Dr Oliver Mathematics
Mathematics Standard Grade: Credit Level
2008 Paper 2: Calculator
1 hour 20 minutes

The total number of marks available is 51.
You must write down all the stages in your working.

1. A local council recycles 42 000 tonnes of waste a year. (4)
   The council aims to increase the amount of waste recycled by 8% each year.
   How much waste does it expect to recycle in 3 years time?
   Give your answer **to three significant figures**.

Solution

$$42\,000 \times 1.08^3 = 52\,907.904 \text{ (FCD)}$$
$$= \underline{\underline{52\,900 \text{ tonnes}}}.$$

2. In a class, 30 pupils sat a test.
   The marks are illustrated by the stem and leaf diagram below.

0	9								
1	6	6	7	8					
2	0	4	5	7	9	9	9		
3	2	2	3	5	5	6	8		
4	0	2	3	4	5	5	7	7	8
5	0	0							

Key: $1|6 = 16$
$n = 30$

(a) Write down the median and the modal mark. (2)

Solution

Median:

$$\frac{30+1}{2} = 15\tfrac{1}{2};$$

so, 15th value is 33 and the 16th value is 35 and take the average:

$$\frac{33+35}{2} = \underline{\underline{34}}.$$

Mode: $\underline{\underline{29}}$.

(b) Find the probability that a pupil selected at random scored **at least** 40 marks. (1)

Solution

$$\begin{aligned} \text{P(at least 40 marks)} &= \frac{9+2}{30} \\ &= \underline{\underline{\tfrac{11}{30}}}. \end{aligned}$$

3. In a sale, all cameras are reduced by 20%. (3)
A camera now costs £45.
Calculate the original cost of the camera.

Solution

$$\begin{aligned} \text{Original cost} &= \frac{45}{0.8} \\ &= \underline{\underline{£56.25}}. \end{aligned}$$

4. Aaron saves 50 pence and 20 pence coins in his piggy bank.
Let x be the number of 50 pence coins in his bank.
Let y be the number of 20 pence coins in his bank.

(a) There are 60 coins in his bank. (1)
Write down an equation in x and y to illustrate this information.

Solution

$\underline{\underline{x+y=60}}$ (1)

(b) The total value of the coins is £17.40. (1)
Write down another equation in x and y to illustrate this information.

Solution

$\underline{\underline{50x + 20y = 1\,740}}$ (2)

(c) Hence find **algebraically** the number of 50 pence coins Aaron has in his piggy bank. (3)

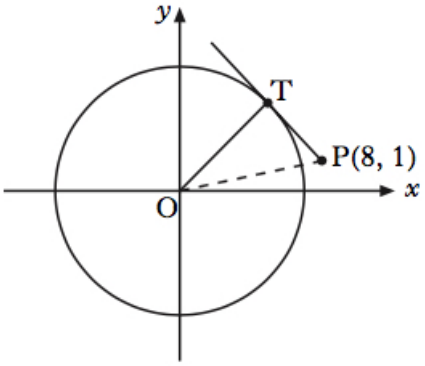
Solution

$$20 \times (1): \quad 20x + 20y = 1\,200 \quad (3)$$
$$(2): \quad 50x + 20y = 1\,740$$

(2) − (3):

$$30x = 540 \Rightarrow \underline{\underline{x = 18}}.$$

5. A circle, centre the origin, is shown.



P is the point $(8, 1)$.

(a) Calculate the length of OP. (2)

Solution

$$OP = \sqrt{8^2 + 1^2}$$
$$= \underline{\underline{\sqrt{65} \text{ or } 8.06 \ \ (3 \text{ sf})}}.$$

The diagram also shows a tangent from P which touches the circle at T.
The radius of the circle is 5 units.

(b) Calculate the length of PT. (2)

Solution

Well, OPT is a right-angled triangle (why?).

$$\begin{aligned} PT &= \sqrt{OP^2 - OT^2} \\ &= \sqrt{(\sqrt{65})^2 - 5^2} \\ &= \sqrt{65 - 25} \\ &= \sqrt{40} \\ &= \underline{\underline{2\sqrt{10} \text{ or } 6.32 \ \ (3 \text{ sf})}}. \end{aligned}$$

6. The distance, d kilometres, to the horizon, when viewed from a cliff top, varies directly as the square root of the height, h metres, of the cliff top above sea level. (5)

From a cliff top 16 metres above sea level, the distance to the horizon is 14 kilometres.

A boat is 20 kilometres from a cliff whose top is 40 metres above sea level.

Is the boat beyond the horizon?
Justify your answer.

Solution

$$d \propto \sqrt{h} \Rightarrow d = k\sqrt{h}$$

for some constant k. Now,

$$14 = k\sqrt{16} \Rightarrow k = \tfrac{7}{2}$$

and so

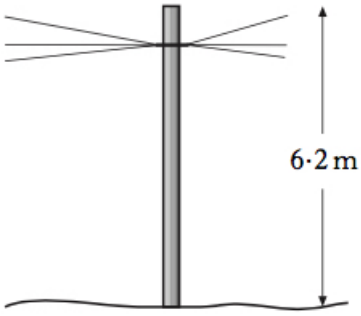
$$d = \tfrac{7}{2}\sqrt{h}.$$

Finally,

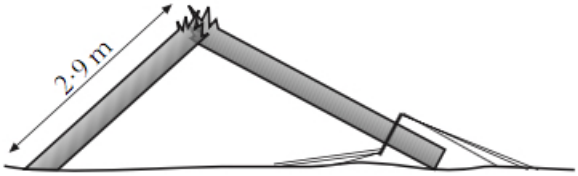
$$\tfrac{7}{2} \times \sqrt{40} = 22.13\ldots$$

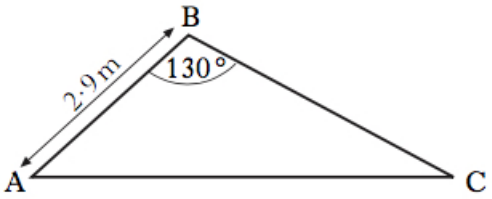
so, <u>no</u>, it is not visible.

7. A telegraph pole is 6.2 metres high. (4)



The wind blows the pole over into the position as shown below.





AB is 2.9 metres and angle ABC is 130°.
Calculate the length of AC.

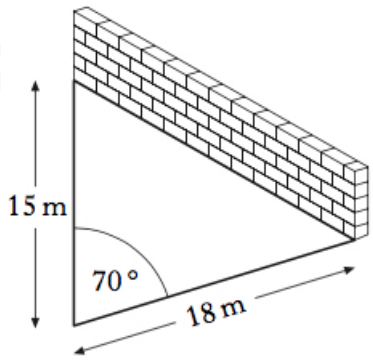
Solution

$$BC = 6.2 - 2.9 = 3.3 \text{ m}$$

and

$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2 - 2 \cdot AB \cdot BC \cdot \cos ABC} \\ &= \sqrt{2.9^2 + 3.3^2 - 2 \times 2.9 \times 3.3 \times \cos 130^\circ} \\ &= 5.621\,650\,545 \text{ (FCD)} \\ &= \underline{\underline{5.62 \text{ m (3 sf)}}}. \end{aligned}$$

8. A farmer builds a sheep-pen using two lengths of fencing and a wall.



The two lengths of fencing are 15 metres and 18 metres long.

(a) Calculate the area of the sheep-pen, when the angle between the fencing is 70°. (3)

Solution

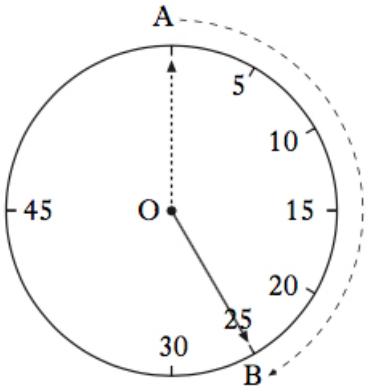
$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 15 \times 18 \times \sin 70^\circ \\ &= 126.858\,503\,8 \text{ (FCD)} \\ &= \underline{\underline{127 \text{ m}^2 \text{ (3 sf)}}}.\end{aligned}$$

(b) What angle between the fencing would give the farmer the largest possible area? (1)

Solution

$\underline{\underline{90^\circ}}$.

9. Contestants in a quiz have 25 seconds to answer a question.
This time is indicated on the clock.
The tip of the clock hand moves through the arc AB as shown.



(a) Calculate the size of angle AOB. (1)

Solution

$$\begin{aligned}\text{Angle} &= \tfrac{5}{12} \times 360 \\ &= \underline{\underline{150^\circ}}.\end{aligned}$$

The length of arc AB is 120 centimetres.

(b) Calculate the length of the clock hand. (4)

Solution

Let r cm be the length of the clock hand. Then

$$\begin{aligned}\frac{5}{12} = \frac{120}{2\pi r} &\Rightarrow r = \frac{120 \times 12}{5 \times 2 \times \pi} \\ &\Rightarrow r = 45.836\,623\,61 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{r = 45.8 \text{ cm (3 sf)}}}.\end{aligned}$$

10. To hire a car costs £25 per day plus a mileage charge.
The first 200 miles are free with each additional mile charged at 12 pence.

(a) Calculate the cost of hiring a car for 4 days when the mileage is 640 miles. (1)

Solution

$$\begin{aligned}\text{Cost} &= (4 \times 25) + [(640 - 200) \times 0.12] \\ &= 100 + (440 \times 0.12) \\ &= 100 + 52.80 \\ &= \underline{\underline{£152.80}}.\end{aligned}$$

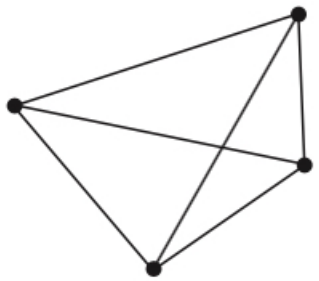
A car is hired for d days and the mileage is m, miles where $m > 200$.

(b) Write down a formula for the cost £C of hiring the car. (3)

Solution

$\underline{\underline{C = 25d + 0.12(m - 200)}}$.

11. The minimum number of roads joining 4 towns to each other is six, as shown.



The minimum number of roads, r, joining n towns to each other is given by the formula

$$r = \tfrac{1}{2}n(n-1).$$

(a) State the minimum number of roads needed to join 7 towns to each other. (1)

Solution

$$\begin{aligned}r &= \tfrac{1}{2} \times 6 \times 7 \\ &= \underline{\underline{21 \text{ roads}}}.\end{aligned}$$

(b) When $r = 55$, show that (2)

$$n^2 - n - 110 = 0.$$

Solution

$$\begin{aligned}\tfrac{1}{2}n(n-1) = 55 &\Rightarrow n(n-1) = 110\\ &\Rightarrow n^2 - n = 110\\ &\Rightarrow \underline{\underline{n^2 - n - 110 = 0}},\end{aligned}$$

as required.

(c) Hence find **algebraically** the value of n. (3)

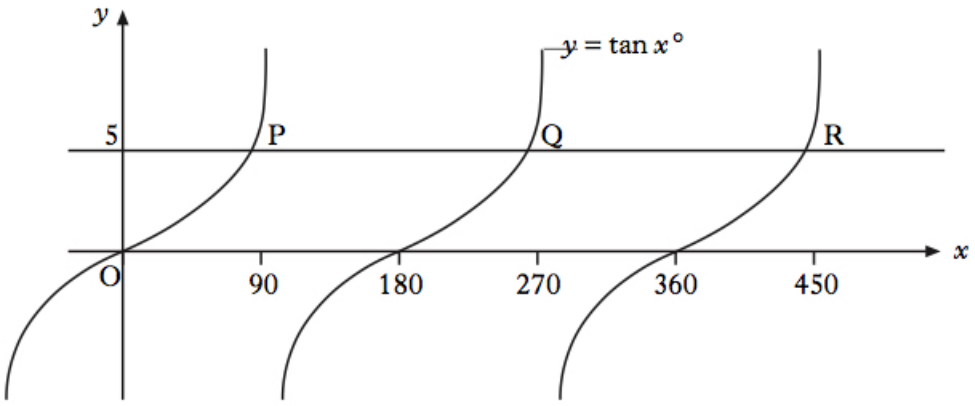
Solution

add to: -1
multiply to: -110
$\Big\}$ $-11, +10$

$$\begin{aligned}n^2 - n - 110 = 0 &\Rightarrow (n-11)(n+10) = 0\\ &\Rightarrow n - 11 = 0 \text{ or } n + 10 = 0\\ &\Rightarrow n = 11 \text{ or } n = -10;\end{aligned}$$

as $n \neq -10$, $\underline{\underline{n = 11}}$.

12. The diagram shows part of the graph of $y = \tan x^\circ$.
The line $y = 5$ is drawn and intersects the graph of $y = \tan x^\circ$ at P and Q.



(a) Find the x-coordinates of P and Q. (3)

Solution

$$\tan x^\circ = 5 \Rightarrow x = 78.690\,067\,53, x = 258.690\,067\,53;$$

hence, P's x-coordinates is 78.7 (1 dp) and Q's x-coordinates is 258.7 (1 dp).

(b) Write down the x-coordinate of the point R, where the line $y = 5$ next intersects the graph of $y = \tan x^\circ$. (1)

Solution

438.7.