Dr Oliver Mathematics
Mathematics
Trigonometry Part 2
Past Examination Questions

This booklet consists of 49 questions across a variety of examination topics.
The total number of marks available is 473.

1. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$. (2)

   (b) Solve, for $0° \leqslant \theta < 360°$, the equation (6)

$$2\tan^2\theta + \sec\theta = 1,$$

   giving your answer to 1 decimal place.

2. (a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that (2)

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

   (b) Show that (4)

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

   (c) Express $4\cos\theta + 6\sin\theta - 3$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

   (d) Hence, for $0 \leqslant x < \pi$, solve (5)

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

   giving your answer in radians to 3 significant figures, where appropriate.

3.

$$f(x) = 12\cos x - 4\sin x.$$

   Given that $f(x) = R\cos(x + \alpha)$ where $R > 0$ and $0° \leqslant \alpha \leqslant 90°$,

   (a) find the value of R and find the value of α. (4)

   (b) Hence solve the equation (5)

$$12\cos x - 4\sin x = 7,$$

   for $0° \leqslant x \leqslant 360°$, giving your answers to 1 decimal place.

   (c) Write down the minimum value of $12\cos x - 4\sin x$. (1)

   (d) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs. (2)

4. (a) Show that

   (i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x$, $x \neq (n - \frac{1}{4})\pi$, $n \in \mathbb{Z}$. (2)

   (ii) $\frac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \frac{1}{2}$. (3)

   (b) Hence, or otherwise, show that the equation (3)

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

   can be written as

$$\sin 2\theta = \cos 2\theta.$$

   (c) Solve, for $0 \leqslant x < 2\pi$, (4)

$$\sin 2\theta = \cos 2\theta,$$

   giving your answers in terms of π.

5. (a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$. (2)

   (b) Hence, or otherwise, prove that (2)

$$\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta.$$

   (c) Solve, for $90^\circ < x < 180^\circ$, (6)

$$\operatorname{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

6. (a) Given that $\cos A = \frac{3}{4}$, $270^\circ \leqslant x < 360^\circ$, find the exact value of $\sin 2A$. (5)

   (b) (i) Show that $\cos(2x + \frac{\pi}{3}) + \cos(2x - \frac{\pi}{3}) \equiv \cos 2x$. (3)

   Given that

$$y = 3\sin^2 x + \cos(2x + \tfrac{\pi}{3}) + \cos(2x - \tfrac{\pi}{3}),$$

   (ii) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sin 2x$. (4)

7. (a) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that (5)

$$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta.$$

   (b) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$. (2)

8.

$$y = \sqrt{3}\cos x + \sin x.$$

   (a) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

(b) Find the values of x, $0 \leqslant x \leqslant 2\pi$, for which $y = 1$. (4)

9. (a) Prove that $\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$. (3)

Given that

$$y = \arccos x,\ -1 \leqslant x \leqslant 1,\ \text{and } 0 \leqslant y \leqslant \pi,$$

(i) express $\arcsin x$ in terms of y. (2)

(ii) Hence evaluate $\arccos x + \arcsin x$. Give your answer in terms of x. (1)

10. (a) Express $3\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

(b) Hence find the greatest value of $(3\sin x + 2\cos x)^4$. (2)

(c) Solve, for $0 < x < 2\pi$, the equation (5)

$$3\sin x + 2\cos x = 1,$$

giving your answers to 3 decimal places.

11. (a) Prove that (4)

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta,\ \theta \neq 90n^\circ.$$

(b) Sketch the graph of $y = 2\operatorname{cosec} 2\theta$ for $0^\circ < \theta < 360^\circ$. (2)

(c) Solve, for $0^\circ < \theta < 360^\circ$, the equation (6)

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3,$$

giving your answers to 1 decimal place.

12. (a) Use the double angle formulae and the identity (4)

$$\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(b) (i) Prove that (4)

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} \equiv 2\sec x,\ x \neq (2n + 1)\frac{\pi}{2}.$$

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of (3)

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 4.$$

13. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x,\ -\pi \leqslant x \leqslant \pi.$$

The point $P(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A. (5)

(b) Express y in the form $R\sin(2x+\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$. Give the value of α to 3 significant figures. (4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places. (4)

14.

$$f(x) = 5\cos x + 12\sin x.$$

Given that $f(x) = R\cos(x-\alpha)$, where $R>0$ and $0<\alpha<\frac{\pi}{2}$,

(a) find the value of R and find the value of α to 3 decimal places. (4)

(b) Hence solve the equation (5)

$$5\cos x + 12\sin x = 6$$

for $0 \leqslant x < 2\pi$.

(c) (i) Write down the maximum value of $5\cos x + 12\sin x$. (1)

(ii) Find the smallest positive value of x for which this maximum value occurs. (2)

15. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$. (2)

(b) Solve, for $0° \leqslant \theta < 180°$, the equation (6)

$$2\cot^2\theta - 9\operatorname{cosec}\theta = 3,$$

giving your answers to 1 decimal place.

16. (a) (i) By writing $3\theta = 2\theta + \theta$, show that (4)

$$\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta.$$

(ii) Hence, or otherwise, for $0<\theta<\frac{\pi}{3}$, solve (5)

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(b) Using $\sin(\theta-\alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that (4)

$$\sin 15° = \tfrac{1}{4}(\sqrt{6}-\sqrt{2}).$$

17. (a) Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta-\alpha)$, where $R>0$ and $0°<\alpha<90°$. (4)

(b) Hence find the maximum value of $3\cos\theta + 4\sin\theta$ and the smallest positive value of θ for which this maximum occurs. (3)

The temperature, $f(t)$, of a warehouse is modelled using the equation

$$f(t) = 10 + 3\cos(15t)° + 4\sin(15t)°,$$

where t is the time in hours from midday and $0 \leqslant t \leqslant 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model. (2)

(d) Find the value of t when this minimum temperature occurs. (3)

18. (a) Use the identity $\sin^2\theta + \cos^2\theta \equiv 1$ to prove that $\tan^2\theta \equiv \sec^2\theta - 1$. (2)

(b) Solve, for $0° \leqslant \theta < 360°$, the equation (6)

$$2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2.$$

19. (a) Use the identity $\cos(A+B) \equiv \cos A\cos B - \sin A \sin B$ to show that (2)

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

The curves C_1 and C_2 have equations

$$C_1 : y = 3\sin 2x \text{ and } C_2 : y = 4\sin^2 x - 2\cos 2x.$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation (3)

$$4\cos 2x + 3\sin 2x = 2.$$

(c) Express $4\cos 2x + 3\sin 2x = 2$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α to 2 decimal places. (3)

(d) Hence find, for $0° \leqslant x < 180°$, all the solutions of (4)

$$4\cos 2x + 3\sin 2x = 2,$$

giving your answers to 1 decimal place.

20. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$. (1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation (5)

$$\operatorname{cosec} x - 8\cos x = 0,$$

giving your answers to 2 decimal places.

21. (a) Express $5\cos x - 3\sin x$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

(b) Hence, or otherwise, solve the equation (5)

$$5\cos x - 3\sin x = 4$$

for $0 \leqslant x < 2\pi$, giving your answers to 2 decimal places.

22. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0° \leqslant x \leqslant 180°$.

23. (a) Show that (2)

$$\frac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan\theta.$$

(b) Hence find, for $-180° \leqslant \theta < 180°$, all the solutions of (3)

$$\frac{2\sin 2\theta}{1 + \cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

24. (a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 4 decimal places. (3)

(b) Find the maximum value of $2\sin\theta - 1.5\cos\theta$ and find the value of θ, for $0 \leqslant x < \pi$, at which this maximum occurs. (3)

Tom models the height of sea water, H metres, on a particular day by the equation

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right),\ 0 \leqslant t < 12,$$

where t hours is the number of hours after midday.

(c) Calculate the maximum value of H predicted by this model and the value of t, to 2 decimal places, when this maximum occurs. (3)

(d) Calculate, to the nearest minute, the times when the height of sea water is predicted, by this model, to be 7 metres. (6)

25. (a) Express $7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 decimal places. (3)

(b) Hence write down the minimum value of $7\cos x - 24\sin x$. (1)

(c) Solve, for $0 \leqslant x < 2\pi$, the equation (5)

$$7\cos x - 24\sin x = 10,$$

giving your answers to 2 decimal place

26. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0° \leqslant \theta < 360°$.

27. (a) Prove that (4)

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} \equiv \tan\theta,\ \theta \neq 90n^\circ,\ n \in \mathbb{Z}.$$

(b) Hence, or otherwise,

(i) show that $\tan 15^\circ = 2 - \sqrt{3}$, (3)

(ii) solve, for $0^\circ < x < 360^\circ$, (5)

$$\operatorname{cosec} 4x - \cot 4x = 1.$$

28. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to to 3 significant figures. (4)

$$\mathrm{f}(x) = \mathrm{e}^{2x}\cos 3x.$$

(b) Show that $\mathrm{f}'(x)$ can be written in the form (5)

$$\mathrm{f}'(x) = R\mathrm{e}^{2x}\cos(3x + \alpha),$$

where R and α are the constants found in part (a).

(c) Hence, or otherwise, find the smallest positive value of x which the curve with equation $y = \mathrm{f}(x)$ has a turning point. (3)

29. Solve, for $0^\circ \leqslant \theta \leqslant 180^\circ$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5.$$

Give your answers in degrees to 1 decimal place.

30. (a) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that (4)

$$\tan(A + B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

(b) Deduce that (3)

$$\tan\left(\theta + \tfrac{\pi}{6}\right) \equiv \frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta}.$$

(c) Hence, or otherwise, solve, for $0 \leqslant x \leqslant \pi$, (6)

$$1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta).$$

Give your answers as multiples of π.

31. (a) Express $4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta$ in terms of $\sin\theta$ and $\cos\theta$. (2)

(b) Hence show that (4)

$$4\,\mathrm{cosec}^2\, 2\theta - \mathrm{cosec}^2\, \theta = \sec^2 \theta.$$

(c) Hence or otherwise solve, for $0 < \theta < \pi$, (3)

$$4\,\mathrm{cosec}^2\, 2\theta - \mathrm{cosec}^2\, \theta = 4,$$

giving your answers in terms of π.

32.

$$\mathrm{f}(x) = 7\cos 2x - 24\sin 2x.$$

Given that $\mathrm{f}(x) = R\cos(2x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$,

(a) find the value of R and find the value of α. (3)

(b) Hence solve the equation (5)

$$7\cos 2x - 24\sin 2x = 12.5,$$

for $0° \leqslant x \leqslant 180°$, giving your answers to 1 decimal place.

(c) Express $14\cos^2 x - 48\sin x\cos x$ in the form $a\cos 2x + b\sin 2x + c$, where a, b, and c are constants to be found. (2)

(d) Hence, using your answers to parts (a) and (c), deduce the maximum value of (2)

$$14\cos^2 x - 48\sin x\cos x.$$

33. (a) Express $6\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 decimal places. (4)

(b)

$$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta},\ 0 \leqslant \theta \leqslant 2\pi.$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$, (2)

(ii) the value of θ at which the maximum occurs. (2)

34. (a) Without using a calculator, find the exact value of (5)

$$(\sin 22.5° + \cos 22.5°)^2.$$

(b) (i) Show that $\cos 2\theta + \sin\theta = 1$ may be written in the form (2)

$$k\sin^2\theta - \sin\theta = 0,$$

stating the value of k.

(ii) Hence solve, for $0^\circ \leqslant \theta < 360^\circ$, the equation (4)

$$\cos 2\theta + \sin\theta = 1.$$

35. Given that

$$2\cos(x+50)^\circ = \sin(x+40)^\circ,$$

(a) show, without a calculator, that (4)

$$\tan x^\circ = \tfrac{1}{3}\tan 40^\circ.$$

(b) Hence solve, for $0 \leqslant \theta < 360$, (4)

$$2\cos(2\theta+50)^\circ = \sin(2\theta+40)^\circ,$$

giving your answers to 1 decimal place.

36. Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 ms^{-1}. Kate is 24 m ahead of John when she starts to cross the road from the fixed point A. John passes her as she reaches the other side of the road at a variable point B, as shown in Figure 1.

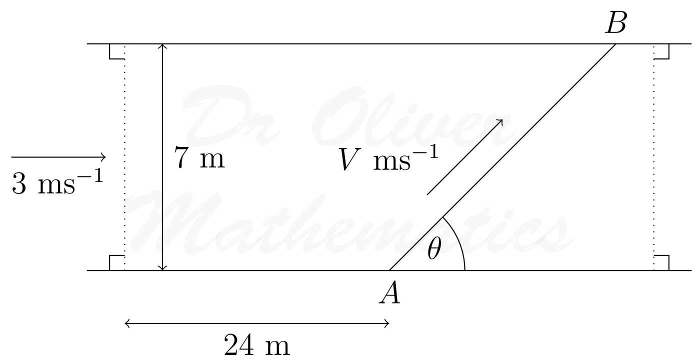

Figure 1: the road

Kate's speed is V ms^{-1} and she moves in a straight line, which makes an angle θ, $0^\circ < \theta < 150^\circ$, with the edge of the road. You may assume that V is given by the formula

$$V = \frac{21}{24\sin\theta + 7\cos\theta},\ 0^\circ < \theta < 150^\circ.$$

(a) Express $24\sin\theta + 7\cos\theta$ in the form $R\cos(\theta-\alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of α to 2 decimal places. (3)

Given that θ varies,

(b) find the minimum value of V. (2)

Given that Kate's speed has the value found in part (b),

(c) find the distance AB. (3)

Given instead that Kate's speed is 1.68 ms^{-1},

(d) find the two possible values of the angle θ, given that $0^\circ < \theta < 150^\circ$. (6)

37.

$$f(x) = 7\cos x + \sin x.$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$,

(a) find the value of R and find the value of α to one decimal place. (3)

(b) Hence solve the equation (5)

$$7\cos x + \sin x = 5$$

for $0^\circ < x < 360^\circ$, giving your answers to one decimal place.

(c) State the values of k for which the equation (2)

$$7\cos x + \sin x = k$$

has only one solution in the interval $0^\circ < x < 360^\circ$.

38. (a) Use an appropriate double angle formula to show that (3)

$$\operatorname{cosec} 2x \equiv \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(b) Solve, for $0 \leqslant \theta < 2\pi$, the equation (6)

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta.$$

Give your answers in terms of π.

39. (a) Show that (5)

$$\operatorname{cosec} 2x + \cot 2x \equiv \cot x,\ x \neq 90^\circ,\ n \in \mathbb{Z}.$$

(b) Hence, or otherwise, solve, for $0^\circ \leqslant \theta < 180^\circ$, (5)

$$\operatorname{cosec}(4\theta + 10)^\circ + \cot(4\theta + 10)^\circ = \sqrt{3}.$$

You must show your working.

40. (a) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 decimal places. (3)

$$\mathrm{H}(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2.$$

Find

(b) (i) the maximum value of $\mathrm{H}(\theta)$, (1)

(ii) the smallest value of θ, for $0 \leqslant \theta < \pi$, at which this maximum value occurs. (2)

(c) (i) the minimum value of $\mathrm{H}(\theta)$, (1)

(ii) the largest value of θ, for $0 \leqslant \theta < \pi$, at which this minimum value occurs. (2)

41. (a) (i) Show that (4)

$$2\tan x - \cot x = 5\,\mathrm{cosec}\,x$$

may be rewritten in the form

$$a\cos^2 x + b\cos x + c = 0,$$

stating the values of the constants a, b, and c.

(ii) Hence solve, for $0 \leqslant x < 2\pi$, the equation (4)

$$2\tan x - \cot x = 5\,\mathrm{cosec}\,x,$$

giving your answers to 3 significant figures.

(b) Show that (4)

$$\tan\theta + \cot\theta \equiv \lambda\,\mathrm{cosec}\,2\theta,\ \theta \neq \tfrac{n\pi}{2},\ n \in \mathbb{Z},$$

stating the value of the constant λ.

42. Figure 2 shows the curve C with equation $y = 6\cos x + 2.5\sin x$ for $0 \leqslant x \leqslant 2\pi$.

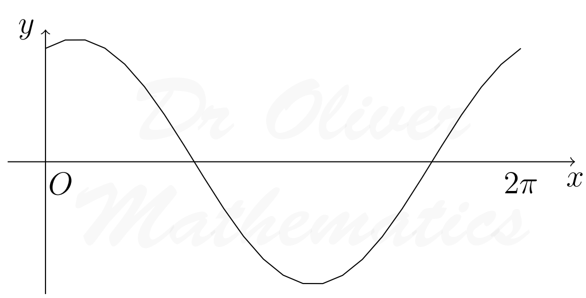

Figure 2: $y = 6\cos x + 2.5\sin x$

(a) Express $6\cos x + 2.5\sin x$ in the form $R\cos(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give the value of α to 3 decimal places. (3)

(b) Find the coordinates of the points on the graph where the curve C crosses the coordinate axes. (3)

A student records the number of hours of daylight each Sunday throughout the year. She starts on the lasts Sunday in May, with a recording of 18 hours, and continues until her final recording 52 weeks later. She models her results with the continuous function given by

$$H = 12 + 6\cos\left(\frac{2\pi t}{52}\right) + 2.5\sin\left(\frac{2\pi t}{52}\right),\ 0 \leqslant t \leqslant 52,$$

where H is the number of hours of daylight and t is the number of weeks since her first recording. Use this function to find

(c) the maximum and minimum values of H predicted by the model, (3)

(d) the values for t when $H = 16$, giving your answers to the nearest whole number. (6)

43. Given that

$$\tan\theta^\circ = p,\ \text{ where } p \text{ is a constant, } p \neq \pm 1,$$

use standard trigonometric identities, to find in terms of p,

(a) $\tan 2\theta^\circ$, (2)

(b) $\cos\theta^\circ$, (2)

(c) $\cot(\theta - 45)^\circ$. (2)

44.

$$\mathrm{g}(\theta) = 4\cos 2\theta + 2\sin 2\theta.$$

Given that $\mathrm{g}(\theta) = R\cos(2\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$,

(a) find the exact value of R and find the exact value of α to 2 decimal places. (3)

(b) Hence solve, for $-90^\circ < \theta < 90^\circ$, (5)

$$4\cos 2\theta + 2\sin 2\theta = 1,$$

giving your answers to one decimal place.

Given that k is a constant and the equation $\mathrm{g}(\theta) = k$ has no solutions,

(c) state the possible values of k. (2)

45. (a) Prove that (5)

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A},\ A \neq \tfrac{(2n+1)\pi}{4},\ n \in \mathbb{Z}.$$

(b) Hence solve, for $0 \leqslant A < 2\pi$, (4)

$$\sec 2A + \tan 2A = \tfrac{1}{2}.$$

Give your answers to 3 decimal places.

46. (a) Express $2\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0° < \alpha < 90°$. Give the exact value for R and give the exact value for α to 2 decimal places. (3)

(b) Hence solve, for $0° < \theta < 360°$, (5)

$$\frac{2}{2\cos\theta - \sin\theta - 1} = 15.$$

Give your answers to one decimal place.

(c) Use your solutions to parts (a) and (b) to deduce the smallest positive value of θ for which (2)

$$\frac{2}{2\cos\theta + \sin\theta - 1} = 15.$$

Give your answer to one decimal place.

47. (a) Prove that (4)

$$2\cot 2x + \tan x \equiv \cot x,\ x \neq \tfrac{n\pi}{2},\ n \in \mathbb{Z}.$$

(b) Hence, or otherwise, solve, for $-\pi \leqslant x < \pi$, (6)

$$6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

48. (a) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 \leqslant \alpha < \frac{\pi}{2}$. Give the exact value of R and give the value of α in radians to 3 decimal places. (3)

(b) Show that the equation (2)

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(c) Hence or otherwise, solve, for $0 \leqslant x < \pi$, (4)

$$5\cot 2x - 3\operatorname{cosec} 2x = 2,$$

giving your answers to 2 decimal places.

49. (a) Prove that (4)

$$\sin 2x - \tan x \equiv \tan x \cos 2x,\ x \neq (2n + 1)90°, n \in \mathbb{Z}.$$

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0° \leqslant x < 360°$, (5)

$$\sin 2x - \tan x = 3\tan x \sin x.$$

Give your answers in degrees to one decimal place where appropriate.