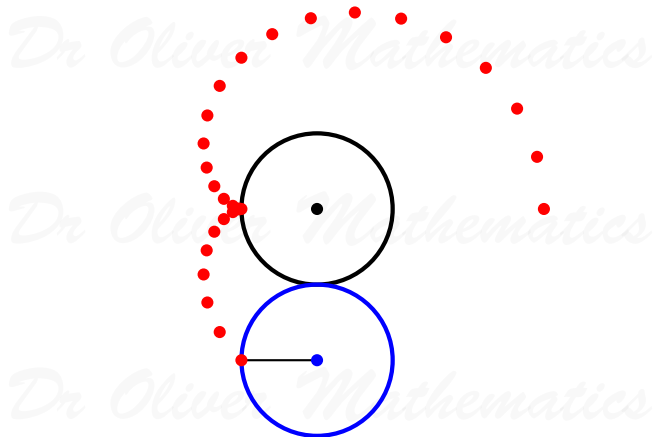
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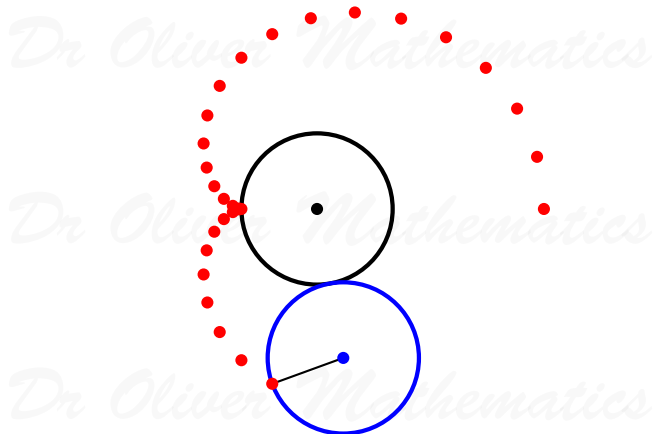
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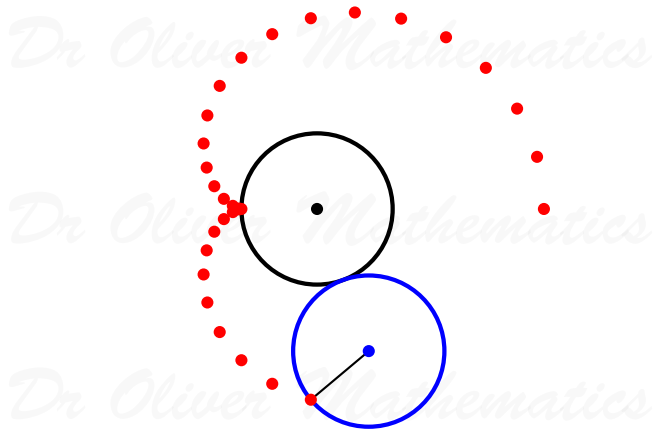
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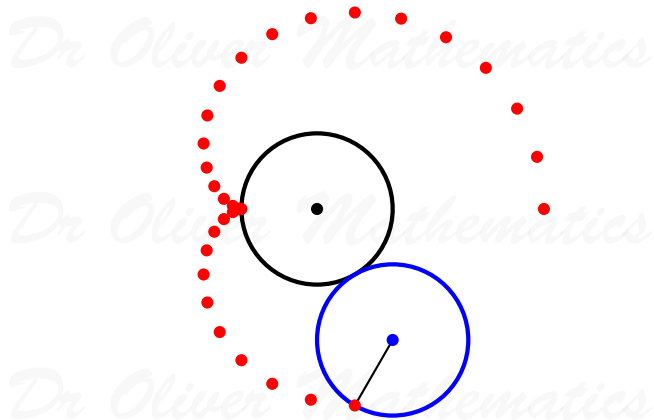
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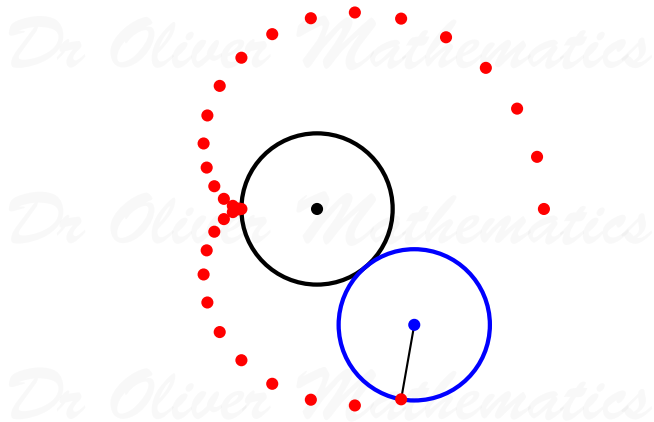
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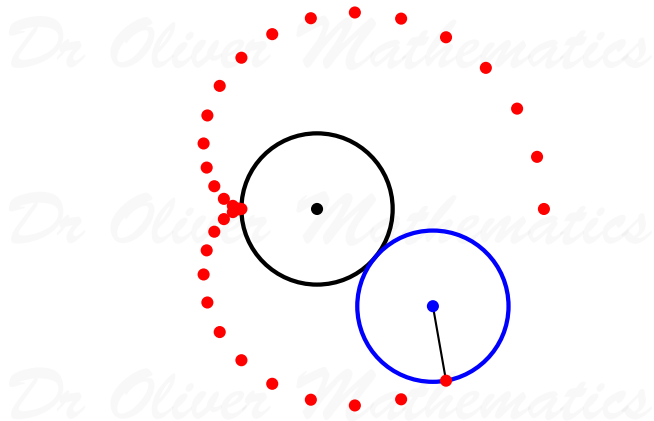
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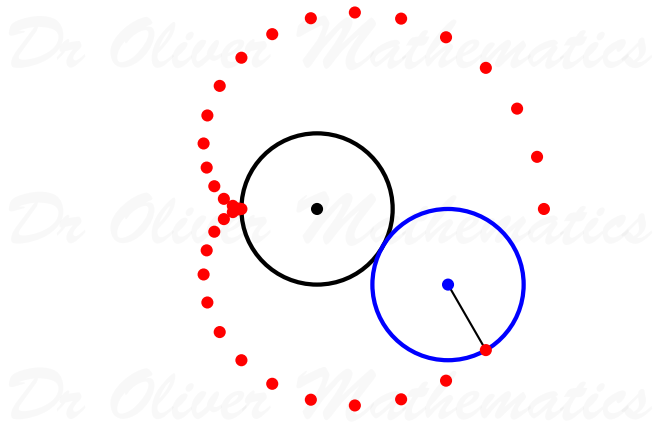
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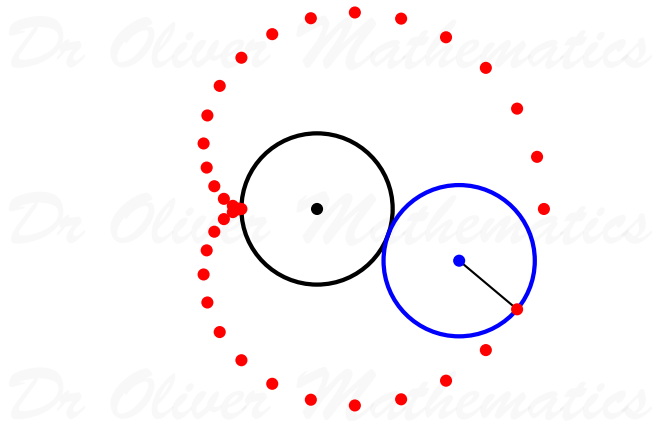
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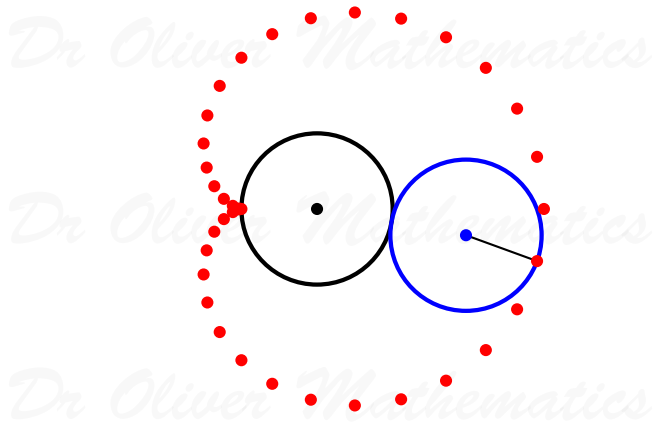
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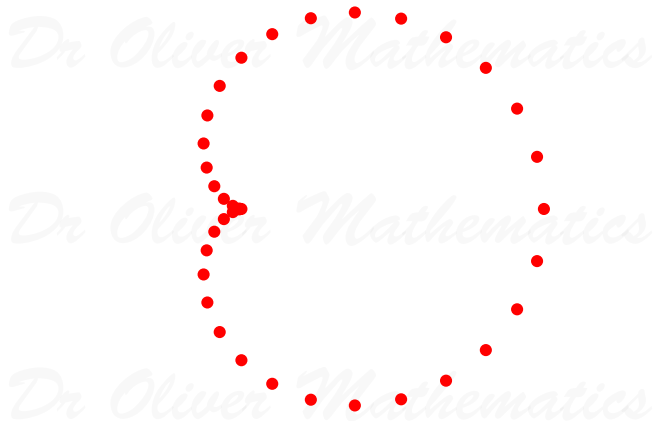
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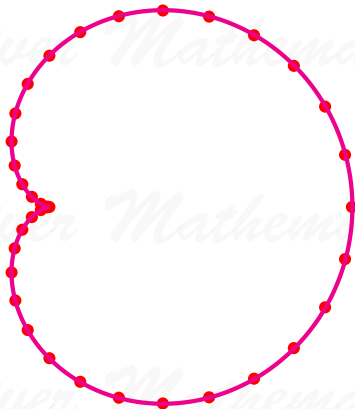
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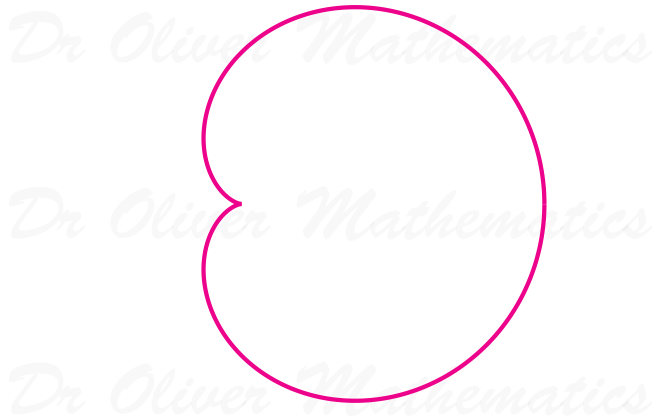
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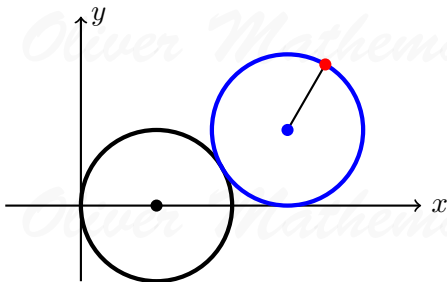


Generating a curve



The equation of the curve

Suppose that the black circle has centre $(0.5, 0)$ and a radius of 0.5 .



$$x = 0.5 + \cos \theta + 0.5 \cos 2\theta$$

and

$$y = 2 \sin \theta + 0.5 \sin 2\theta$$

The equation of the curve

$$\begin{aligned}x &= 0.5 + \cos \theta + 0.5 \cos 2\theta \\&= 0.5 + \cos \theta + 0.5 (2 \cos^2 \theta - 1) \\&= 0.5 + \cos \theta + \cos^2 \theta - 0.5 \\&= \cos \theta + \cos^2 \theta \\&= \cos \theta (1 + \cos \theta)\end{aligned}$$

and

$$\begin{aligned}y &= \sin \theta + 0.5 \sin 2\theta \\&= \sin \theta + 0.5 (2 \sin \theta \cos \theta) \\&= \sin \theta + \sin \theta \cos \theta \\&= \sin \theta (1 + \cos \theta)\end{aligned}$$

and hence the polar equation of the curve is $r = 1 + \cos \theta$