

**Dr Oliver Mathematics**  
**Mathematics**  
**Coordinates Part 2**  
**Past Examination Questions**

This booklet consists of 23 questions across a variety of examination topics. The total number of marks available is 178.

1. The points  $A$  and  $B$  have coordinates  $(5, -1)$  and  $(13, 11)$  respectively.  
(a) Find the coordinates of the mid-point of  $AB$ . (2)

Given that  $AB$  is a diameter of the circle  $C$ ,

- (b) find an equation for  $C$ . (4)
2. The circle  $C$ , with centre at the point  $A$ , has equation  $x^2 + y^2 - 10x + 9 = 0$ . Find  
(a) the coordinates of  $A$ , (2)  
(b) the radius of  $C$ , (2)  
(c) the coordinates of the points at which  $C$  crosses the  $x$ -axis. (2)

Given that the line  $l$  with gradient  $\frac{7}{2}$  is a tangent to  $C$ , and that  $l$  touches  $C$  at the point  $T$ ,

- (d) find an equation of the line which passes through  $A$  and  $T$ . (3)
3. In Figure 1,  $A(4, 0)$  and  $B(3, 5)$  are the end points of a diameter of the circle  $C$ .

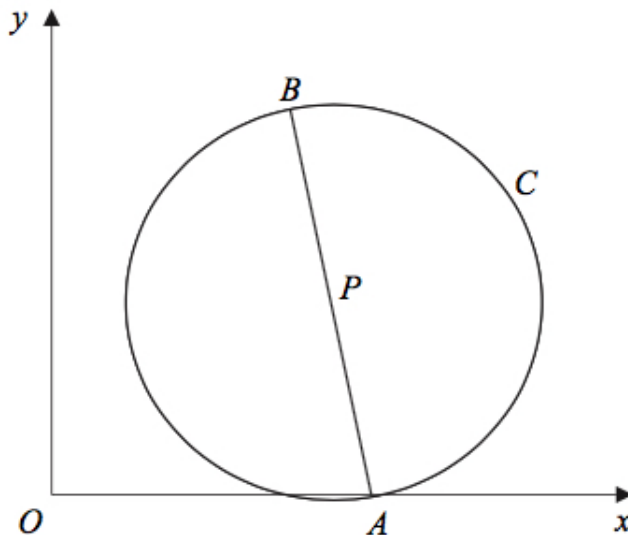


Figure 1: a circle  $C$

Find

- (a) the exact length of  $AB$ , (2)
  - (b) the coordinates of the midpoint  $P$  of  $AB$ , (2)
  - (c) an equation for the circle  $C$ . (3)
4. The line  $y = 3x - 4$  is a tangent to the circle  $C$ , touching  $C$  at the point  $P(2, 2)$ , as shown in Figure 2.

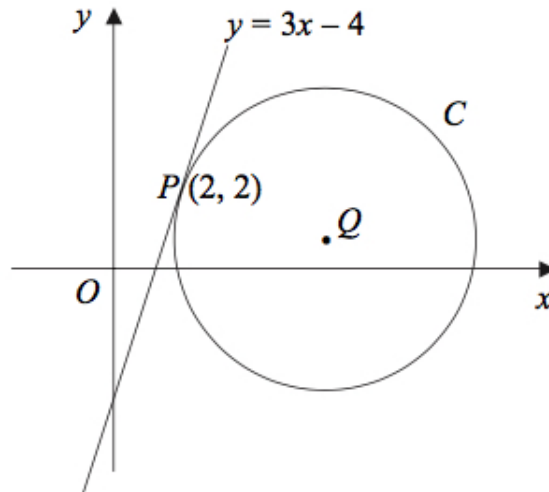


Figure 2:  $y = 3x - 4$  is a tangent to the circle  $C$

The point  $Q$  is the centre of  $C$ .

- (a) Find an equation of the straight line through  $P$  and  $Q$ . (3)
- Given that  $Q$  lies on the line  $y = 1$ ,
- (b) show that the  $x$ -coordinate of  $Q$  is 5, (1)
  - (c) find an equation for  $C$ . (4)
5. The line joining the points  $(-1, 4)$  and  $(3, 6)$  is a diameter of the circle  $C$ . Find an equation for  $C$ . (6)
6. The points  $A$  and  $B$  lie on a circle with centre  $P$ , as shown in Figure 3.

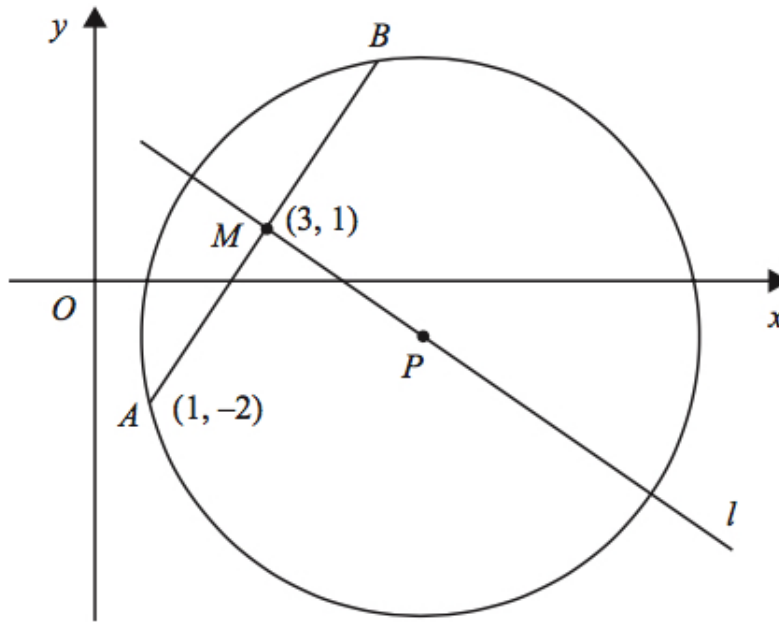


Figure 3: the points  $A$  and  $B$

The point  $A$  has coordinates  $(1, -2)$  and the mid-point  $M$  of  $AB$  has coordinates  $(3, 1)$ . The line  $l$  passes through the points  $M$  and  $P$ .

- (a) Find an equation for  $l$ . (4)

Given that the  $x$ -coordinate of  $P$  is 6,

- (b) use your answer to part (a) to show that the  $y$ -coordinate of  $P$  is  $-1$ , (1)

- (c) find an equation for the circle. (4)

7. A circle  $C$  has centre  $M(6, 4)$  and radius 3. Write down the equation of the circle in the form (2)

$$(x - a)^2 + (y - b)^2 = r^2.$$

8. The circle  $C$  has centre  $(3, 1)$  and passes through the point  $P(8, 3)$ .

- (a) Find an equation for  $C$ . (4)

- (b) Find an equation for the tangent to  $C$  at  $P$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$ , and  $c$  are integers. (5)

9. The points  $P(-3, 2)$ ,  $Q(9, 10)$ , and  $R(a, 4)$  lie on the circle  $C$ , as shown in Figure 4.

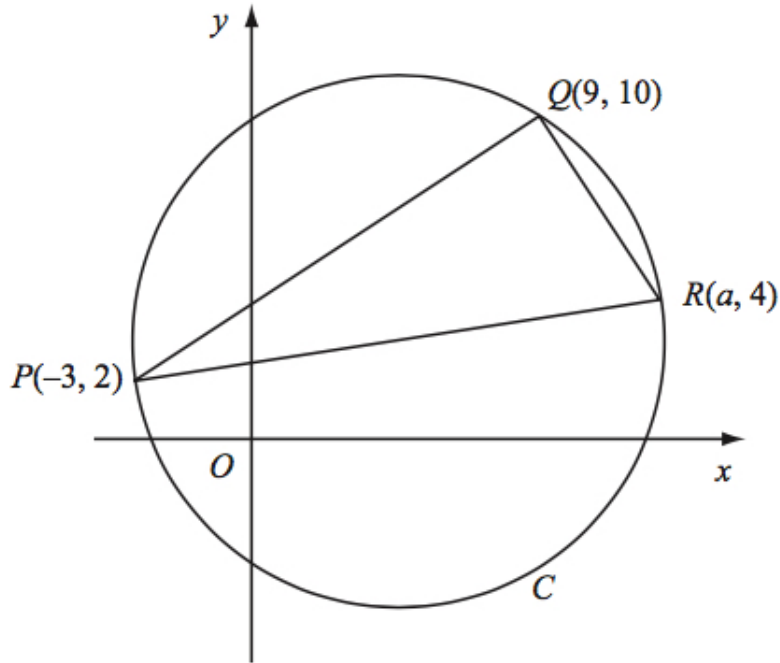


Figure 4: the points  $P(-3, 2)$ ,  $Q(9, 10)$ , and  $R(a, 4)$

Given that  $PR$  is a diameter of  $C$ ,

(a) show that  $a = 13$ , (3)

(b) find an equation for  $C$ . (5)

10. The circle  $C$  has equation

$$x^2 + y^2 - 6x + 4y = 12.$$

(a) Find the centre and the radius of  $C$ . (5)

The point  $P(-1, 1)$  and the point  $Q(7, -5)$  both lie on  $C$ .

(b) Show that  $PQ$  is a diameter of  $C$ . (2)

The point  $R$  lies on the positive  $y$ -axis and the angle  $PRQ = 90^\circ$ .

(c) Find the coordinates of  $R$ . (4)

11. Figure 5 shows a sketch of the circle  $C$  with centre  $N$  and equation

$$(x - 2)^2 + (y + 1)^2 = \frac{169}{4}.$$

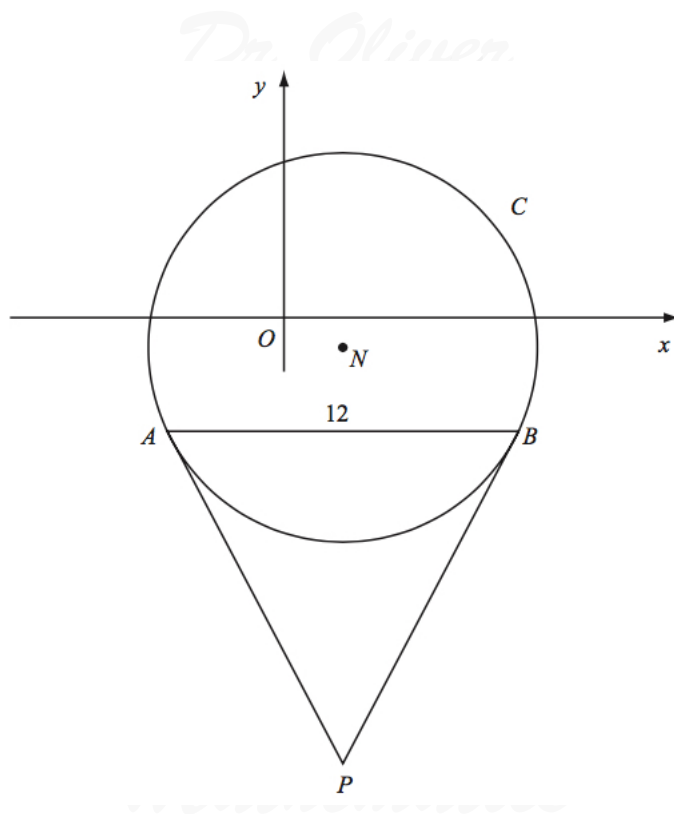


Figure 5:  $(x - 2)^2 + (y + 1)^2 = \frac{169}{4}$

- (a) Write down the coordinates of  $N$ . (2)
- (b) Find the radius of  $C$ . (1)

The chord  $AB$  of  $C$  is parallel to the  $x$ -axis, lies below the  $x$ -axis and is of length 12 units, as shown in Figure 5.

- (c) Find the coordinates of  $A$  and the coordinates of  $B$ . (5)
12. The circle  $C$  has centre  $A(2, 1)$  and passes through the point  $B(10, 7)$ .

- (a) Find an equation for  $C$ . (4)

The line  $l_1$  is the tangent to  $C$  at the point  $B$ .

- (b) Find an equation for  $l_1$ . (4)

The line  $l_2$  is parallel to  $l_1$  and passes through the mid-point of  $AB$ . Given that  $l_2$  intersects  $C$  at the points  $P$  and  $Q$ ,

- (c) find the length of  $PQ$ , giving your answer in its simplest surd form. (3)

13. The points  $A$  and  $B$  have coordinates  $(-2, 11)$  and  $(8, 1)$  respectively. Given that  $AB$  is a diameter of the circle  $C$ ,

- (a) show that the centre of  $C$  has coordinates  $(3, 6)$ , (1)

- (b) find an equation for  $C$ . (4)
- (c) Verify that the point  $(10, 7)$  lies on  $C$ . (1)
- (d) Find an equation of the tangent to  $C$  at the point  $(10, 7)$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are constants. (4)

14. The circle  $C$  has equation

$$x^2 + y^2 + 4x - 2y - 11 = 0.$$

Find

- (a) the coordinates of the centre of  $C$ , (2)
- (b) the radius of  $C$ , (2)
- (c) the coordinates of the points where  $C$  crosses the  $y$ -axis, giving your answers as simplified surds. (4)
15. A circle  $C$  has centre  $(-1, 7)$  and passes through the point  $(0, 0)$ . Find an equation for  $C$ . (4)
16. The circle  $C$  with centre  $T$  and radius  $r$  has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0.$$

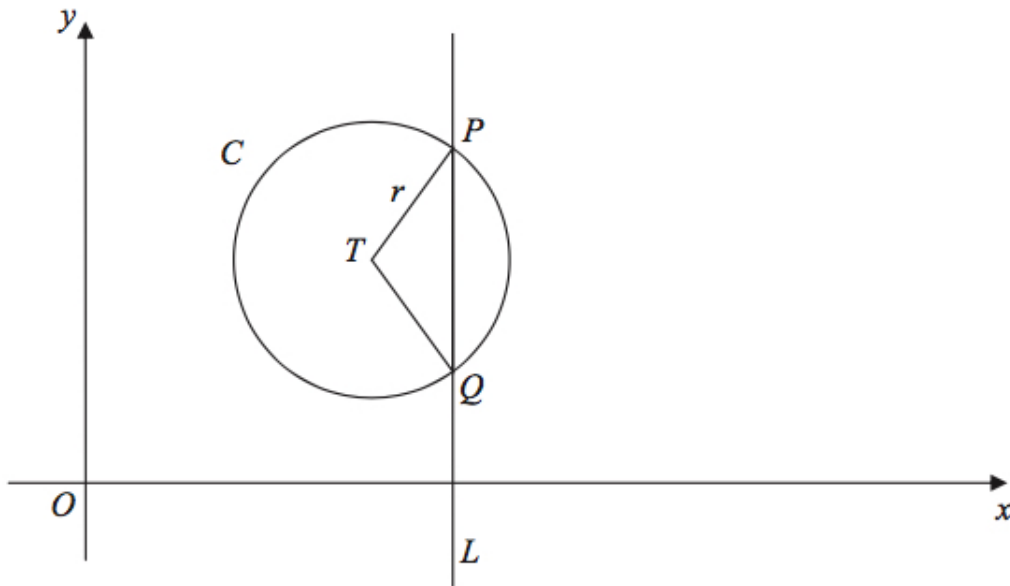


Figure 6: the circle  $C$  with centre  $T$  and radius  $r$

- (a) Find the coordinates of the centre of  $C$ . (3)

(b) Show that  $r = 5$ . (2)

The line  $L$  has equation  $x = 13$  and crosses  $C$  at the points  $P$  and  $Q$  as shown in Figure 6.

(c) Find the  $y$ -coordinate of  $P$  and the  $y$ -coordinate of  $Q$ . (3)

Given that, to 3 decimal places, the angle  $PTQ$  is 1.855 radians,

(d) find the perimeter of the sector  $PTQ$ . (3)

17. The circle  $C$  has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0.$$

The centre of  $C$  is at the point  $M$ .

(a) Find (5)

(i) the coordinates of the point  $M$ ,

(ii) the radius of the circle  $C$ .

$N$  is the point with coordinates  $(25, 32)$ .

(b) Find the length of the line  $MN$ . (2)

The tangent to  $C$  at a point  $P$  on the circle passes through point  $N$ .

(c) Find the length of the line  $NP$ . (2)

18. The circle  $C$  has radius 5 and touches the  $y$ -axis at the point  $(0, 9)$ , as shown in Figure 7.

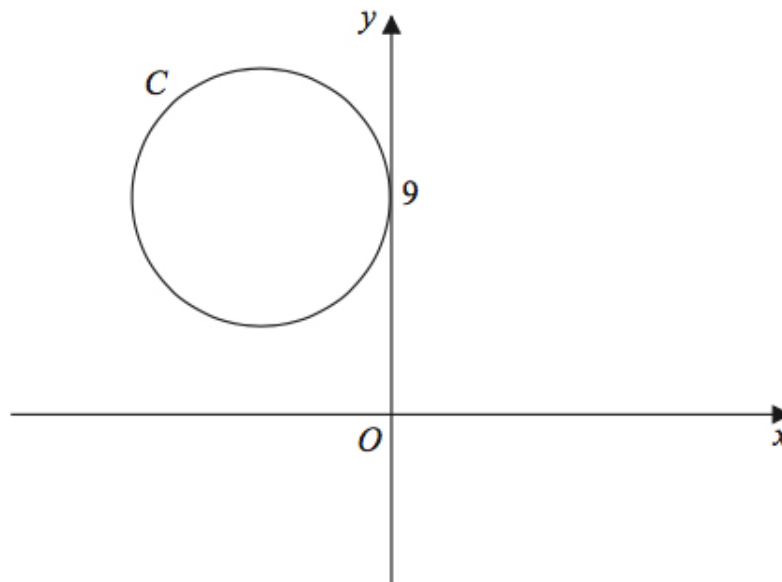


Figure 7: the circle  $C$  has radius 5

- (a) Write down an equation for the circle  $C$ , that is shown in Figure 7. (3)

A line through the point  $P(8, -7)$  is a tangent to the circle  $C$  at the point  $T$ .

- (b) Find the length of  $PT$ . (3)

19. Figure 8 shows a circle  $C$  with centre  $Q$  and radius 4 and the point  $T$  which lies on  $C$ .

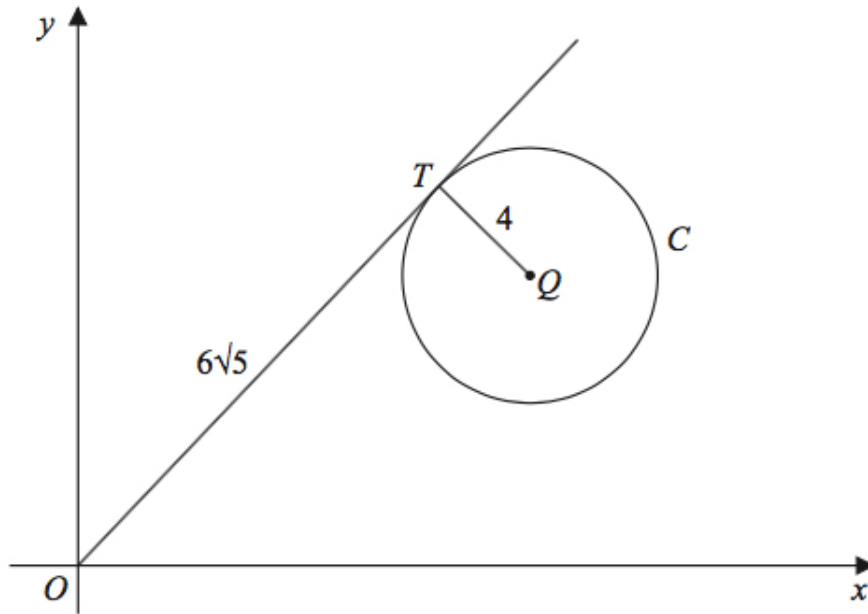


Figure 8: a circle  $C$  with centre  $Q$  and radius 4

The tangent to  $C$  at the point  $T$  passes through the origin  $O$  and  $OT = 6\sqrt{5}$ . Given that the coordinates of  $Q$  are  $(11, k)$ , where  $k$  is a positive constant,

- (a) find the exact value of  $k$ , (3)

- (b) find an equation for  $C$ . (2)

20. The circle  $C$ , with centre  $A$ , passes through the point  $P$  with coordinates  $(-9, 8)$  and the point  $Q$  with coordinates  $(15, -10)$ . Given that  $PQ$  is a diameter of the circle  $C$ ,

- (a) find the coordinates of  $A$ , (2)

- (b) find an equation for  $C$ . (3)

A point  $R$  also lies on the circle  $C$ . Given that the length of the chord  $PR$  is 20 units,

- (c) find the length of the shortest distance from  $A$  to the chord  $PR$ . Give your answer as a surd in its simplest form. (2)

- (d) Find the size of the angle  $ARQ$ , giving your answer to the nearest 0.1 of a degree. (2)



21. A circle  $C$  with centre at the point  $(2, -1)$  passes through the point  $A$  at  $(4, -5)$ .
- (a) Find an equation for the circle  $C$ . (3)
- (b) Find an equation of the tangent to the circle  $C$  at the point  $A$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$ , and  $c$  are integers. (4)
22. The circle  $C$  has centre  $P(7, 8)$  and passes through the point  $Q(10, 13)$ , as shown in Figure 9.

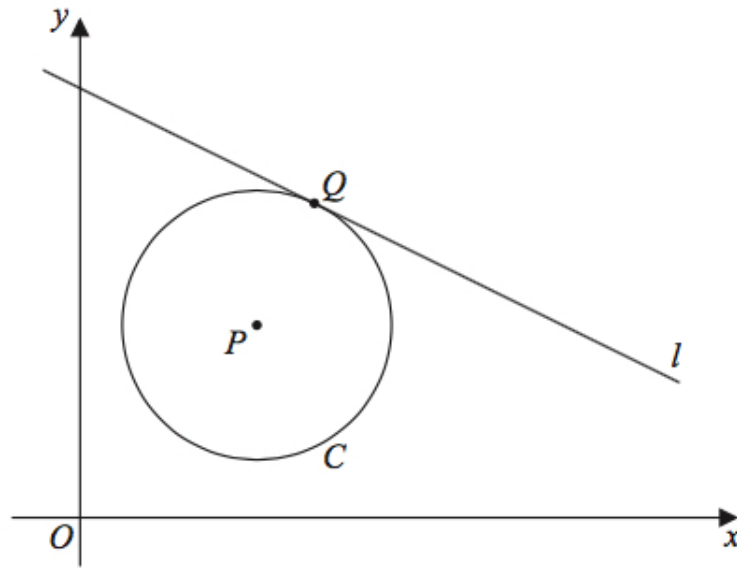


Figure 9: the circle  $C$  has centre  $P(7, 8)$

- (a) Find the length  $PQ$ , giving your answer as an exact value. (2)
- (b) Hence write down an equation for  $C$ . (2)

The line  $l$  is a tangent to  $C$  at the point  $Q$ , as shown in Figure 9.

- (c) Find an equation for  $l$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$ , and  $c$  are integers. (4)

23. The circle  $C$  has equation

$$x^2 + y^2 - 10x + 6y + 30 = 0.$$

Find

- (a) the coordinates of the centre of  $C$ , (2)
- (b) the radius of  $C$ , (2)
- (c) the  $y$ -coordinates of the points where the circle  $C$  crosses the line with equation  $x = 4$ , giving your answers as simplified surds. (3)