Dr Oliver Mathematics
Worked Examples
Radius of a Circle 2

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1. A square, with side length x cm, is drawn.

   A circle, with radius r cm is drawn as follows: the circle is tangent to the bottom and left sides of the square and passes through its top-right corner, as shown in Figure 2.

   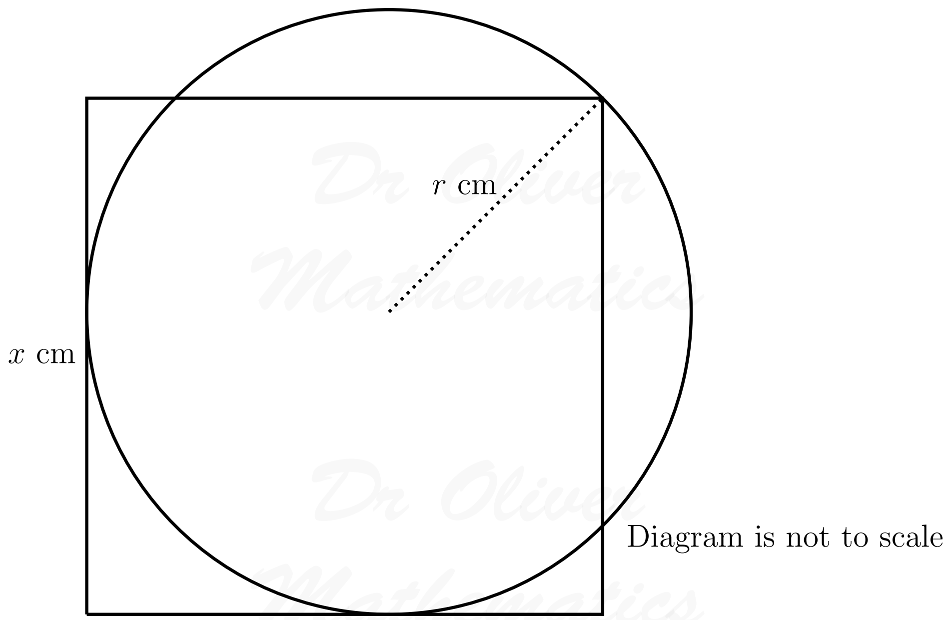

   Figure 1: a square and a circle

   Find the length of the radius of the circle.

   Solution

   Let O be the centre of the circle. We add in the new dimensions:

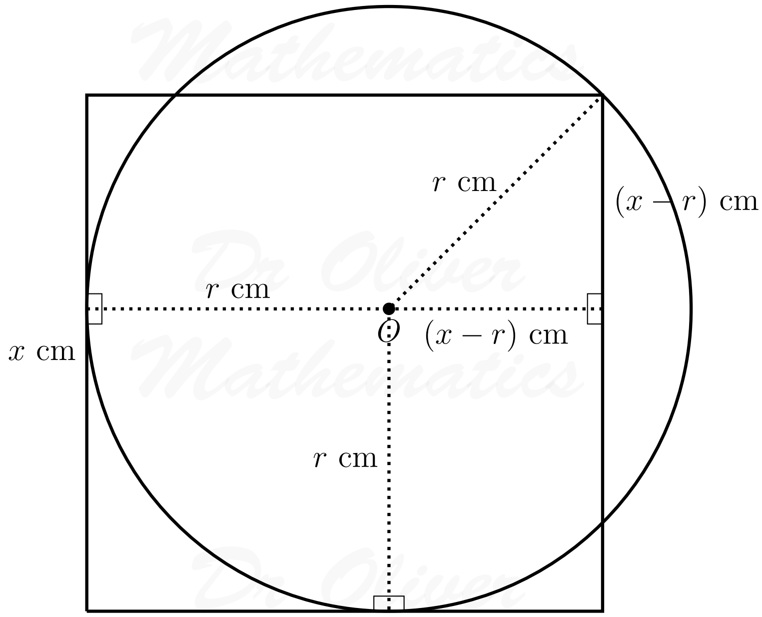

Figure 2: with the dimension added on

Pythagoras' Theorem:

$$\begin{aligned}(x-r)^2+(x-r)^2=r^2 &\Rightarrow 2(x-r)^2=r^2\\ &\Rightarrow 2(x^2-2rx+r^2)=r^2\\ &\Rightarrow 2x^2-4rx+2r^2=r^2\\ &\Rightarrow 2x^2-4rx+r^2=0\\ &\Rightarrow r^2-4rx=-2x^2\\ &\Rightarrow r^2-4rx+(2x)^2=-2x^2+(2x)^2\\ &\Rightarrow (r-2x)^2=-2x^2+4x^2\\ &\Rightarrow (r-2x)^2=2x^2\\ &\Rightarrow r-2x=\pm x\sqrt{2}\\ &\Rightarrow r=2x\pm x\sqrt{2}\\ &\Rightarrow r=(2\pm\sqrt{2})x.\end{aligned}$$

$r=(2+\sqrt{2})x$? Look at picture: $(x-r)$ is a *length* which means it is bigger than zero:

$$x-r>0\Rightarrow x>r.$$

So $r\neq(2+\sqrt{2})x$. Hence, we need the other solution:

$$\underline{\underline{r=(2-\sqrt{2})x}}.$$