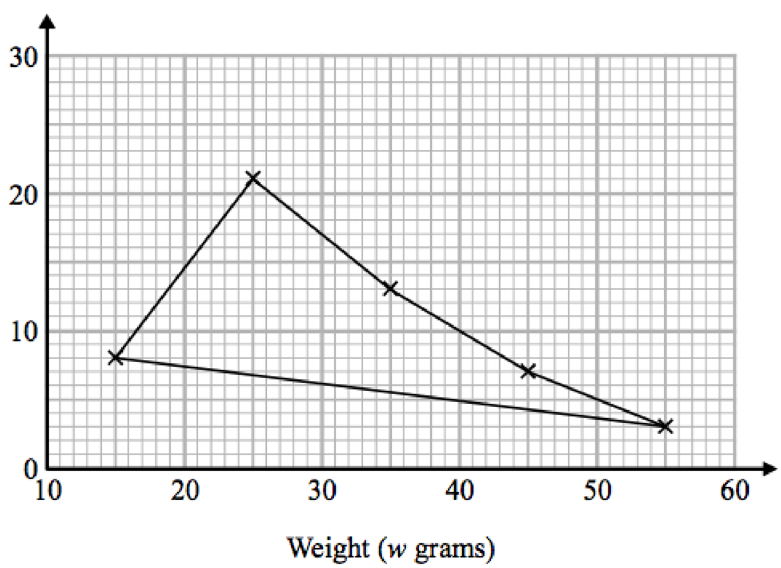
Dr Oliver Mathematics
GCSE Mathematics
2019 November Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. The table shows some information about the weights of 50 potatoes. (2)

Weight (w grams)	Frequency
$10 < w \leqslant 20$	6
$20 < w \leqslant 30$	21
$30 < w \leqslant 40$	13
$40 < w \leqslant 50$	7
$50 < w \leqslant 60$	3

Iveta drew this frequency polygon for the information in the table.
The frequency polygon is not fully correct.



Write down **two** things that are wrong with the frequency polygon.

Solution

E.g., there is no 'Frequency' label for the y-axis, the first point is joined to $(15, 8)$ and not $(15, 6)$, the line joining $(15, 8)$ and $(55, 3)$ should not be there.

2. The length of a pencil is 128 mm correct to the nearest millimetre. (2)

   Complete the error interval for the length of the pencil.

Solution

$$\underline{\underline{127.5 \leqslant \text{length} < 128.5}}.$$

3. Tom and Adam have a total of 240 stamps. (4)
   The ratio of the number of Tom's stamps to the number of Adam's stamps is 3 : 7.

   Tom buys some stamps from Adam.
   The ratio of the number of Tom's stamps to the number of Adam's stamps is now 3 : 5.

   How many stamps does Tom buy from Adam?
   You must show all your working.

Solution

Before The number of Tom's stamps is

$$\frac{3}{3+7} \times 240 = \frac{3}{10} \times 240$$
$$= 72$$

and Adam has

$$240 - 72 = 168.$$

After The number of Tom's stamps is

$$\frac{3}{3+5} \times 240 = \frac{3}{8} \times 240$$
$$= 90$$

and Adam has

$$240 - 90 = 150.$$

Hence, Tom buy 18 stamps.

4. Each person in a fitness club is going to get a free gift.
Stan is going to order the gifts.

Stan takes a sample of 50 people in the fitness club.
He asks each person to tell him the gift they would like.

The table shows information about his results.

Gift	Number of people
Sports bag	17
Gym towel	7
Headphones	11
Voucher	15

There are 700 people in the fitness club.

(a) Work out how many sports bags Stan should order. (2)

Solution

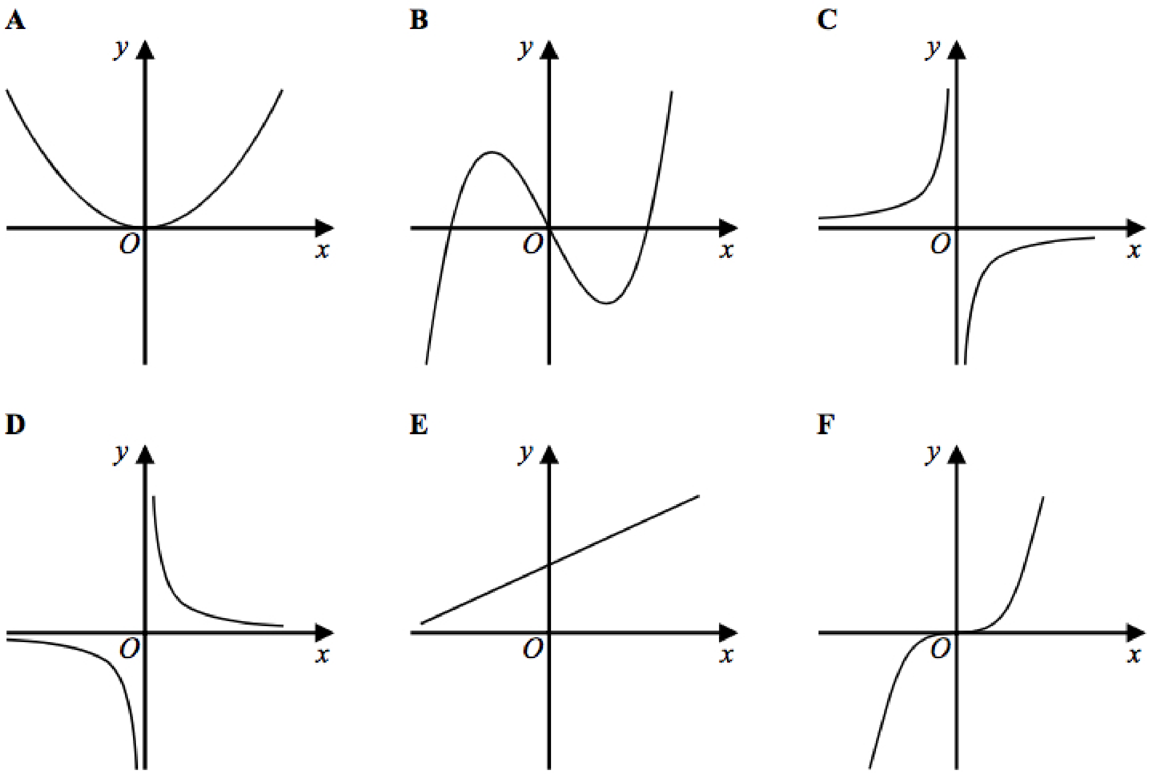
$$\frac{17}{50} \times 700 = \underline{\underline{238}}.$$

(b) Write down any assumption you made **and** explain how this could affect your answer. (1)

Solution

E.g., that the sample is representative, a random sample, that there was nobody else asked for another present (squash balls, shuttlecocks, etc), that there was nobody else asked for no present, that there was nobody else asked for two or more presents, etc - it could made the answer either lower or higher.

5. Here are six graphs.



Write down the letter of the graph that could have the equation

(a) $y = x^3$ (1)

Solution

$\underline{\underline{F}}$.

(b) $y = \dfrac{1}{x}$. (1)

Solution

$\underline{\underline{D}}$.

6. The nth term of a sequence is (3)

$$2n^2 - 1.$$

The nth of a different sequence is

$$40 - n^2.$$

Show that there is only one number that is in both of these sequences.

Solution

$$\begin{aligned} 2n^2 - 1 &: \quad 1,\ 7,\ 17,\ 31,\ \ldots \\ 40 - n^2 &: \quad 39,\ 36\,31,\ \ldots \end{aligned}$$

Well, $2n^2 - 1$ is increasing but $40 - n^2$ is decreasing. Hence, $\underline{\underline{31}}$ is the only number that appears in both of these sequences

7. Work out (2)

$$(3.42 \times 10^{-7}) \div (7.5 \times 10^{-6}).$$

Give your answer in standard form.

Solution

$$\begin{aligned} (3.42 \times 10^{-7}) \div (7.5 \times 10^{-6}) &= \frac{3.42 \times 10^{-7}}{7.5 \times 10^{-6}} \\ &= 0.045\,6 \\ &= \underline{\underline{4.56 \times 10^{-2}}}. \end{aligned}$$

8. The number of days, d, that it will take to build a house is given by (3)

$$d = \frac{720}{n},$$

where n is the number of workers used each day.

Ali's company will take 40 days to build the house.
Hayley's company will take 30 days to build the house.

Hayley's company will have to use more workers each day than Ali's company.

How many more?

Solution

Ali will take

$$\frac{720}{40} = 18$$

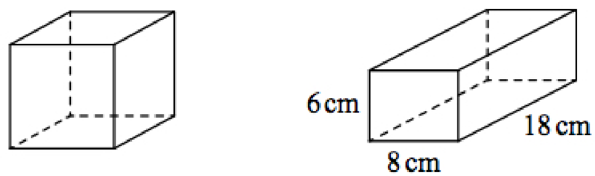
whereas Hayley will take

$$\frac{720}{30} = 24.$$

Hence, Hayley will take

$$24 - 18 = \underline{\underline{6 \text{ more days}}}.$$

9. The diagram shows a cube and a cuboid. (5)



The total surface area of the cube is equal to the total surface area of the cuboid.

Janet says, "The volume of the cube is equal to the volume of the cuboid."

Is Janet correct?
You must show how you get your answer.

Solution

The total surface area of the cuboid is

$$\begin{aligned} 2[(6 \times 8) + (6 \times 18) + (8 \times 18)] &= 2[48 + 108 + 144] \\ &= 2 \times 300 \\ &= 600 \text{ cm}^2. \end{aligned}$$

Suppose the cube is of radius r cm. Then

$$\begin{aligned} 6r^2 = 600 &\Rightarrow r^2 = 100 \\ &\Rightarrow r = 10 \text{ cm}. \end{aligned}$$

Now, the volume of the cube is

$$10^3 = 1\,000 \text{ cm}^3$$

and the volume of the cuboid is

$$6 \times 8 \times 18 = 864 \text{ cm}^3.$$

Hence, Jane is $\underline{\underline{\text{wrong}}}$.

10. Make k the subject of the formula (2)

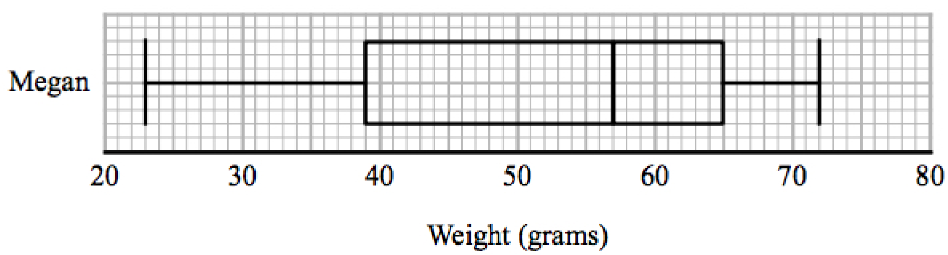
$$y = \sqrt{2m - k}.$$

Solution

$$y = \sqrt{2m-k} \Rightarrow y^2 = 2m - k$$
$$\Rightarrow \underline{\underline{k = 2m - y^2}}.$$

11. Megan grows potatoes.

The box plot below shows information about the weights of Megan's potatoes.



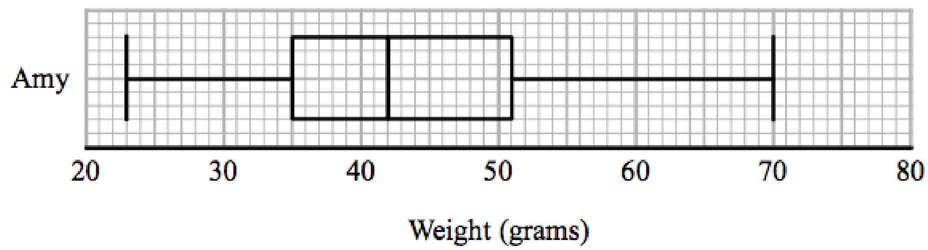
Megan says that half of her potatoes weigh less than 50 grams each.

(a) Is Megan correct? (1)
Give a reason for your answer.

Solution
<u>No</u>: her median is at 57 g.

Amy also grows potatoes.

The box plot below shows information about the weights of Amy's potatoes.



(b) Compare the distribution of the weights of Megan's potatoes with the distribution of the weights of Amy's potatoes. (2)

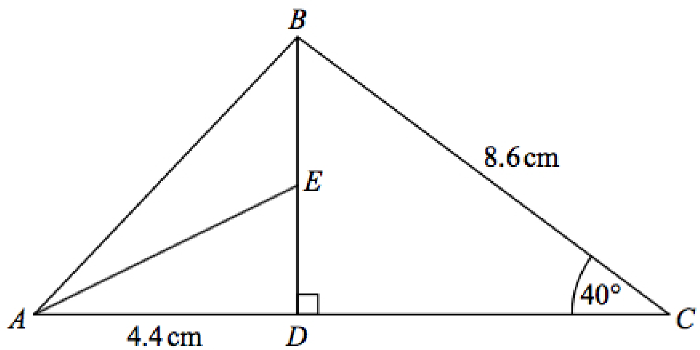
Solution

Quantity	Megan	Amy
Median	57	42
IQR	$65 - 39 = 26$	$51 - 35 = 16$
Range	$72 - 23 = 49$	$70 - 23 = 47$

Average
Since the median for Megan is higher than the median for Amy, Megan's potatoes have more mass on average.

Spread
Since the range for Megan is larger than the range for Amy, Megan's potatoes have less consistent mass.
OR
Since the IQR for Megan is smaller than the IQR for Amy, Megan's potatoes have more consistent mass.

12. The diagram shows triangle ABC. (4)



ADC and DEB are straight lines.

$AD = 4.4$ cm.
$BC = 8.6$ cm.
E is the midpoint of DB.

Angle $CDB = 90°$.
Angle $DCB = 40°$.

Work out the size of angle EAD.
Give your answer correct to 1 decimal place.
You must show all your working.

Solution

$$\sin = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 40^\circ = \frac{BD}{8.6}$$
$$\Rightarrow BD = 8.6 \sin 40^\circ$$
$$\Rightarrow ED = 4.3 \sin 40^\circ$$

and

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan EAD = \frac{4.3 \sin 40^\circ}{4.4}$$
$$\Rightarrow \angle EAD = 32.136\,167\,35 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle EAD = 32.1^\circ \text{ (1 dp)}}}.$$

13. Sakira invested £3 550 in a savings account for 3 years. (3)

She was paid 2.6% per annum compound interest for each of the first 2 years.
She was paid R% interest for the third year.

Sakira had £3 819.21 in her savings account at the end of the 3 years.

Work out the value of R.
Give your answer correct to 1 decimal place.

Solution

Well,

$$\begin{aligned}
& (1.026)^2 \times \left(1 + \frac{R}{100}\right) \times 3\,550 = 3\,819.21 \\
\Rightarrow\quad & 1 + \frac{R}{100} = \frac{3\,819.21}{(1.026)^2 \times 3\,550} \\
\Rightarrow\quad & \frac{R}{100} = \frac{3\,819.21}{(1.026)^2 \times 3\,550} - 1 \\
\Rightarrow\quad & R = 100\left[\frac{3\,819.21}{(1.026)^2 \times 3\,550} - 1\right] \\
\Rightarrow\quad & R = 2.199\,898\,432 \text{ (FCD)} \\
\Rightarrow\quad & \underline{\underline{R = 2.2 \text{ (1 dp)}}}.
\end{aligned}$$

14. Sadia is going to buy a new car. (2)
For the car, she can choose one body colour, one roof colour, and one wheel type.

She can choose from

- 19 different body colours and
- 25 different wheel types.

The total number of ways Sadia can choose the body colour and the roof colour and the wheel type is 3 325.

Work out the number of different roof colours that Sadia can choose from.

Solution

$$19 \times 25 \times \text{roof colour} = 3\,325 \Rightarrow \text{roof colour} = \frac{3\,325}{19 \times 25}$$
$$\Rightarrow \underline{\underline{\text{roof colour} = 7}}.$$

15. Expand and simplify (3)

$$(3x + 2)(2x + 1)(x - 5).$$

Solution

$\times$	$2x$	$+1$
$3x$	$6x^2$	$+3x$
$+2$	$+4x$	$+2$

So

$$(3x + 2)(2x + 1)(x - 5) = (6x^2 + 7x + 2)(x - 5).$$

$\times$	$6x^2$	$+7x$	$+2$
x	$6x^3$	$+7x^2$	$+2x$
-5	$-30x^2$	$-35x$	-10

So

$$(3x + 2)(2x + 1)(x - 5) = \underline{\underline{6x^3 - 23x^2 - 33x - 10}}.$$

16. Marek has 9 cards. (3)
There is a number on each card.

1	2	3	4	5	6	7	8	9

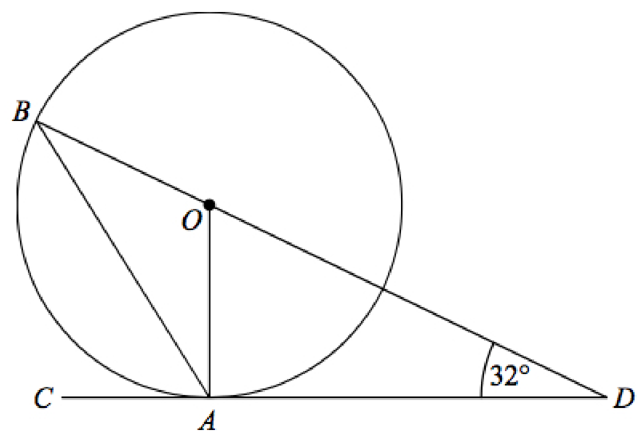
Marek takes at random two of the cards.
He works out the product of the numbers on the two cards.

Work out the probability that the product is an even number.

Solution

$$\begin{aligned} \text{P(product is an even number)} &= 1 - \text{P(odd, odd)} \\ &= 1 - (\tfrac{5}{9} \times \tfrac{4}{8}) \\ &= 1 - \tfrac{20}{72} \\ &= \tfrac{52}{72} \\ &= \underline{\underline{\tfrac{13}{18}}}. \end{aligned}$$

17. A and B are points on a circle with centre O. (3)
CAD is the tangent to the circle at A.
BOD is a straight line.



Angle $ODA = 32°$.

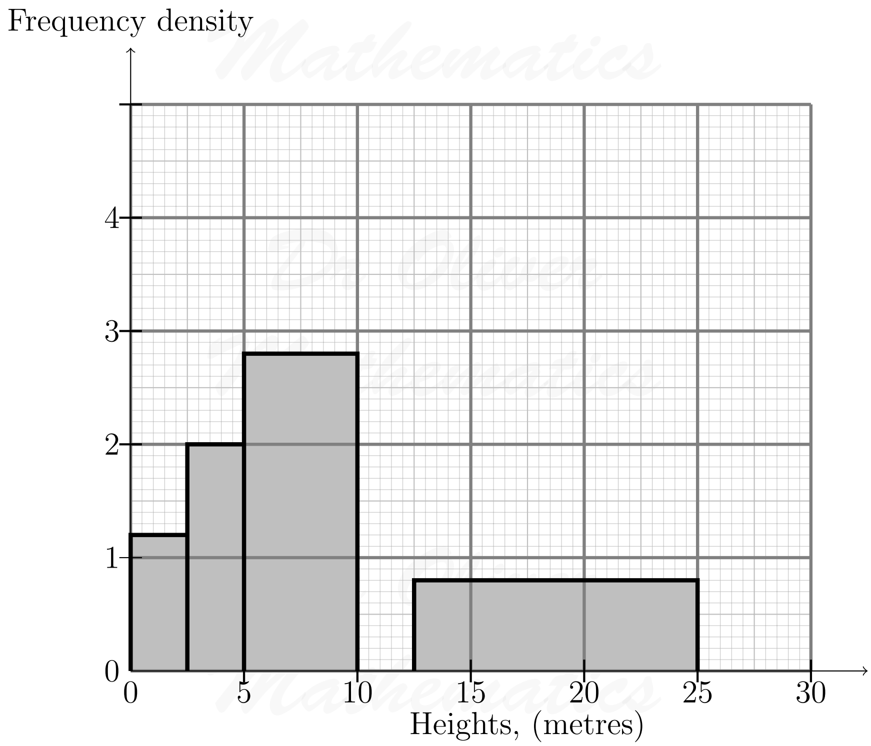
Work out the size of angle CAB.
You must show all your working.

Solution

$\angle DAO = 90°$ (it is a right-angle)
$\angle AOD = 180 - (90 + 32) = 58°$ (completing the triangle)

$\angle BOA = 180 - 58 = 122°$ (supplementary angles)
$\angle BAO = \frac{1}{2}(180 - 122) = 29°$ (base angles)
$\angle CAB = 90 - 29 = \underline{\underline{61°}}$ (it is what completes the right-angle)

18. The histogram gives information about the heights, in metres, of the trees in a park. (3)
The histogram is incomplete.



20% of the trees in the park have a height between 10 metres and 12.5 metres.
None of the trees in the park have a height greater than 25 metres.

Complete the histogram.

Solution

Height	Frequency	Width	Frequency Density
$0 - 2.5$	$2.5 \times 1.2 = 3$	2.5	1.2
$2.5 - 5$	$2.5 \times 2 = 5$	2.5	2
$5 - 10$	$5 \times 2.8 = 14$	5	2.8
$10 - 12.5$		2.5	
$12.5 - 25$	$12.5 \times 0.8 = 10$	12.5	0.8

Now,

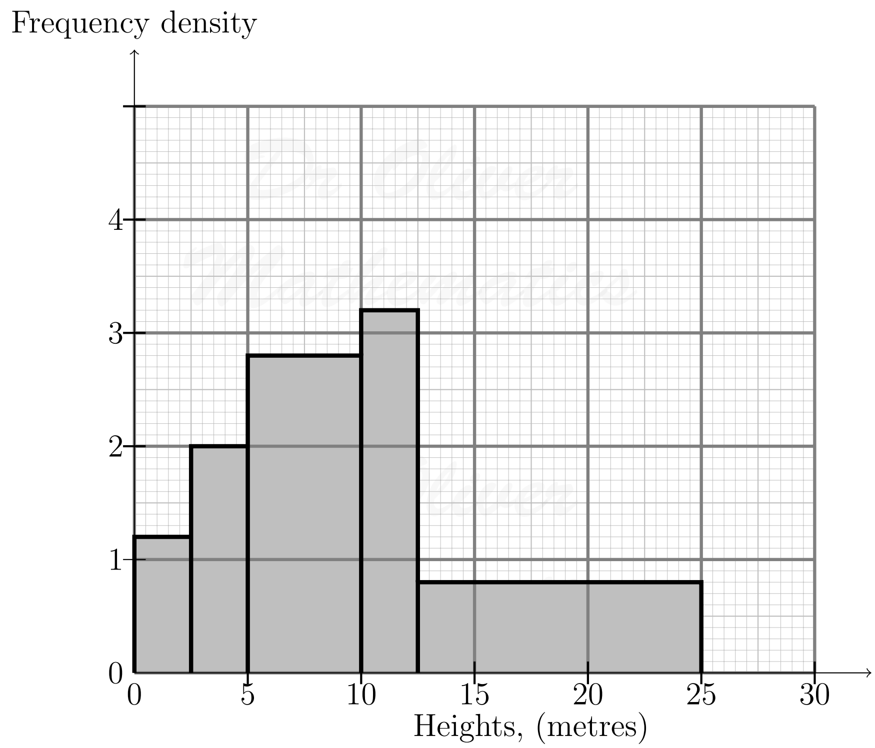
$$3 + 5 + 14 + 10 = 32$$

which means

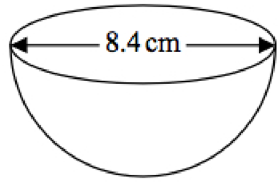
$$\tfrac{5}{4} \times 32 = 40$$

were planted. Hence, for $10 - 12.5$, we have the Frequency Density

$$\frac{8}{2.5} = 3.2.$$



19. The diagram shows a hemisphere with diameter 8.4 cm. (2)



Work out the volume of the hemisphere.
Give your answer correct to 3 significant figures.

Solution

$$\text{Radius} = \frac{8.4}{2} = 4.2 \text{ cm}$$

and the

$$\begin{aligned} \text{volume} &= \tfrac{1}{2} \times \tfrac{4}{3} \times \pi \times (4.2^3) \\ &= 155.169\,544\,3 \text{ (FCD)} \\ &= \underline{\underline{155 \text{ cm}^3 \text{ (3 sf)}}}. \end{aligned}$$

(4)

20.

$$d = \tfrac{1}{8}c^3.$$

$c = 10.9$, correct to 3 significant figures.

By considering bounds, work out the value of d to a suitable degree of accuracy.
Give a reason for your answer.

Solution

$$10.85 \leqslant c < 10.95$$

and

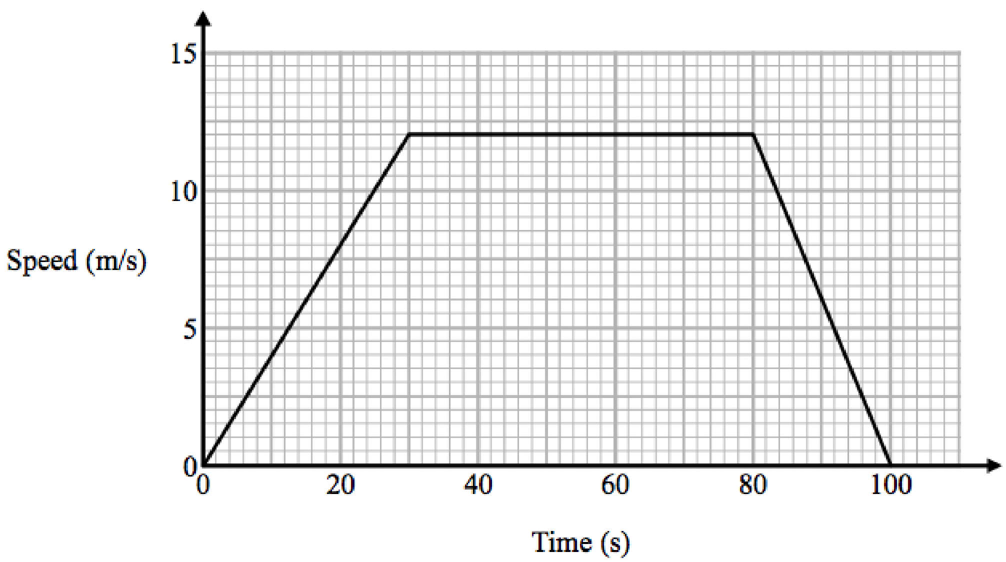
$$\begin{aligned} \text{lower bound} &= \tfrac{1}{8}(10.85^3) \\ &= 159.661\,140\,6 \text{ (FCD)} \\ \text{upper bound} &= \tfrac{1}{8}(10.85^3) \\ &= 164.116\,546\,9 \text{ (FCD)}. \end{aligned}$$

Accuracy	LB	UB	Agree
1 sf	200	200	✓
2 sf	160	160	✓
3 sf	160	164	✗

So, the LB and UB agree to 2 sf but not to 3 sf. Hence,

$$\underline{\underline{d = 160 \quad (2 \text{ sf})}}.$$

21. Here is a speed-time graph for a train journey between two stations.
The journey took 100 seconds.



(a) Calculate the time taken by the train to travel half the distance between the two stations. (4)
You must show all your working.

Solution

$$\begin{aligned}\text{Total distance} &= \tfrac{1}{2}(50 + 100)(12) \\ &= 900 \text{ m.}\end{aligned}$$

When the train is accelerating, it does a

$$\begin{aligned}\text{distance} &= \tfrac{1}{2} \times 30 \times 12 \\ &= 180 \text{ m.}\end{aligned}$$

and so

$$(\tfrac{1}{2} \times 900) - 180 = 270 \text{ m}$$

when the train is going its fastest. Hence,

$$\begin{aligned}\text{half the distance} &= 30 + \frac{270}{12} \\ &= 30 + 22.5 \\ &= \underline{\underline{52.5 \text{ s}}}.\end{aligned}$$

(b) Compare the acceleration of the train during the first part of its journey with the acceleration of the train during the last part of its journey. (1)

Solution

$$\begin{aligned}\text{Acceleration (first part)} &= \frac{12}{30} \\ &= 0.4 \text{ m s}^{-2}\end{aligned}$$

and

$$\begin{aligned}\text{Deceleration (second part)} &= \frac{12}{20} \\ &= 0.6 \text{ m s}^{-2}.\end{aligned}$$

Hence, the train decelerates faster than it accelerates by a factor 1.5.

22. The number of rabbits on a farm at the end of month n is P_n.

The number of rabbits at the end of the next month is given by

$$P_{n+1} = 1.2P_n - 50.$$

At the end of March there are 200 rabbits on the farm.

(a) Work out how many rabbits there will be on the farm at the end of June. (3)

Solution

There are three months (April, May, and June) and

$$\begin{aligned} \text{April} &= 1.2 \times 200 - 50 = 190 \\ \text{May} &= 1.2 \times 190 - 50 = 178 \\ \text{June} &= 1.2 \times 178 - 50 = 163.6; \end{aligned}$$

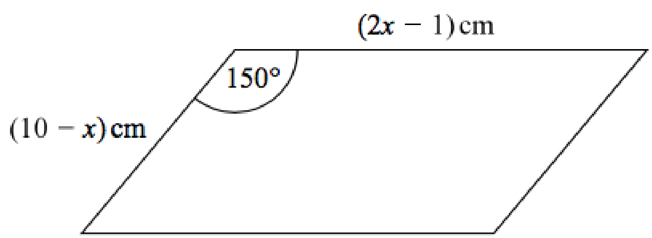
hence, there will be either 163 or 164 rabbits.

(b) Considering your results in part (a), suggest what will happen to the number of rabbits on the farm after a long time. (1)

Solution

The number of rabbits goes down to zero as there are fewer rabbits.

23. The diagram shows a parallelogram.



The area of the parallelogram is greater than 15 cm^2.

(a) Show that (3)

$$2x^2 - 21x + 40 < 0.$$

Solution

$$\text{Area} = 2 \times \tfrac{1}{2} \times (2x - 1) \times (10 - x) \times \sin 150^\circ$$

$\times$	$2x$	-1
10	$20x$	-10
$-x$	$-2x^2$	$+x$

$$= \tfrac{1}{2}(-2x^2 + 21x - 10).$$

So, if the area of the parallelogram is greater than 15 cm^2,

$$\begin{aligned} \tfrac{1}{2}(-2x^2 + 21x - 10) > 15 &\Rightarrow -2x^2 + 21x - 10 > 30 \\ &\Rightarrow 2x^2 - 21x + 10 < -30 \\ &\Rightarrow \underline{\underline{2x^2 - 21x + 40 < 0}}, \end{aligned}$$

as required.

(b) Find the range of possible values of x. (3)

Solution

$$\left.\begin{array}{ll} \text{add to:} & -21 \\ \text{multiply to:} & (+2) \times (-40) = -80 \end{array}\right\} -5, -16$$

Now, e.g.,

$$\begin{aligned} 2x^2 - 21x + 40 = 0 &\Rightarrow 2x^2 - 5x - 16x + 40 = 0 \\ &\Rightarrow x(2x - 5) - 8(2x - 5) = 0 \\ &\Rightarrow (x - 8)(2x - 5) = 0 \\ &\Rightarrow x = 8 \text{ or } x = 2\tfrac{1}{2}. \end{aligned}$$

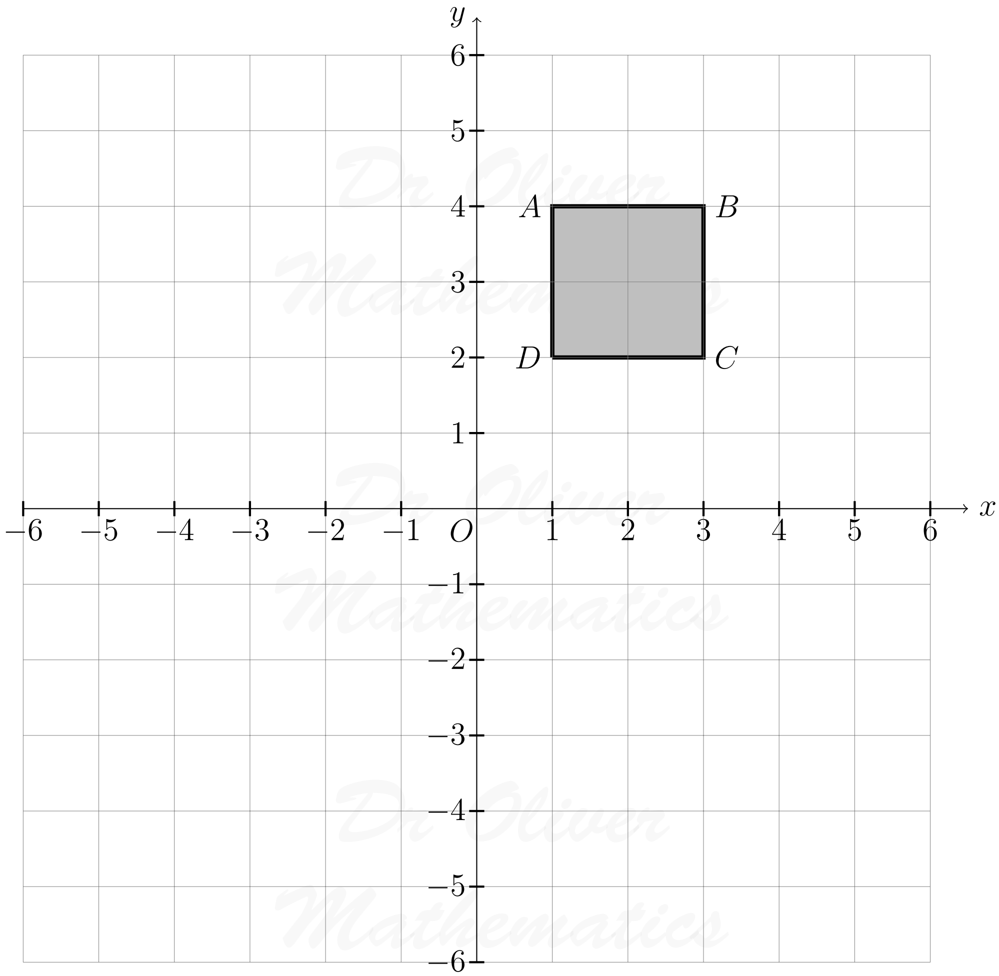
We now do a 'table of signs':

	$x < 2\frac{1}{2}$	$x = 2\frac{1}{2}$	$2\frac{1}{2} < x < 8$	$x = 8$	$x > 8$
$2x - 5$	$-$	0	$+$	$+$	$+$
$x - 8$	$-$	$-$	$-$	0	$+$
$(x - 8)(2x - 5)$	$+$	0	$-$	0	$+$

Hence,

$$\underline{\underline{2\tfrac{1}{2} < x < 8}}.$$

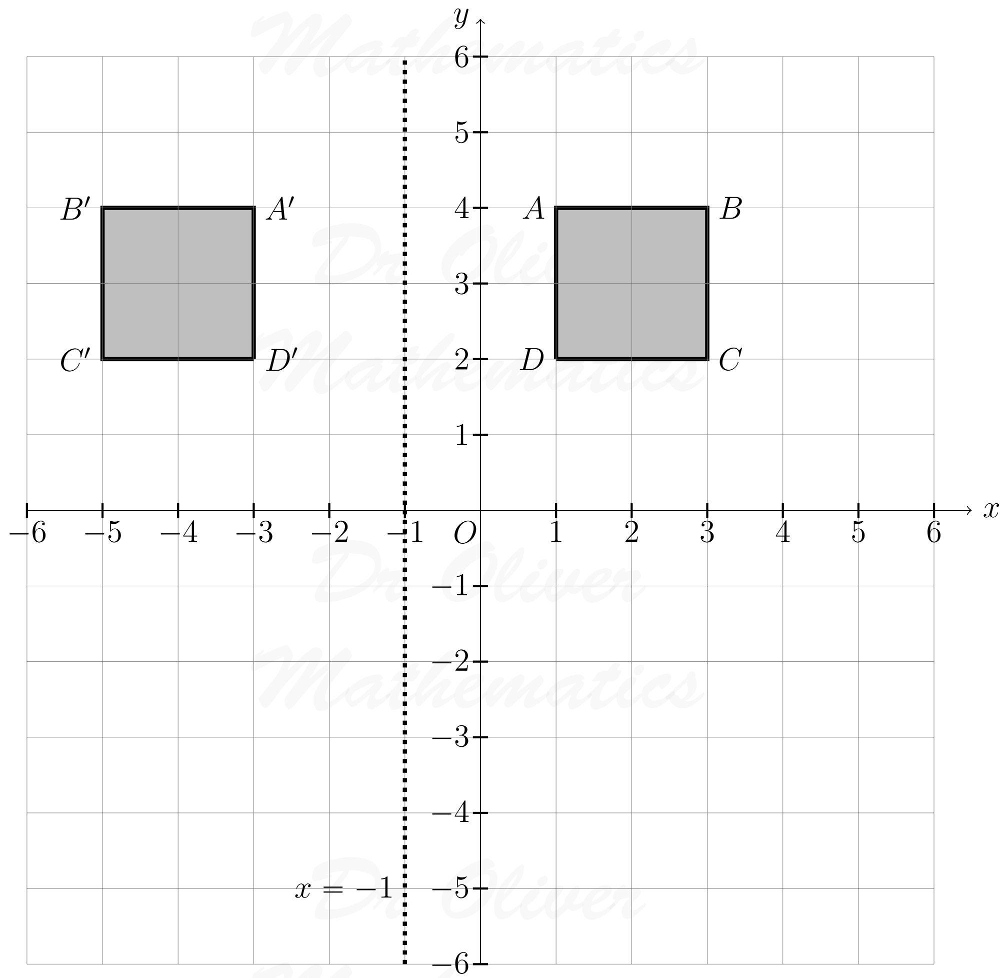
24. Square $ABCD$ is transformed by a combined transformation of a reflection in the line $x = -1$ followed by a rotation. (2)



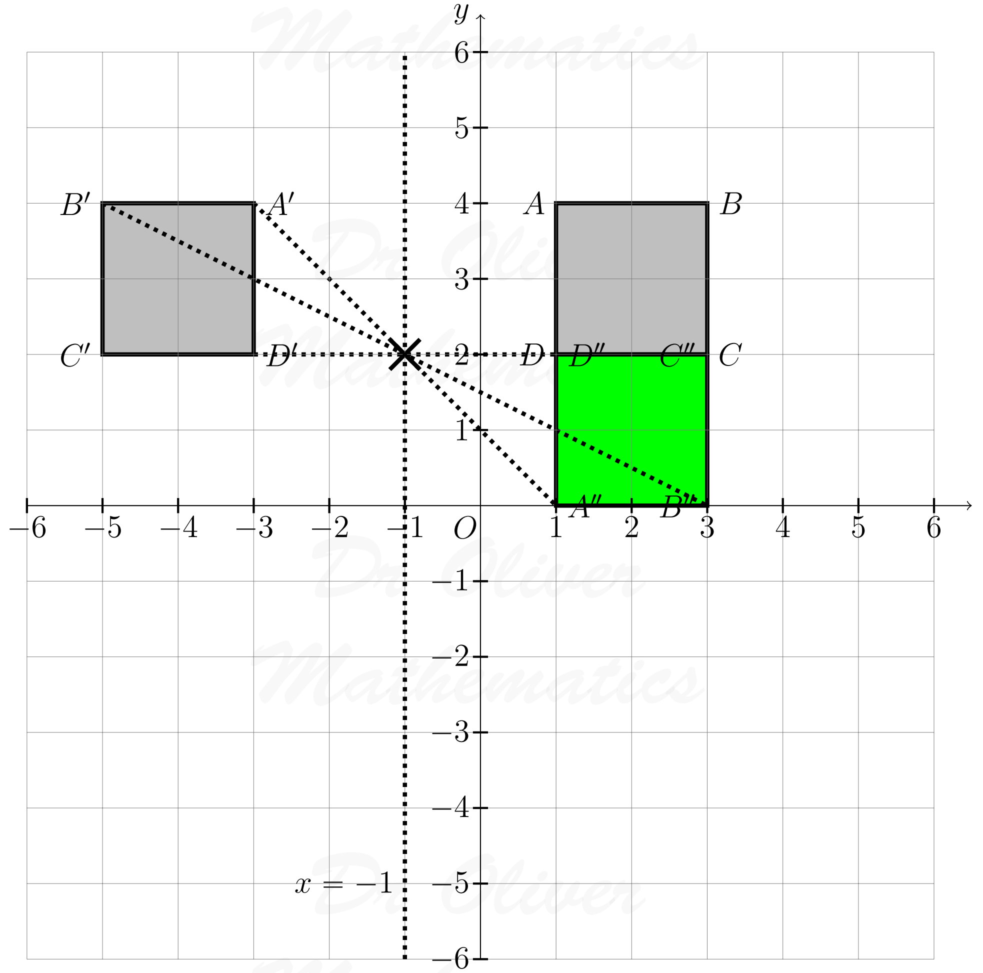
Under the combined transformation, two vertices of the square $ABCD$ are invariant.

Describe fully one possible rotation.

Solution

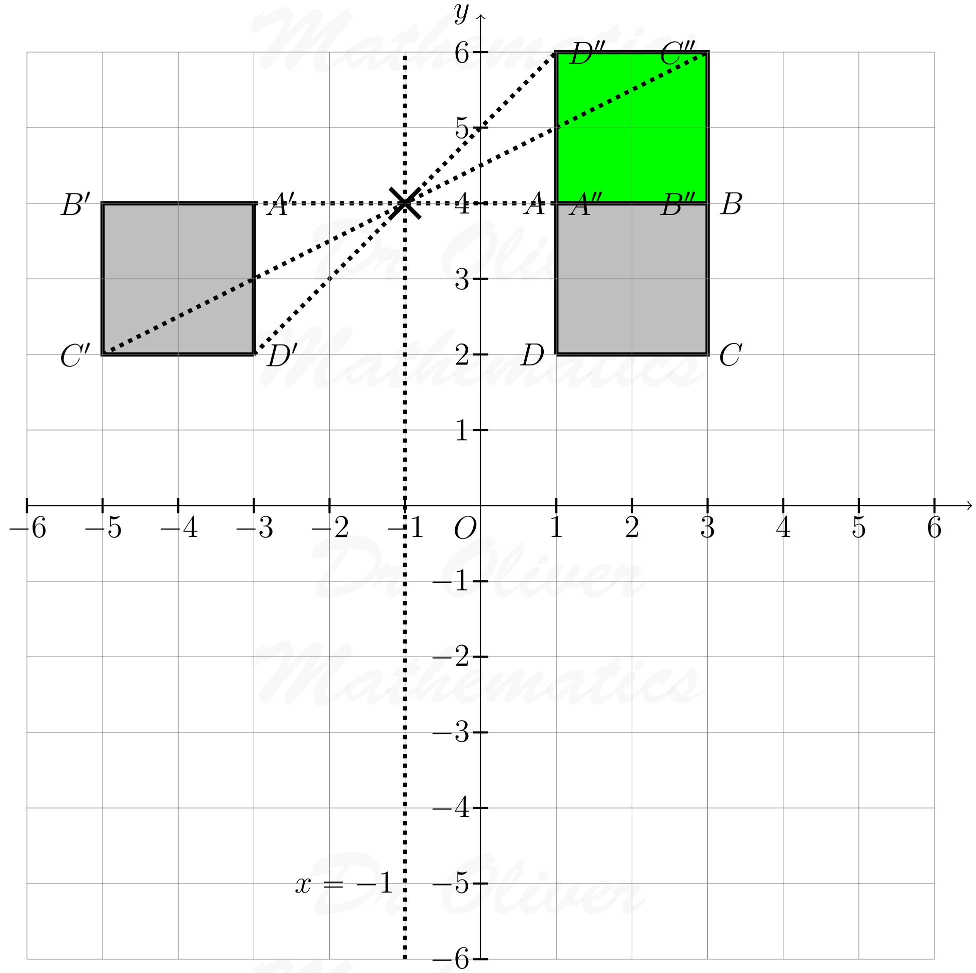


There are four possibilities:
Case 1:



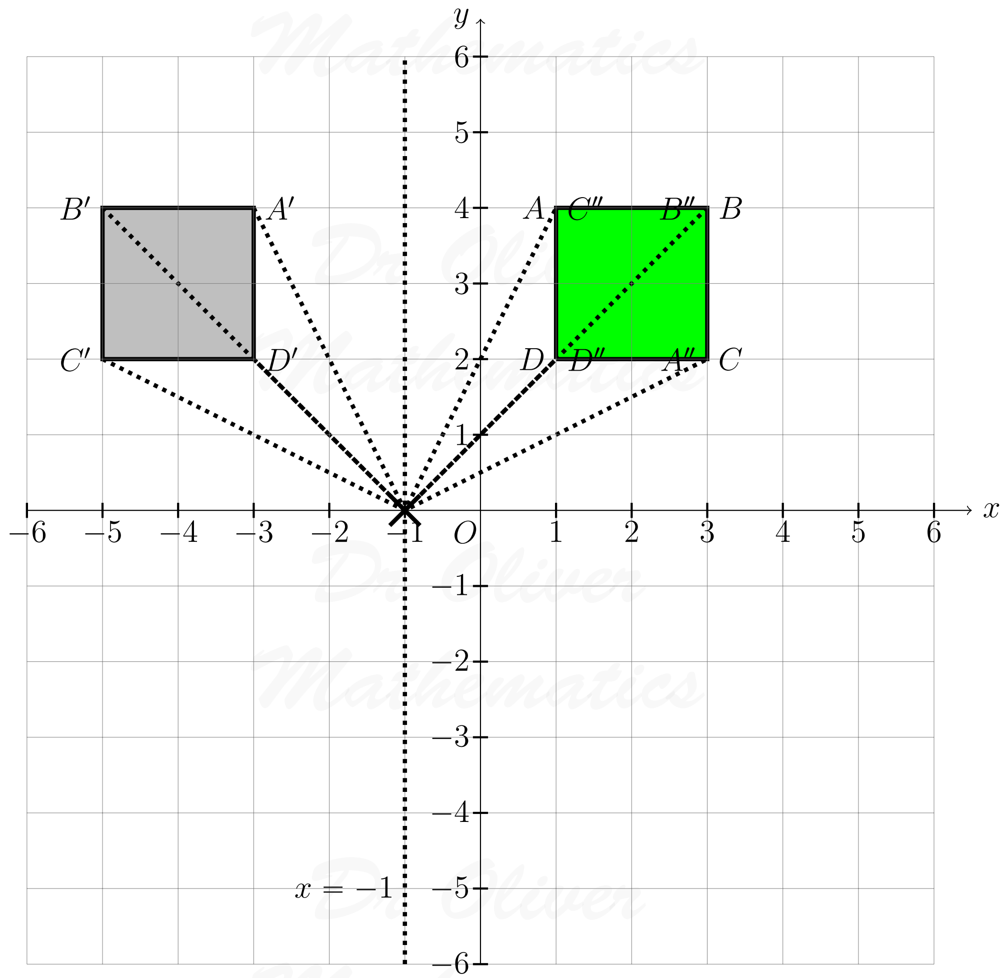
C and D stay the same: Rotation, about $180°$, centre $(-1, 2)$

Case 2:



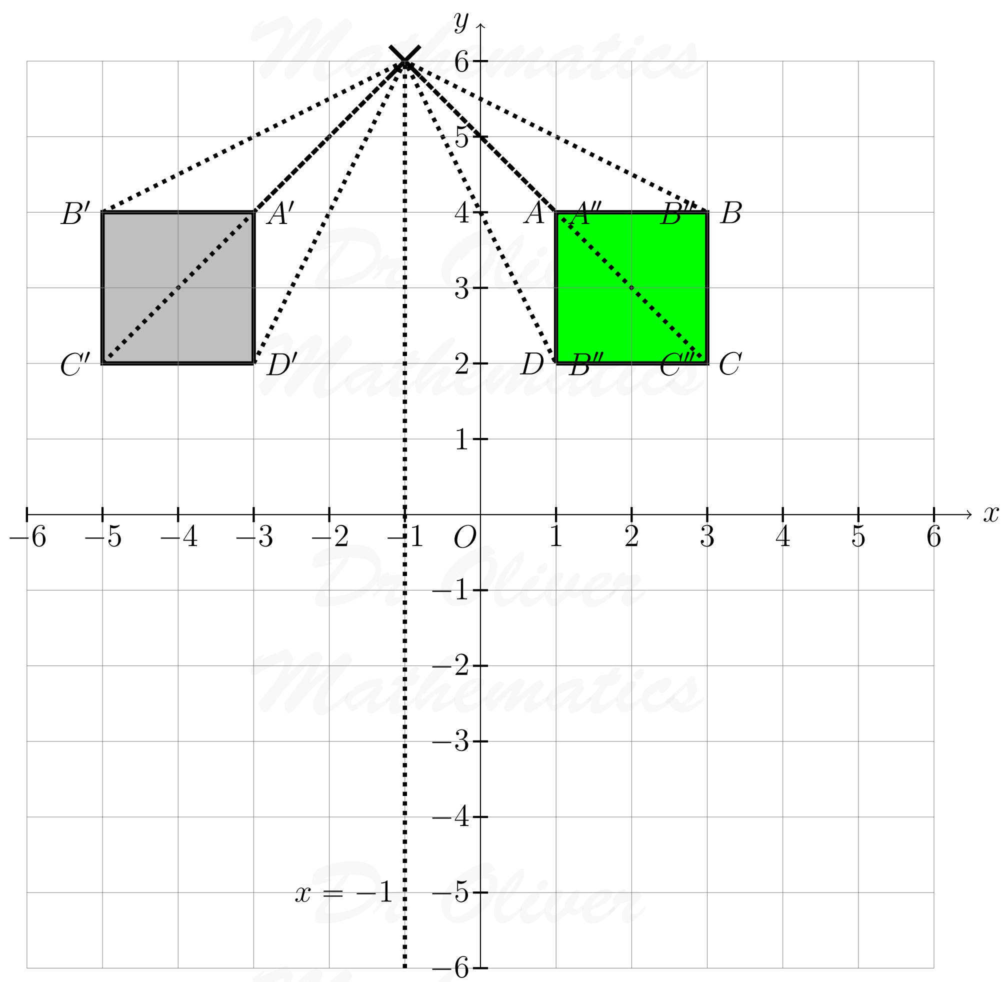
A and B stay the same: Rotation, about $180°$, centre $(-1, 4)$

Case 3:



B and D stay the same: Rotation, about 90°, clockwise, centre $(-1, 0)$.

Case 4:



A and C stay the same: Rotation, about 90° anti-clockwise, centre $(-1, 6)$.

25. The straight line **L** has equation (5)

$$3x + 2y = 17.$$

The point A has coordinates $(0, 2)$.
The straight line **M** is perpendicular to **L** and passes through A.

Line **L** crosses the y-axis at the point B.
Lines **L** and **M** intersect at the point C.

Work out the area of triangle ABC.
You must show all your working.

Solution

$$3x + 2y = 17 \Rightarrow 2y = -3x + 17$$
$$\Rightarrow y = -\tfrac{3}{2}x + \tfrac{17}{2} \quad (1)$$

and the gradient of the line **M** is

$$\frac{-1}{-\frac{3}{2}} = \tfrac{2}{3}.$$

Now, the equation of the line **M** is

$$y = \tfrac{2}{3}x + c,$$

for some constant c. Next,

$$x = 0, y = 2 \Rightarrow 2 = 0 + c \Rightarrow c = 2$$

and we have

$$y = \tfrac{2}{3}x + 2 \quad (2).$$

For B,

$$x = 0 \Rightarrow 0 + 2y = 17$$
$$\Rightarrow y = \tfrac{17}{2}$$

and so $B(0, \frac{17}{2})$. Do (1) = (2):

$$-\tfrac{3}{2}x + \tfrac{17}{2} = \tfrac{2}{3}x + 2 \Rightarrow \tfrac{13}{6}x = \tfrac{13}{2}$$
$$\Rightarrow x = 3$$
$$\Rightarrow y = 4;$$

hence, $C(3, 4)$. Hence,

$$\text{area} = \tfrac{1}{2} \times (\tfrac{17}{2} - 2) \times 3$$
$$= \tfrac{1}{2} \times \tfrac{13}{2} \times 3$$
$$= \underline{\underline{9\tfrac{3}{4}}}.$$