

Dr Oliver Mathematics

Worked Examples

Proportion 2

From: Examples in Mathematics for GCSE (Higher Tier)

1. The force, F , acting on a mass moving in a circle varies directly as the square of its speed (v) and inversely as the radius of the circle (r) in which it moves.

Find the increase (or decrease) in the force F if

- (a) the speed is doubled and the radius is halved,

Solution

Well,

$$F \propto \frac{v^2}{r} \Rightarrow F = \frac{kv^2}{r},$$

for some constant k .

‘The speed is doubled and the radius is halved’:

$$\begin{aligned} F &= \frac{k(2v)^2}{\frac{1}{2}r} \\ &= \frac{k(4v^2)}{\frac{1}{2}r} \\ &= 8 \cdot \frac{kv^2}{r}; \end{aligned}$$

hence, F goes up by a factor of 8: the force is $8F$.

- (b) the speed is halved and the radius is doubled.

Solution

‘The speed is halved and the radius is doubled’:

$$\begin{aligned} F &= \frac{k(\frac{1}{2}v)^2}{2r} \\ &= \frac{k(\frac{1}{4}v^2)}{2r} \\ &= \frac{1}{8} \cdot \frac{kv^2}{r}; \end{aligned}$$

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hence, F goes down by a factor of 8: the force is $\frac{1}{8}F$.

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