Dr Oliver Mathematics
Mathematics: Higher
2012 Paper 2: Calculator
1 hour 10 minutes

The total number of marks available is 60.
You must write down all the stages in your working.

1. Functions f and g are defined on the set of real numbers by

$$\mathrm{f}(x) = x^2 + 3,$$
$$\mathrm{g}(x) = x + 4.$$

   (a) Find expressions for: (3)
      (i) $\mathrm{f}(\mathrm{g}(x))$,
      (ii) $\mathrm{g}(\mathrm{f}(x))$.

   (b) Show that (3)

$$\mathrm{f}(\mathrm{g}(x)) + \mathrm{g}(\mathrm{f}(x)) = 0$$

   has no real roots.

2. Relative to a suitable set of coordinate axes, Diagram 1 shows the line

$$2x - y + 5 = 0$$

   intersecting the circle

$$x^2 + y^2 - 6x - 2y - 30 = 0$$

   at the points P and Q.

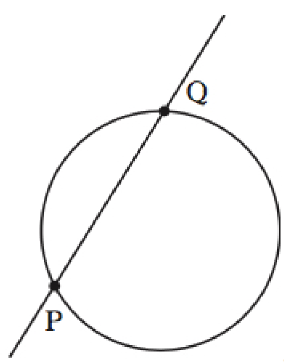


Diagram 1

(a) Find the coordinates of P and Q. (6)

Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

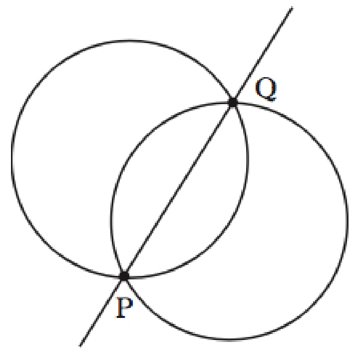


Diagram 2

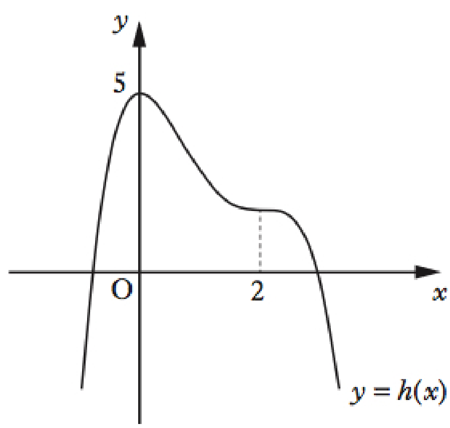
(b) Determine the equation of this second circle. (6)

3. A function f is defined on the domain $0 \leqslant x \leqslant 3$ by (7)

$$f(x) = x^3 - 2x^2 - 4x + 6.$$

Determine the maximum and minimum values of f.

4. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(a) $y = h'(x)$, (3)

(b) $y = 2 - h'(x)$ (3)

5. A is the point $(3, -3, 0)$, B is $(2, -3, 1)$, and C is $(4, k, 0)$.

   (a) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form. (7)

   (ii) Show that

$$\cos\angle ABC = \frac{3}{\sqrt{2(k^2 + 6k + 14)}}.$$

   (b) If angle $ABC = 30°$, find the possible values of k. (5)

6. For $0 < x < \frac{1}{2}\pi$, sequences can be generated using the recurrence relation

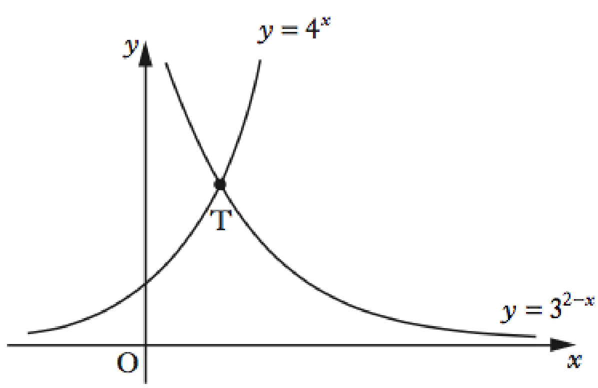
$$u_{n+1} = (\sin x)u_n + \cos 2x, \quad \text{with } u_0 = 1.$$

   (a) Why do these sequences have a limit? (2)

   (b) The limit of one sequence generated by this recurrence relation is $\frac{1}{2}\sin x$. Find the value(s) of x. (7)

7. The diagram shows the curves with equations

$$y = 4^x \text{ and } y = 3^{2-x}.$$



The graphs intersect at the point T.

   (a) Show that the x-coordinate of T can be written in the form (6)

$$\frac{\log_a p}{\log_a q},$$

   for all $a > 1$.

   (b) Calculate the y-coordinate of T, giving your answer to 1 decimal place. (2)