# Dr Oliver Mathematics
# AQA GCSE Mathematics
# 2020 June Paper 3: Calculator
# 1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. What does (1)

$$A \cup B$$

represent in

$$\mathrm{P}(A \cup B)?$$

Circle your answer.

$A$ or $B$ or both  $A$ but not $B$  not $A$ and not $B$  $A$ and $B$

**Solution**

$\underline{\underline{A \text{ or } B \text{ or both}}}$  $A$ but not $B$  not $A$ and not $B$  $A$ and $B$

2. Circle the equation of the line that is parallel to (1)

$$y = \tfrac{1}{2}x + 3.$$

$y = -2x$  $y = 2x$  $y = \frac{1}{2}x$  $y = -\frac{1}{2}x$

**Solution**

$y = -2x$  $y = 2x$  $\underline{\underline{y = \frac{1}{2}x}}$  $y = -\frac{1}{2}x$

3. Work out 320 as a percentage of 80. (1)
Circle your answer.

25%  75%  300%  400%

**Solution**

Well,

$$\frac{320}{80} \times 100\% = 400\%$$

so

$$25\% \quad 75\% \quad 300\% \quad \underline{\underline{400\%}}$$

4. A fair coin is spun four times. (1)
   Circle the probability of getting four Heads.

$$\tfrac{1}{2} \quad 2 \quad \tfrac{1}{8} \quad \tfrac{1}{16}$$

**Solution**

Well,

$$(\tfrac{1}{2})^4 = \tfrac{1}{16}$$

so

$$\tfrac{1}{2} \quad 2 \quad \tfrac{1}{8} \quad \underline{\underline{\tfrac{1}{16}}}$$

5. To the nearest 1 000, there are 18 000 people at a festival.
   (a) Write down the minimum possible number of people at the festival. (1)

**Solution**

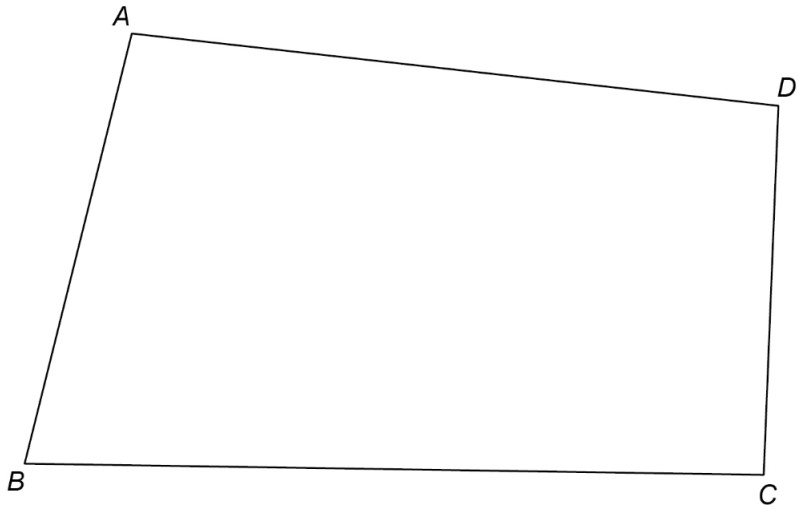
$\underline{17\,500}$.

   (b) Write down the maximum possible number of people at the festival. (1)

**Solution**

$\underline{18\,499}$.

6. $ABCD$ represents the plan of a field. (2)




There is a path across the field that

- starts at $B$ and
- is the same distance from $BA$ and $BC$.

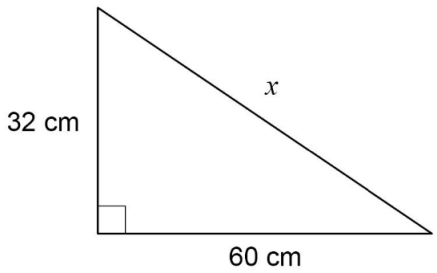
Using ruler and compasses, show the position of the path.

**Solution**

We have to produce the angle bisector.

- Open your compasses to a little than half the distance from $B$ to $A$.
- Insert your compass at $B$.
- Then draw a faint (but visible!) arc, starting at 12 o'clock and finishes up at 4 o'clock.
- Mark the point $E$ (where the arc crosses $BA$) and the point $F$ (where the arc crosses $BC$).
- Insert your compass at $E$.
- Then draw a faint (but visible!) arc, starting at $BE$ and finishes up $BC$.
- Insert your compass at $F$.
- Then draw a faint (but visible!) arc, starting at $BC$ and finishes up $BE$.
- Draw a straight line from $B$ until you cross the point where your arcs have intersected.

7. Use Pythagoras' theorem to work out the value of $x$. (3)



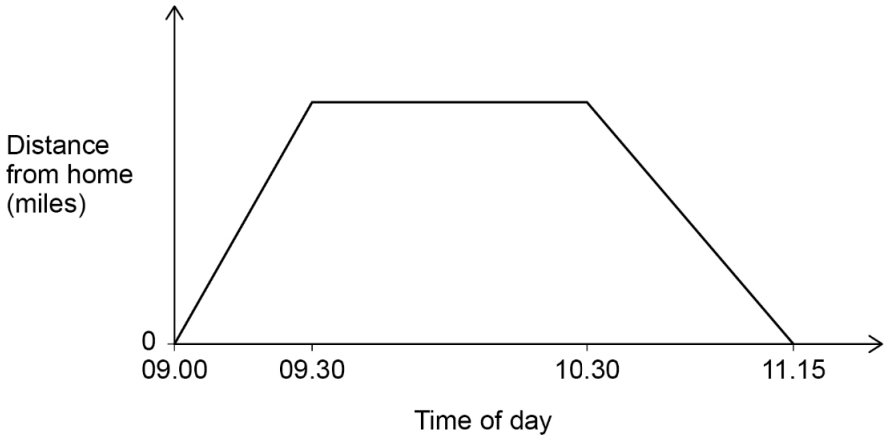

**Solution**

$$x^2 = 32^2 + 60^2 \Rightarrow x^2 = 1\,024 + 3\,600$$
$$\Rightarrow x^2 = 4\,624$$
$$\Rightarrow \underline{\underline{x = 68 \text{ cm}}}.$$

8. Chris visits a library. (3)

- He cycles to the library in half an hour at a speed of 12 miles per hour.
- He stays at the library for one hour.
- He then cycles home.

The sketch graph represents his visit.




Work out the speed, in miles per hour, at which Chris cycles home.

**Solution**

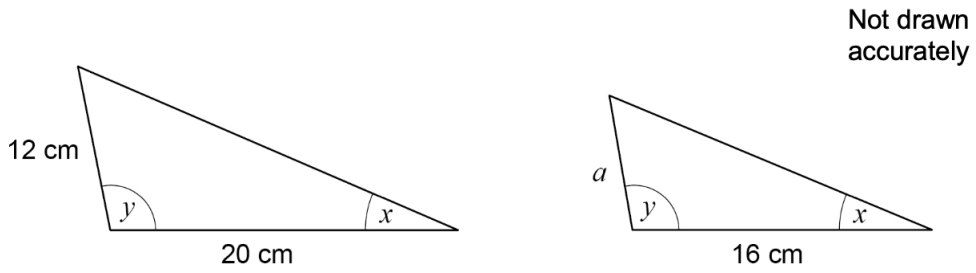
Well, the distance he cycles is

$$12 \times \tfrac{1}{2} = 6 \text{ miles}$$

and the distance he cycles back is

$$\frac{6}{\frac{3}{4}} = \underline{\underline{8 \text{ miles per hour}}}.$$

9. These two triangles are similar. (2)




Work out the value of $a$.

**Solution**

Similar triangles:

$$\frac{a}{16} = \frac{12}{20} \Rightarrow a = \tfrac{3}{5} \times 16$$
$$\Rightarrow \underline{\underline{a = 9\tfrac{3}{5} \text{ cm}}}.$$

10. Expand and simplify fully (2)

$$4(2c + 3) - (5c - 1).$$

**Solution**

Well,

$$\begin{aligned} 4(2c+3)-(5c-1) &= 8c+12-5c+1 \\ &= \underline{\underline{3c+13}}. \end{aligned}$$

11. A spinner can land on red, blue, or green. (3)

After 350 spins,

- relative frequency of red = 0.18 and
- relative frequency of blue = 0.62.

Work out the number of times the spinner landed on green.

**Solution**

Well,

$$\begin{aligned} \text{lands on green} &= 350 \times (1-0.18-0.62) \\ &= 350 \times 0.2 \\ &= \underline{\underline{70 \text{ times}}}. \end{aligned}$$

12. Here is some information about 26 houses. (3)
$a$, $b$, and $c$ are all different numbers.

| Number of bedrooms | Number of houses |
|---|---|
| 1 | 7 |
| 2 | $a$ |
| 3 | $b$ |
| 4 | $c$ |
| 5 | 8 |

The median number of bedrooms is 3.5.
Work out a possible set of values for $a$, $b$, and $c$.

**Solution**

E.g.,

| $a$ | $b$ | $c$ |
|---|---|---|
| 2 | 4 | 5 |
| 4 | 2 | 5 |

13. (a) Simplify (2)

$$\frac{25a}{8} \times \frac{2a}{5}.$$

Give your answer as a single fraction in its simplest form.

**Solution**

$$\frac{25a}{8} \times \frac{2a}{5} = \frac{5a}{4} \times \frac{a}{1}$$
$$= \underline{\underline{\tfrac{5}{4}a^2}}.$$

(b) Sofia is trying to simplify (1)

$$\frac{6c + 10}{2}.$$

Her method is

- divide $6c$ by 2 then
- then add 10.

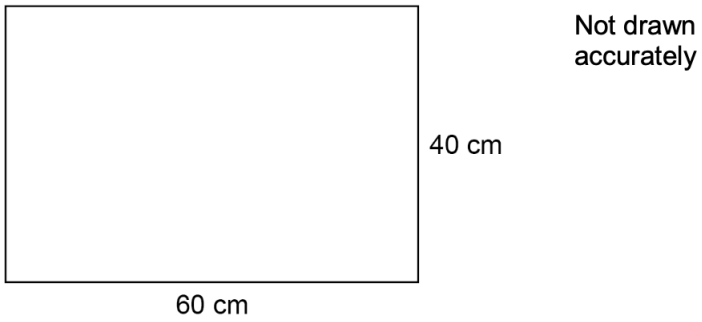
Evaluate her method.

**Solution**

E.g., she should $\underline{\underline{\text{added}}}$

$$\frac{10}{2} = 5.$$

14. A rectangle has length 60 cm and width 40 cm. (4)




- The length decreases by 15%.
- The width decreases by 10%.

Sue says, "The perimeter decreases by 25% because 15% + 10% is 25%."

Is she correct?
You **must** show calculations to support your answer.

**Solution**

Well,

$$\begin{aligned}\text{perimeter} &= 2([60 \times (1 - 0.15)] + [40 \times (1 - 0.1)]) \\ &= 2(51 + 36) \\ &= 174 \text{ cm}\end{aligned}$$

but

$$2(60 + 40) \times 0.75 = 150 \text{ cm}.$$

Hence, the perimeter does not decreases by 25% because 15% + 10% is 25%.

15. Solve (2)

$$4 > 11 - \frac{x}{3}.$$

**Solution**

$$\begin{aligned}4 > 11 - \frac{x}{3} &\Rightarrow \frac{x}{3} > 11 - 4 \\ &\Rightarrow \frac{x}{3} > 7 \\ &\Rightarrow x > 21.\end{aligned}$$

16. The number of goals scored by 20 players in a season is shown. (3)

| Number of goals | Frequency | Midpoint |
|---|---|---|
| 0 to 4 | 6 | |
| 5 to 9 | 11 | |
| 10 to 14 | 3 | |
| | Total = 20 | |

Work out an estimate of the mean number of goals per player.
Give your answer as a decimal.

**Solution**

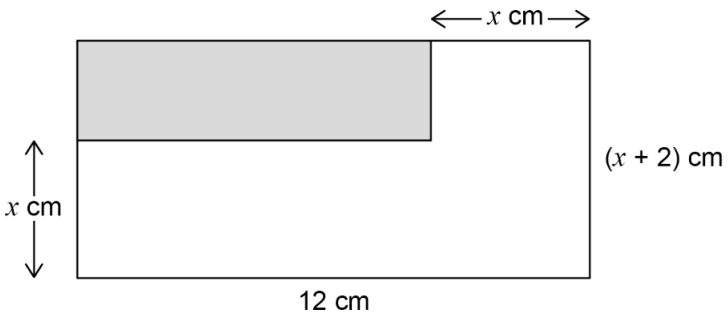
| Number of goals | Frequency | Midpoint | Frequency × Midpoint |
|---|---|---|---|
| 0 to 4 | 6 | 2 | $6 \times 2 = 12$ |
| 5 to 9 | 11 | 7 | $11 \times 7 = 77$ |
| 10 to 14 | 3 | 12 | $3 \times 12 = 36$ |
| | Total = 20 | | Total = 125 |

Now,

$$\begin{aligned} \text{mean} &= \frac{\sum fx}{\sum f} \\ &\approx \frac{125}{20} \\ &= \underline{\underline{6.25}}. \end{aligned}$$

17. Here are two rectangles. (4)

Not drawn accurately




The area of the shaded rectangle is $\frac{1}{4}$ the area of the large rectangle.

Work out the value of $x$.

**Solution**

Well, the shaded rectangle is

$$(12 - x) \times 2$$

and

$$\begin{aligned} \frac{2(12-x)}{12(x+2)} = \tfrac{1}{4} &\Rightarrow 8(12-x) = 12(x+2) \\ &\Rightarrow 96 - 8x = 12x + 24 \\ &\Rightarrow 72 = 20x \\ &\Rightarrow \underline{\underline{x = 3.6}}. \end{aligned}$$

18. The pressure in a tyre is 30 pounds per square inch. (3)

| |
|---|
| Use 1 pound = 0.45 kilograms<br>and<br>1 inch = 2.54 centimetres |

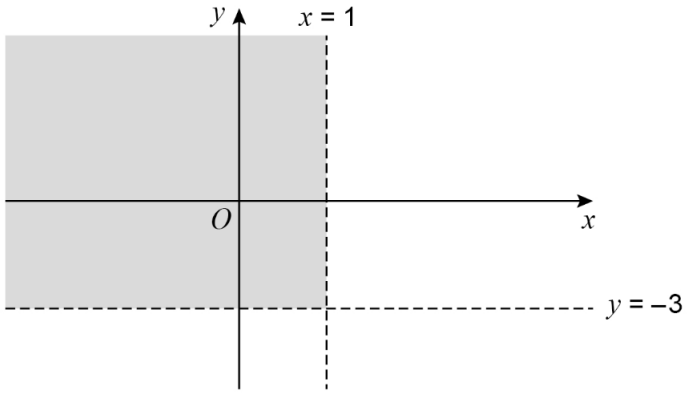
Convert the pressure into kilograms per square centimetre.

**Solution**

Well,

$$\begin{aligned}\text{pressure} &= \frac{30 \text{ pounds}}{1 \text{ inch}^2} \\ &= \frac{30 \times 0.45\text{kg}}{2.54^2 \text{ cm}^2} \\ &= 2.092\,504\,185 \text{ (FCD)} \\ &= \underline{\underline{2.09 \text{ kilograms per square centimetre (3 sf)}}}.\end{aligned}$$

19. The sketch shows the lines $x = 1$ and $y = -3$. (1)




Which pair of inequalities describes the shaded region?
Tick **one** box.

- ☐ $x < 1$ and $y < -3$
- ☐ $x < 1$ and $y > -3$
- ☐ $x > 1$ and $y > -3$
- ☐ $x > 1$ and $y < -3$

**Solution**

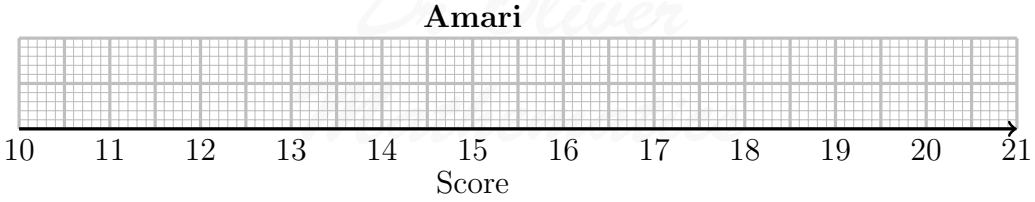
Tick $x < 1$ and $y > -3$.

20. Amari and Ben each play a game.

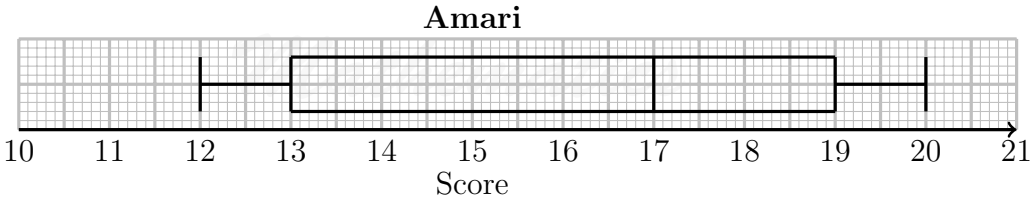
(a) Here is some information about Amari's scores. (2)

- Lowest: 12.
- Highest: 20.
- Lower quartile: 13.
- Upper quartile: 19
- Median: 17.

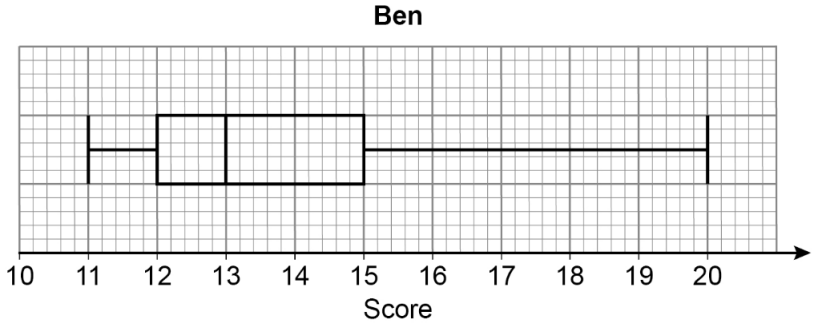
Draw a box plot to represent his scores.




**Solution**




(b) This box plot represents Ben's scores. (2)



Who had more consistent scores, Amari or Ben?
Work out the interquartile ranges to support your answer.

**Solution**

In terms of IQR, Amari's is

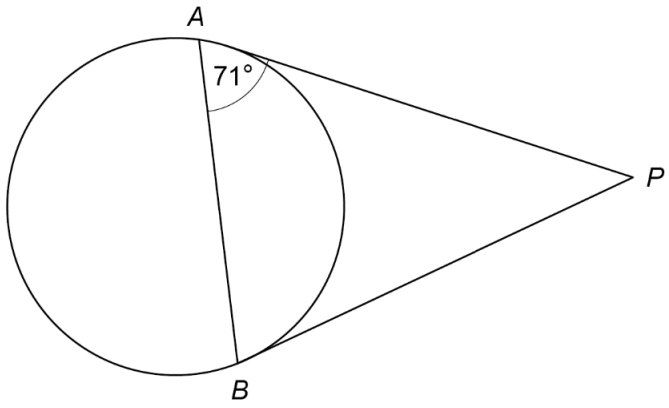
$$19 - 13 = 6$$

and Ben's is

$$15 - 12 = 3.$$

Therefore, Ben had more consistent scores as the IQR is lower.

21. (a) $A$ and $B$ are points on a circle. (2)
$PA$ and $PB$ are tangents.



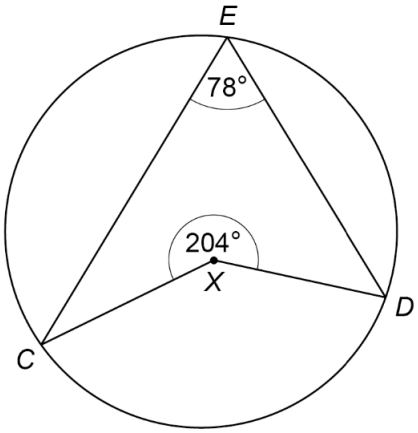

Not drawn accurately

Work out the size of angle $APB$.

**Solution**

$$\begin{aligned} \angle APB &= 180 - (2 \times 71) \\ &= 180 - 142 \\ &= 38^\circ. \end{aligned}$$

(b) $C$, $D$, and $E$ are points on a different circle. (2)




Not drawn accurately

Is $X$ the centre of the circle?
Tick a box.

☐ Yes ☐ No

Show working to support your answer.

**Solution**
Well,

$$\begin{aligned} \text{minor } CXD &= 2 \times 78 \\ &= 156^\circ \end{aligned}$$

and

$$\begin{aligned} \text{minor } CXD + \text{major } CXD &= 156 + 204 \\ &= 360^\circ; \end{aligned}$$

so, yes it is.

22. Visitors to a museum buy a child ticket or an adult ticket. (4)
Here is some information about two groups of visitors.

| | |
|---|---|
| Group $X$ | 250 visitors, including 120 children |
| Group $Y$ | number of children : number of adults = 17 : 15 |

One visitor from each group is picked at random.

Is this statement correct?

| Probability of picking two children > probability of picking two adults. |
|---|

You **must** show your working.

**Solution**

Well, Group $X$ has 130 adults and 120 children whereas Group $Y$ is made up of

$$\begin{aligned} \text{adults}: &\quad \frac{15}{17+15} = \tfrac{15}{32} \\ \text{chidren}: &\quad \frac{17}{17+15} = \tfrac{17}{32}. \end{aligned}$$

Now,

$$\begin{aligned} \text{P(two children)} &= \tfrac{120}{250} \times \tfrac{17}{32} \\ &= \tfrac{51}{200} \\ &= 0.255 \end{aligned}$$

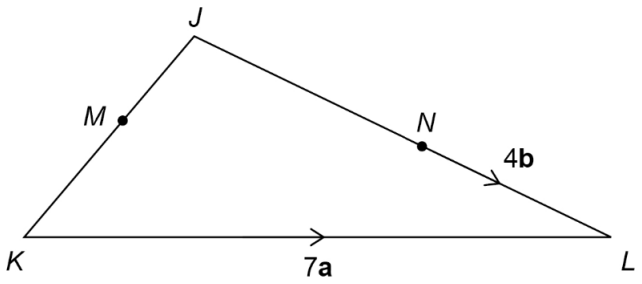
and

$$\begin{aligned} \text{P(two adults)} &= \tfrac{130}{250} \times \tfrac{15}{32} \\ &= \tfrac{39}{160} \\ &= 0.243\,75; \end{aligned}$$

hence, the statement is correct.

23. In triangle $JKL$, (3)

- $M$ is the midpoint of $JK$,
- $JN : NL = 3 : 2$,
- $\overrightarrow{KL} = 7\mathbf{a}$, and
- $\overrightarrow{NL} = 4\mathbf{b}$.




Not drawn accurately

Work out $\overrightarrow{JM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Give your answer in its simplest form.

**Solution**

Well,

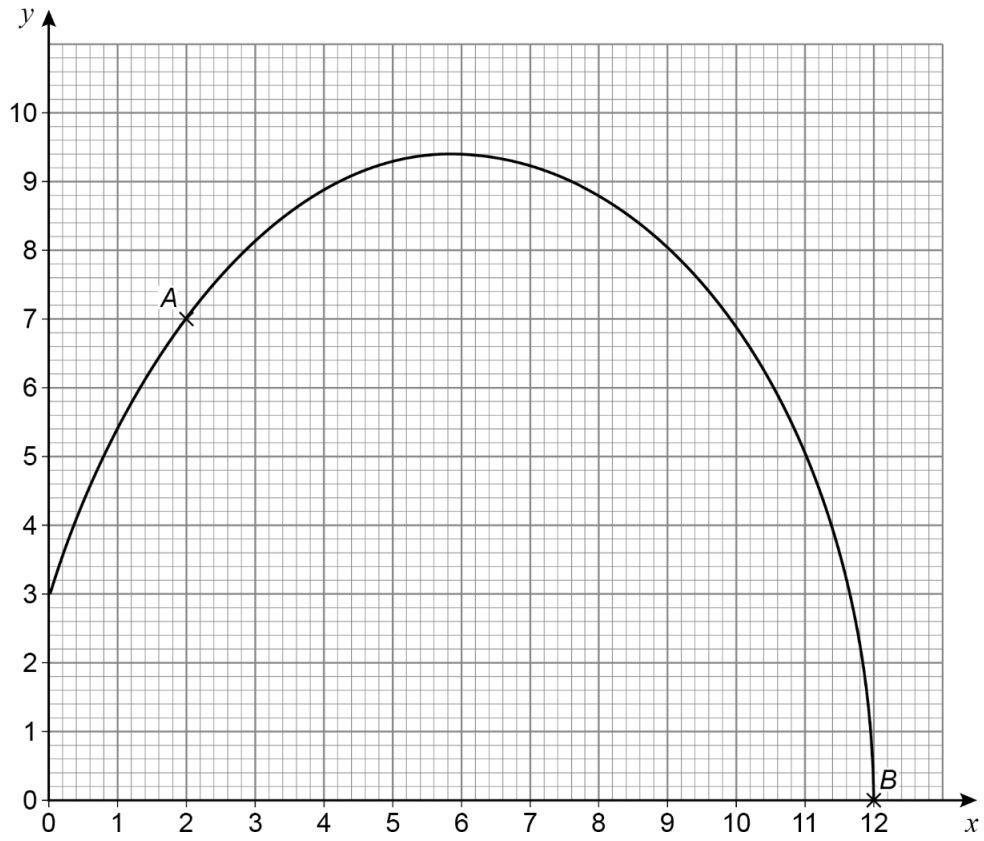
$$\begin{aligned} JN : NL = 3 : 2 &\Rightarrow \overrightarrow{JN} : \overrightarrow{NL} = 3\mathbf{b} : 2\mathbf{b} \\ &\Rightarrow \overrightarrow{JN} : \overrightarrow{NL} = 6\mathbf{b} : 4\mathbf{b} \end{aligned}$$

and

$$\begin{aligned} \overrightarrow{JL} &= \overrightarrow{JN} + \overrightarrow{NL} \\ &= 6\mathbf{b} + 4\mathbf{b} \\ &= 10\mathbf{b}. \end{aligned}$$

$$\begin{aligned} \overrightarrow{JM} &= \overrightarrow{JL} + \overrightarrow{LK} + \overrightarrow{KM} \\ &= \overrightarrow{JL} - \overrightarrow{KL} - \overrightarrow{MK} \\ &= \overrightarrow{JL} - \overrightarrow{KL} - \tfrac{1}{2}\overrightarrow{JK} \\ &= 10\mathbf{b} - 7\mathbf{a} - \tfrac{1}{2}(10\mathbf{b} - 7\mathbf{a}) \\ &= 10\mathbf{b} - 7\mathbf{a} - 5\mathbf{b} + \tfrac{7}{2}\mathbf{a} \\ &= \underline{\underline{5\mathbf{b} - \tfrac{7}{2}\mathbf{a}}}. \end{aligned}$$

24. $A$ and $B$ are points on a curve.
$A$ is $(2, 7)$ and $B$ is $(12, 0)$.




(a) Work out the instantaneous rate of change of $y$ with respect to $x$ at point $A$. (2)

**Solution**

The tangent at $(2, 7)$ goes through $(0, 4)$ and $(4, 9.4)$ so

$$\begin{aligned} m &= \frac{9.4 - 4}{4 - 0} \\ &= \frac{5.4}{4} \\ &= \underline{\underline{1.35}}. \end{aligned}$$

(b) The average rate of change of $y$ with respect to $x$ between points $A$ and $B$ is worked out. (1)

Which statement is correct?
Tick **one** box.

☐ It is positive.

☐ It is zero.

☐ It is negative.

☐ You cannot tell if it is positive or negative.

**Solution**

Tick It is negative.

25. The equation of a circle is (1)

$$x^2 + y^2 = 9.$$

Work out the length of the diameter.
Circle your answer.

3 6 9 18

**Solution**

Well, the radius is

$$\sqrt{9} = 3$$

and we double to find the diameter:

$$2 \times 3 = 6.$$

So

3 6 9 18

26. Prove algebraically that (3)

$$3.4\dot{7} = \tfrac{313}{90}.$$

**Solution**

Let

$$x = 3.4\dot{7}.$$

Then

$$10x = 34.\dot{7} \quad (1)$$
$$100x = 347.\dot{7} \quad (2).$$

Do (2) − (1):

$$90x = 313 \Rightarrow \underline{\underline{x = \tfrac{313}{90}}},$$

as required.

27. The equation of a curve is (1)

$$y = (x-1)^2 - 6.$$

Circle the coordinates of the turning point.

$$(-1,-6) \quad (1,6) \quad (-1,6) \quad (1,-6)$$

**Solution**

$$(-1,-6) \quad (1,6) \quad (-1,6) \quad \underline{\underline{(1,-6)}}$$

28. Line $A$ has equation (4)

$$y = 4x - 1.$$

Line $B$ is

- perpendicular to line $A$ and
- passes through the point $(8,5)$.

Work out the coordinates of the point where line $B$ intersects the $x$-axis.

**Solution**

Well, the line $B$ has gradient $-\frac{1}{4}$ and the equation is

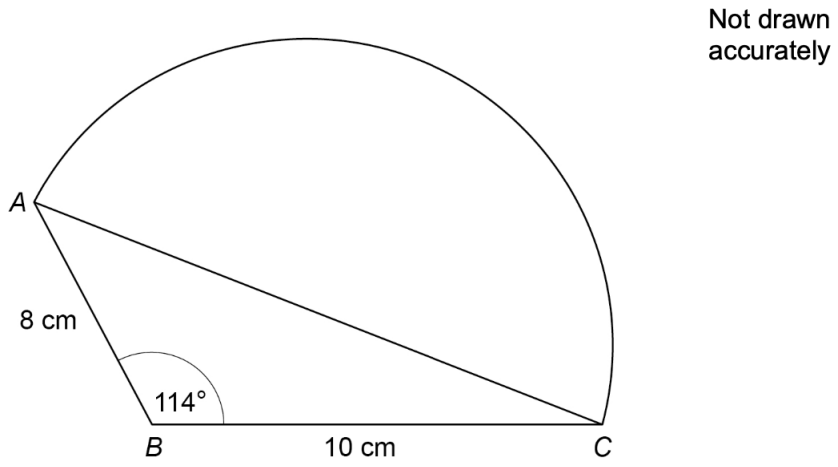
$$y - 5 = -\tfrac{1}{4}(x - 8).$$

Now,

$$\begin{aligned} y = 0 \Rightarrow -5 &= -\tfrac{1}{4}(x - 8) \\ \Rightarrow 20 &= x - 8 \qquad\qquad \Rightarrow x = 28; \end{aligned}$$

hence, the coordinates of the point where line $B$ intersects the $x$-axis are $\underline{\underline{(28, 0)}}$.

29. A shape is made by joining triangle $ABC$ to a semicircle with diameter $AC$. (5)




Work out the **total** area of the shape.

**Solution**

Let $D$ be the midpoint of $AC$. Now,

$$\begin{aligned} & AC^2 = AB^2 + BC^2 - 2 \times AB \times BC \times \cos ABC \\ \Rightarrow \quad & AC^2 = 8^2 + 10^2 - 2 \times 8 \times 10 \times \cos 114^\circ \\ \Rightarrow \quad & AC^2 = 229.077\,862\,2 \text{ (FCD)} \\ \Rightarrow \quad & AC = 15.135\,318\,39 \text{ (FCD)} \\ \Rightarrow \quad & AD = 7.567\,659\,197 \text{ (FCD)}. \end{aligned}$$

Finally,

$$\begin{aligned}\text{area of the shape} &= \text{triangle} + \text{semi-circle}\\ &= (\tfrac{1}{2} \times 8 \times 10 \times \sin 114^\circ) + (\tfrac{1}{2} \times \pi \times 7.567\ldots^2)\\ &= 126.500\,484\,7 \text{ (FCD)}\\ &= \underline{\underline{127 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

30. (4)

$$\mathrm{f}(x) = \tfrac{1}{2}x \text{ and } \mathrm{g}(x) = x - x^2.$$

Solve

$$\mathrm{f}^{-1}(x) = \mathrm{g}\,\mathrm{f}(x).$$

**Solution**

Now,

$$\mathrm{f}(x) = \tfrac{1}{2}x \Rightarrow \mathrm{f}^{-1}(x) = 2x$$

and

$$\begin{aligned}\mathrm{g}\,\mathrm{f}(x) &= \mathrm{g}(\mathrm{f}(x))\\ &= \mathrm{g}(\tfrac{1}{2}x)\\ &= \tfrac{1}{2}x - (\tfrac{1}{2}x)^2\\ &= \tfrac{1}{2}x - \tfrac{1}{4}x^2.\end{aligned}$$

Next,

$$\begin{aligned}\mathrm{f}^{-1}(x) = \mathrm{g}\,\mathrm{f}(x) &\Rightarrow 2x = \tfrac{1}{2}x - \tfrac{1}{4}x^2\\ &\Rightarrow \tfrac{1}{4}x^2 + \tfrac{3}{2}x = 0\\ &\Rightarrow x^2 + 6x = 0\\ &\Rightarrow x(x+6) = 0\\ &\Rightarrow \underline{\underline{x = -6 \text{ or } x = 0}}.\end{aligned}$$