Dr Oliver Mathematics
Worked Examples
Probability 10

From: My Own Example

1. There are only black counters and white counters in a box. (5)

   $\frac{5}{9}$ of the counters are black.

   Denise takes at random two counters from the box.

   The probability that the two counters are the same colour is $\frac{86}{171}$.

   Work out the number of white counters in the box.

Solution

We should take the number of counters to be 5 and the numbers of black counters to be 4, right?

No: that is not what the question says. The second line says, "$\frac{5}{9}$ of the counters are black."

In particular,

- 9 counters: 5 black, 4 white,
- 18 counters: 10 black, 8 white,
- 27 counters: 15 black, 12 white, etc,

each follow $\frac{5}{9}$ of the counters are black.

Let the number of all of the counters be $9n$.

Then the number of black counters will be $5n$ and the number of white counters is

$$9n - 5n = 4n.$$

$\underline{\mathrm{P}(BB)}$:

- Pick any black counter — $5n$ — out of the total number — $9n$:

$$\frac{5n}{9n}.$$

- And then pick any black counter — $(5n-1)$ — out of the remaining counters — $(9n-1)$:

$$\frac{5n-1}{9n-1}.$$

- Multiply them:

$$\left(\frac{5n}{9n}\right) \times \left(\frac{5n-1}{9n-1}\right).$$

<u>P(WW)</u>:

- Pick any white counter — $4n$ — out of the total number — $9n$:

$$\frac{4n}{9n}.$$

- And then pick any white counter — $(4n-1)$ — out of the remaining counters — $(9n-1)$:

$$\frac{4n-1}{9n-1}.$$

- Multiply them:

$$\left(\frac{4n}{9n}\right) \times \left(\frac{4n-1}{9n-1}\right).$$

Now,

$$\begin{aligned} & \text{P(same colour)} = \tfrac{86}{171} \\ \Rightarrow \quad & \text{P}(BB) + \text{P}(WW) = \tfrac{86}{171} \\ \Rightarrow \quad & \left(\frac{5n}{9n}\right) \times \left(\frac{5n-1}{9n-1}\right) + \left(\frac{4n}{9n}\right) \times \left(\frac{4n-1}{9n-1}\right) = \tfrac{86}{171} \end{aligned}$$

cross-multiply:

$$
\begin{aligned}
&\Rightarrow \quad 171[5n(5n-1)+4n(4n-1)] = 86(9n)(9n-1)\\
&\Rightarrow \quad 171(25n^2-5n+16n^2-4n) = 774n(9n-1)\\
&\Rightarrow \quad 171(41n^2-9n) = 6\,966n^2-774n\\
&\Rightarrow \quad 7\,011n^2-1\,539n = 6\,966n^2-1-774n\\
&\Rightarrow \quad 45n^2-1\,539n = 6\,966n^2-774n\\
&\Rightarrow \quad 45n^2-765n = 0\\
&\Rightarrow \quad 45n(n-17) = 0\\
&\Rightarrow \quad n = 0 \text{ (we need at least 9 counters!) or } n-17 = 0\\
&\Rightarrow \quad n = 17\\
&\Rightarrow \quad 4n = 68;
\end{aligned}
$$

hence, the number of yellow counters in the box is $\underline{\underline{68}}$.