Dr Oliver Mathematics Worked Examples Proportion 3

(6)

From: CCEA 2023 M8 Paper 2 (Calculator)

1. P is inversely proportional to the square of x.

When the value of x is increased from a to (a + 1), the value of P is halved.

Find the exact values of a.

Solution

Well,

$$P \propto \frac{1}{x^2}$$

which means

$$P = \frac{k}{x^2},$$

for some value of k.

Now,

$$P(a) = \frac{k}{a^2}$$

and

$$P(a) = \frac{k}{a^2}$$

$$P(a+1) = \frac{k}{(a+1)^2}$$

and the question tells us "the value of x is increased from a to (a + 1), the value of P is halved."

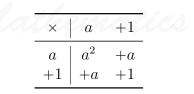
So

$$P(a) = 2P(a+1) \Rightarrow \frac{k}{a^2} = 2 \times \frac{k}{(a+1)^2}$$
$$\Rightarrow \frac{\cancel{k}}{a^2} = \frac{2\cancel{k}}{(a+1)^2}$$
$$\Rightarrow \frac{1}{a^2} = \frac{2}{(a+1)^2}$$

cross-multiply:

$$\Rightarrow (a+1)^2 = 2a^2$$

Dr Oliver



$$\Rightarrow a^{2} + 2a + 1 = 2a^{2}$$

$$\Rightarrow 1 = a^{2} - 2a$$

$$\Rightarrow 1 + 1 = (a^{2} - 2a) + 1$$

$$\Rightarrow 2 = (a - 1)^{2}$$

$$\Rightarrow \pm \sqrt{2} = a - 1$$

and

$$\underline{a=1\pm\sqrt{2}}.$$

Dr Oliver

Dr Oliver Mathematics

Dr Oliver Mathematics