Dr Oliver Mathematics
Worked Examples
Proportion 3

From: CCEA 2023 M8 Paper 2 (Calculator)

1. P is inversely proportional to the square of x. (6)

   When the value of x is increased from a to $(a+1)$, the value of P is halved.

   Find the exact values of a.

Solution

Well,

$$P \propto \frac{1}{x^2}$$

which means

$$P = \frac{k}{x^2},$$

for some value of k.

Now,

$$P(a) = \frac{k}{a^2}$$

and

$$P(a+1) = \frac{k}{(a+1)^2}$$

and the question tells us "the value of x is increased from a to $(a+1)$, the value of P is halved."

So

$$\begin{aligned} P(a) = 2P(a+1) &\Rightarrow \frac{k}{a^2} = 2 \times \frac{k}{(a+1)^2} \\ &\Rightarrow \frac{\cancel{k}}{a^2} = \frac{2\cancel{k}}{(a+1)^2} \\ &\Rightarrow \frac{1}{a^2} = \frac{2}{(a+1)^2} \end{aligned}$$

cross-multiply:

$$\Rightarrow (a+1)^2 = 2a^2$$

$\times$	a	$+1$
a	a^2	$+a$
$+1$	$+a$	$+1$

$$\begin{aligned}
&\Rightarrow a^2 + 2a + 1 = 2a^2 \\
&\Rightarrow 1 = a^2 - 2a \\
&\Rightarrow 1 + 1 = (a^2 - 2a) + 1 \\
&\Rightarrow 2 = (a-1)^2 \\
&\Rightarrow \pm\sqrt{2} = a - 1
\end{aligned}$$

and

$$\underline{\underline{a = 1 \pm \sqrt{2}}}.$$