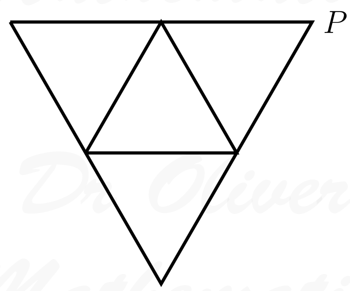
Dr Oliver Mathematics
GCSE Mathematics
2006 June Paper 6H: Calculator
2 hours

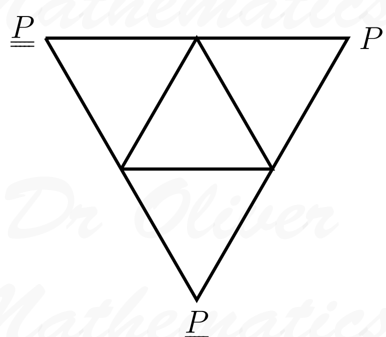
The total number of marks available is 100.
You must write down all the stages in your working.

1. Here is the net of a 3-D shape. (2)



The net is folded to make the 3-D shape.
Two other vertices meet at P.
Mark each of these vertices with the letter P.

Solution



2. Amy, Beth and Colin share 36 sweets in the ratio 2 : 3 : 4. (3)
Work out the number of sweets that each of them receives.

Solution

$$2 + 3 + 4 = 9$$

and so the number of sweets that each of them receives is

Amy:

$$\frac{2}{9} \times 36 = \underline{\underline{8 \text{ sweets}}}.$$

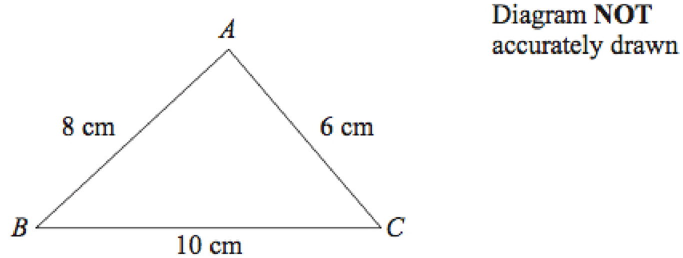
Beth:

$$\frac{3}{9} \times 36 = \underline{\underline{12 \text{ sweets}}}.$$

Colin:

$$\frac{4}{9} \times 36 = \underline{\underline{16 \text{ sweets}}}.$$

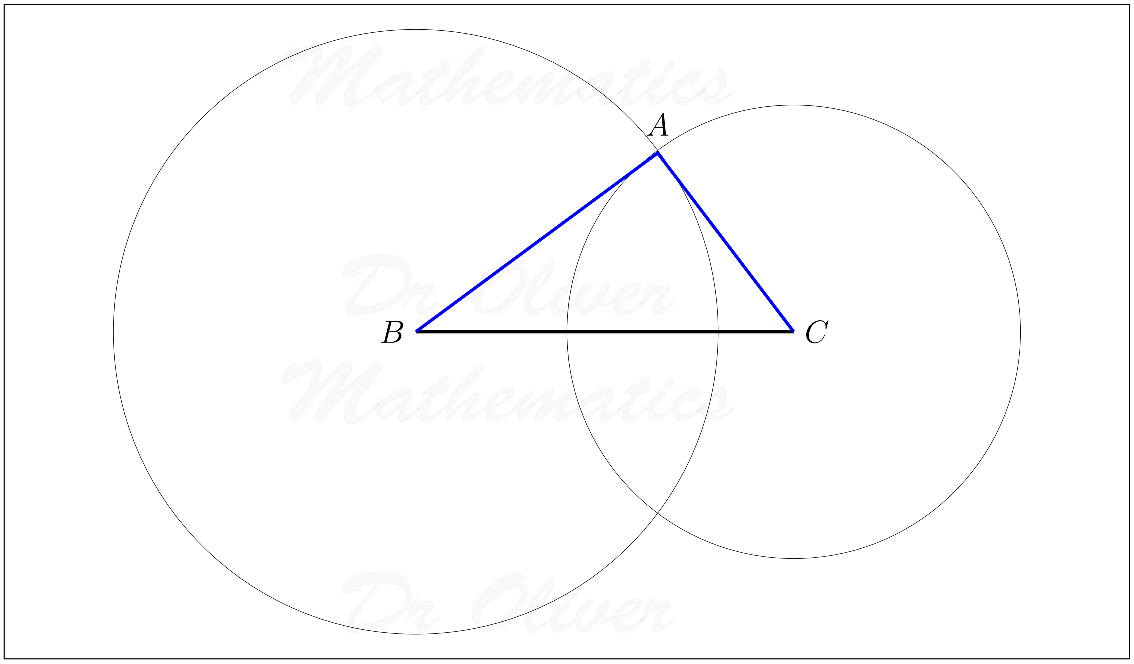
3. ABC is a triangle.



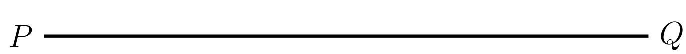
$AB = 8$ cm.
$AC = 6$ cm.
$BC = 10$ cm.

(a) Use ruler and compasses to construct an accurate drawing of triangle ABC. (2)
The line BC has been drawn for you.
You must show all your construction lines.

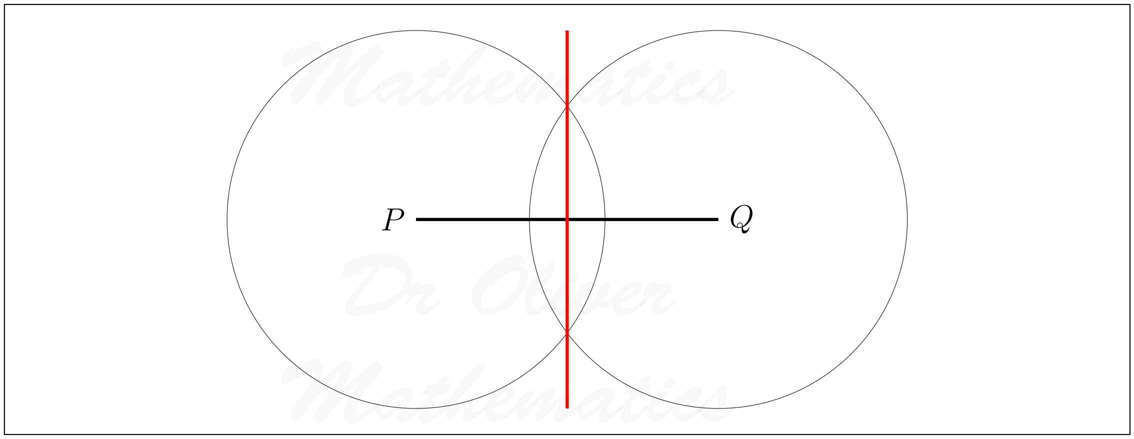
Solution



(b) Use ruler and compasses to construct the perpendicular bisector of the line PQ. You must show all your construction lines. (2)



Solution



4. Sophie says, "For any whole number, n, the value of $6n - 1$ is always a prime number." (2)
Sophie is wrong.
Give an example to show that Sophie is wrong.

Solution

E.g., $n = 6$:

$$6 \times 6 - 1 = 35 = 5 \times 7$$

is **not** prime.

5. This item appeared in a newspaper. (3)

Cows produce 3% more milk.
A farmer found that when his cow listened to classical music the milk it produced increased by 3%.
This increase of 3% represented 0.72 litres of milk.

Calculate the amount of milk produced by the cow when it listened to classical music.

Solution

$$\frac{103}{3} \times 0.72 = \underline{\underline{24.72 \text{ litres}}}.$$

6. (a) Simplify (5)
   (i) $x^4 \times x^5$,

Solution

$$x^4 \times x^5 = \underline{\underline{x^9}}.$$

(ii) $\dfrac{p^8}{p^3}$,

Solution

$$\frac{p^8}{p^3} = \underline{\underline{p^5}}.$$

(iii) $3s^2t^3 \times 4s^4t^2$,

Solution

$$3s^2t^3 \times 4s^4t^2 = \underline{\underline{12s^6t^5}}.$$

(iv) $(q^3)^4$.

Solution

$$(q^3)^4 = \underline{\underline{q^{12}}}.$$

(b) Expand (1)

$$3(2g - 1).$$

Solution

$$3(2g - 1) = \underline{\underline{6g - 3}}.$$

(c) Expand (2)

$$2d(d + 3).$$

Solution

$$2d(d + 3) = \underline{\underline{2d^2 + 6d}}.$$

(d) Expand and simplify (2)

$$(x + 2)(x + 3).$$

Solution

$\times$	x	$+2$
x	x^2	$+2x$
$+3$	$+3x$	$+6$

$$(x+2)(x+3) = \underline{\underline{x^2+5x+6}}.$$

7. PQR is a right-angled triangle. (3)

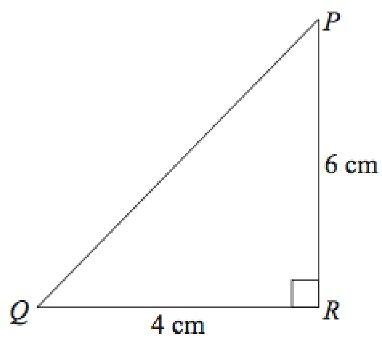


Diagram **NOT** accurately drawn

$PR = 6$ cm.
$QR = 4$ cm.
Work out the length of PQ.
Give your answer correct to 3 significant figures

Solution

$$\begin{aligned} PQ &= \sqrt{PR^2 + QR^2} \\ &= \sqrt{P6^2 + 4^2} \\ &= \sqrt{52} \\ &= 7.211\,102\,551 \text{ (FCD)} \\ &= \underline{\underline{7.21 \text{ cm (3 sf)}}}. \end{aligned}$$

8. Bill recorded the times, in minutes, taken to complete his last 40 homeworks.
This table shows information about the times.

Time (t minutes)	Frequency
$20 \leqslant t < 25$	8
$25 \leqslant t < 30$	3
$30 \leqslant t < 35$	7
$35 \leqslant t < 40$	7
$40 \leqslant t < 45$	15

(a) Find the class interval in which the median lies. (1)

Solution

Time (t minutes)	Frequency	Time (t minutes)	Cum Freq
$20 \leqslant t < 25$	8	$20 \leqslant t < 25$	8
$25 \leqslant t < 30$	3	$20 \leqslant t < 30$	$8 + 3 = 11$
$30 \leqslant t < 35$	7	$20 \leqslant t < 35$	$11 + 7 = 18$
$35 \leqslant t < 40$	7	$20 \leqslant t < 40$	$18 + 7 = 25$
$40 \leqslant t < 45$	15	$20 \leqslant t < 45$	$25 + 15 = 40$

The median is in the

$$\frac{40+1}{2} = 20\tfrac{1}{2}\text{th}$$

place and the median is $\underline{\underline{35 \leqslant t < 40}}$.

(b) Calculate an estimate of the mean time it took Bill to complete each homework. (4)

Solution

Time (t minutes)	Frequency	Midpoint	Frequency×Midpoint
$20 \leqslant t < 25$	8	22.5	180
$25 \leqslant t < 30$	3	27.5	82.5
$30 \leqslant t < 35$	7	32.5	227.5
$35 \leqslant t < 40$	7	37.5	262.5
$40 \leqslant t < 45$	15	42.5	637.5
Total	40		1 390

$$\begin{aligned}\text{Mean} &= \frac{\Sigma ft}{\Sigma f} \\ &\approx \frac{1\,390}{40} \\ &= \underline{\underline{34.75 \text{ minutes}}}.\end{aligned}$$

9. Work out

$$\frac{\sqrt{2.56 + 3.50}}{8.765 - 6.78}.$$

(a) Write down all the figures on your calculator display. (2)

Solution

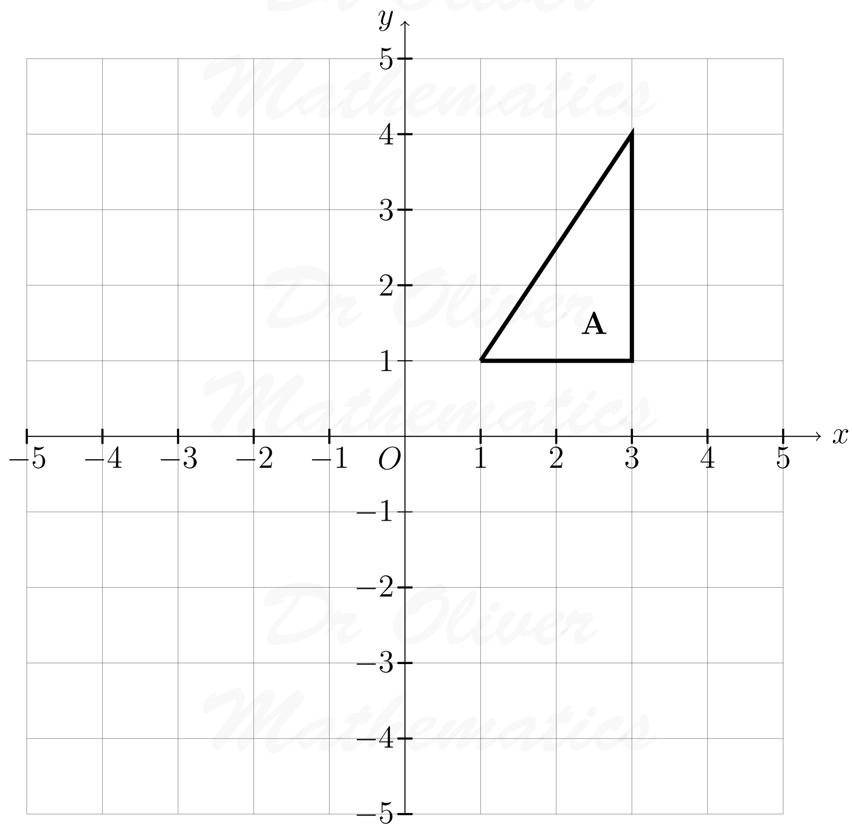
$$\begin{aligned}\frac{\sqrt{2.56 + 3.50}}{8.765 - 6.78} &= \frac{\sqrt{6.06}}{1.985} \\ &= \underline{\underline{1.240\,154\,521 \text{ (FCD)}}}.\end{aligned}$$

(b) Give your answer to part (a) to an appropriate degree of accuracy. (1)

Solution

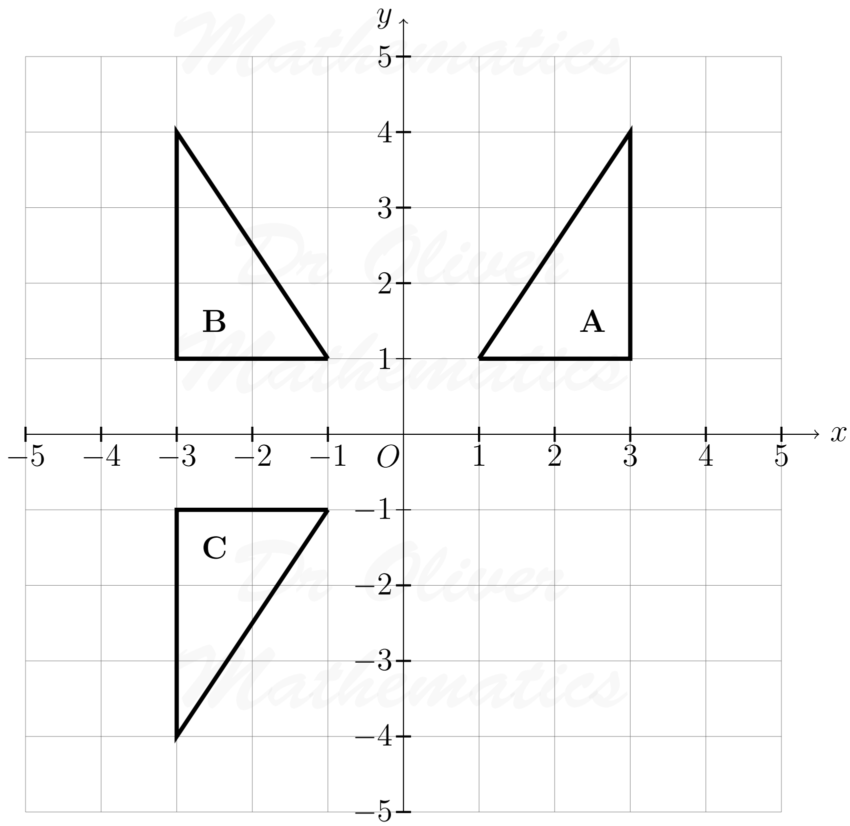
$\underline{\underline{1.24 \text{ (3 sf)}}}$.

10. Here is a picture. (3)



Triangle **A** is reflected in the y-axis to give triangle **B**.
Triangle **B** is then reflected in the x-axis to give triangle **C**.
Describe the **single** transformation that takes triangle **A** to triangle **C**.

Solution



Rotation, centre $(0, 0)$, about $180°$.

11. Solve the simultaneous equations (3)

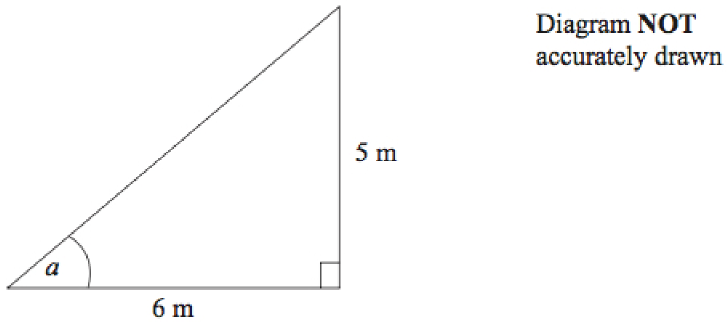
$$5a + 3b = 9$$
$$2a - 3b = 12.$$

Solution

Add:

$$7a = 21 \Rightarrow \underline{\underline{a = 3}}$$
$$\Rightarrow 15 + 3b = 9$$
$$\Rightarrow 3b = -6$$
$$\Rightarrow \underline{\underline{b = -2}}.$$

12. (a) Calculate the size of angle a in this right-angled triangle. (3)

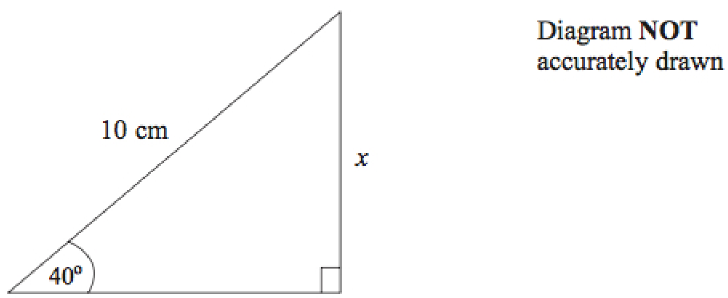


Give your answer correct to 3 significant figures.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan a = \tfrac{5}{6}$$
$$\Rightarrow a = 39.805\,571\,09 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{a = 39.8^\circ \text{ (3 sf)}}}.$$

(b) Calculate the size of angle x in this right-angled triangle. (3)

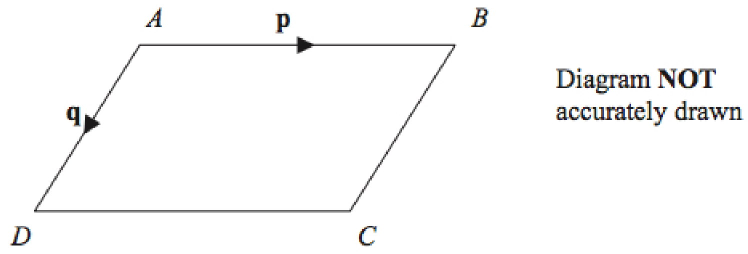


Give your answer correct to 3 significant figures.

Solution

$$\text{opp} = \text{hyp} \times \sin \Rightarrow x = 10 \sin 40^\circ$$
$$\Rightarrow x = 6.427\,876\,097 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 6.43 \text{ cm (3 sf)}}}.$$

13. $ABCD$ is a parallelogram.



AB is parallel to DC.
AD is parallel to BC.
$\overrightarrow{AB} = \mathbf{p}$.
$\overrightarrow{AD} = \mathbf{q}$.

(a) Express, in terms of $\mathbf{p}$ and $\mathbf{q}$, (2)

(i) $\overrightarrow{AC}$,

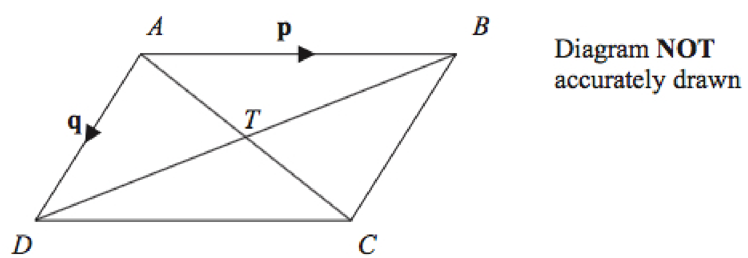
Solution

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \underline{\underline{\mathbf{p} + \mathbf{q}}}.$$

(ii) $\overrightarrow{BD}$.

Solution

$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD} = \underline{\underline{\mathbf{q} - \mathbf{p}}}.$$



AC and BD are diagonals of parallelogram $ABCD$.
AC and BD intersect at T.

(b) Express $\overrightarrow{AT}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. (1)

Solution

$$\overrightarrow{AT} = \tfrac{1}{2}\overrightarrow{AC} = \underline{\underline{\tfrac{1}{2}(\mathbf{p} + \mathbf{q})}}.$$

14. Jim makes a model of his school. (2)
He uses a scale of 1 : 50.
The area of the door on his model is 8 cm^2.
Work out the area of the door on the real school.

Solution

The length scale ratio (LSR) is 50 and the area scale ratio (ASR) is

$$50^2 = 2\,500.$$

Finally, the area of the door on the real school is

$$8 \times 2\,500 = \underline{\underline{20\,000 \text{ cm}^2}}.$$

15. (a) List all the possible integer values of n such that (2)

$$-2 \leqslant n < 3.$$

Solution

$\underline{\underline{-2, -1, 0, 1, 2}}$.

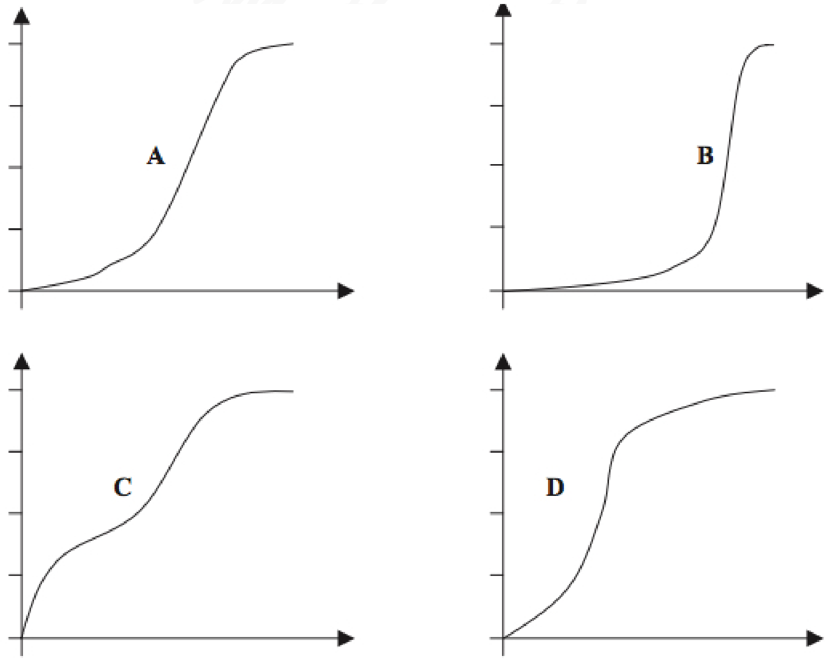
(b) Solve the inequality (2)

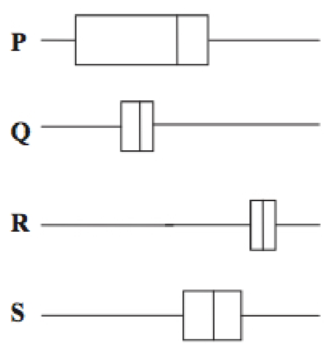
$$4p - 8 < 7 - p.$$

Solution

$$\begin{aligned} 4p - 8 < 7 - p &\Rightarrow 5p < 15 \\ &\Rightarrow \underline{\underline{p < 3}}. \end{aligned}$$

16. Here are four cumulative frequency diagrams. (2)



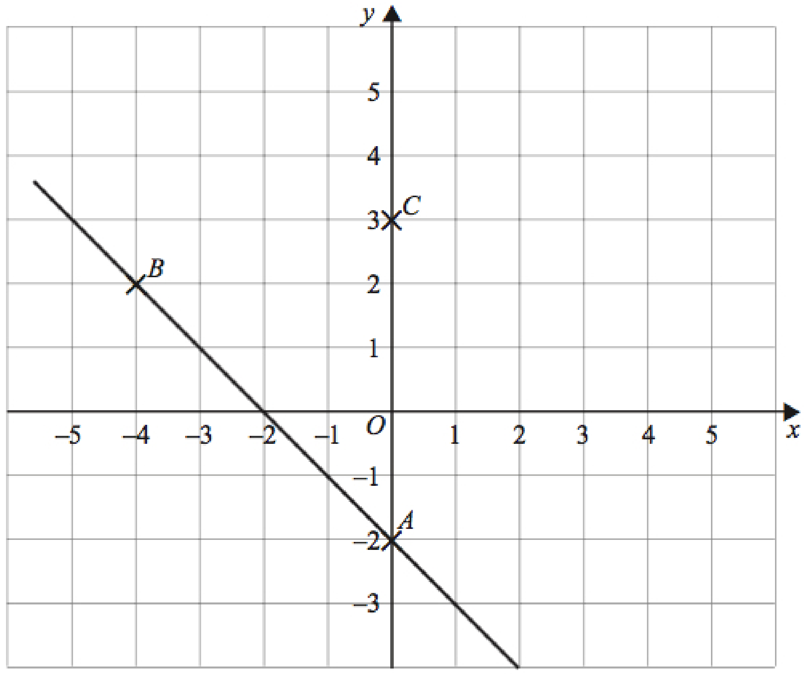


For each box plot, write down the letter of the appropriate cumulative frequency diagram.

Solution

P and **C**.
Q and **D**.
R and **B**.
S and **A**.

17. In the diagram, A is the point $(0, -2)$, B is the point $(-4, 2)$, and C is the point $(0, 3)$. (4)



Find an equation of the line that passes through C and is parallel to AB.

Solution

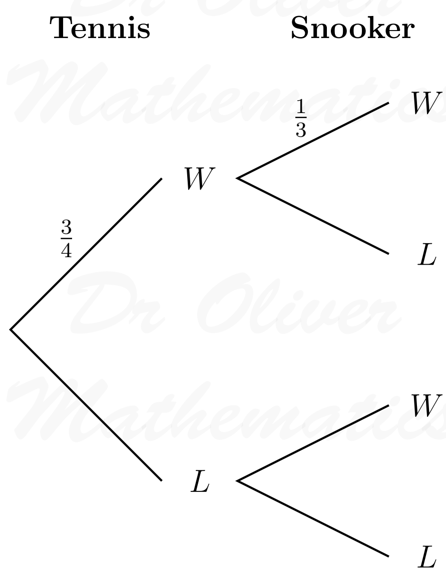
$$\text{Gradient of } AB = \frac{2-(-2)}{-4-0} = -1$$

and so the equation is

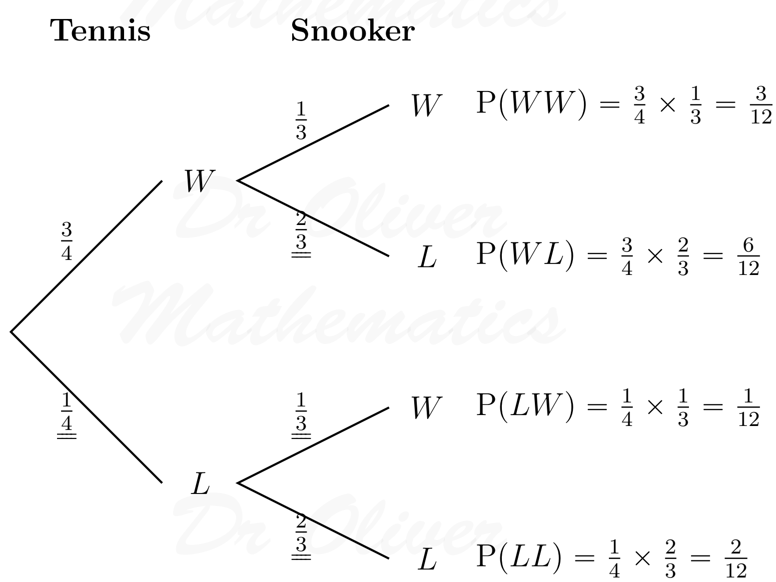
$$y - 3 = -1(x - 0) \Rightarrow y - 3 = -x$$
$$\Rightarrow \underline{\underline{y = -x + 3}}.$$

18. Simon plays one game of tennis and one game of snooker.
The probability that Simon will win at tennis is $\frac{3}{4}$.
The probability that Simon will win at snooker is $\frac{1}{3}$.

(a) Complete the probability tree diagram below. (2)



Solution



(b) Work out the probability that Simon wins both games. (2)

Solution

$$\mathrm{P}(WW) = \frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \underline{\underline{\frac{1}{4}}}.$$

(c) Work out the probability that Simon will win only one game. (3)

Solution

From part (a),

$$P(WL) + P(LW) = \tfrac{6}{12} + \tfrac{1}{12} = \underline{\underline{\tfrac{7}{12}}}.$$

19. The length of a rectangle is 6.7 cm, correct to 2 significant figures.

(a) For the length of the rectangle write down (2)

(i) the upper bound,

Solution

$\underline{\underline{6.75\text{ cm}}}$.

(ii) the lower bound.

Solution

$\underline{\underline{6.65\text{ cm}}}$.

The area of the rectangle is 26.9 cm^2, correct to 3 significant figures.

(b) (i) Calculate the upper bound for the width of the rectangle. (3)
Write down all the figures on your calculator display.

Solution

$$\frac{26.95}{6.65} = \underline{\underline{4.052\,631\,579\text{ cm (FCD)}}}.$$

(ii) Calculate the lower bound for the width of the rectangle.
Write down all the figures on your calculator display.

Solution

$$\frac{26.85}{6.75} = \underline{\underline{3.977\,777\,778\text{ cm (FCD)}}}.$$

(c) (i) Write down the width of the rectangle to an appropriate degree of accuracy. (2)

Solution

Degree of accuracy	Lower bound	Upper bound
1 sf	4	4
2 sf	4.0	4.1

Hence, the width of the rectangle to an appropriate degree of accuracy is 4 cm (1 sf).

(ii) Give a reason for your answer.

Solution
The lower and upper bounds agree to 1 significant figures but not to 2 significant figures.

20. (a) Simplify fully (2)

$$(3x^2y^4)^3.$$

Solution

$$(3x^2y^4)^3 = \underline{\underline{27x^6y^{12}}}.$$

(b) Expand and simplify (2)

$$(2x+5)(3x-2).$$

Solution

×	$2x$	$+5$
$3x$	$6x^2$	$+15x$
-2	$-4x$	-10

Hence,

$$(2x+5)(3x-2) = \underline{\underline{6x^2+11x-10}}.$$

(c) Simplify fully (2)

$$\frac{x^2+5x+6}{x^2+2x}.$$

Solution

$$\begin{aligned}\frac{x^2+5x+6}{x^2+2x} &= \frac{(x+2)(x+3)}{x(x+2)} \\ &= \underline{\underline{\frac{x+3}{x}}}.\end{aligned}$$

21. Solve this quadratic equation (3)

$$x^2 - 5x - 8 = 0.$$

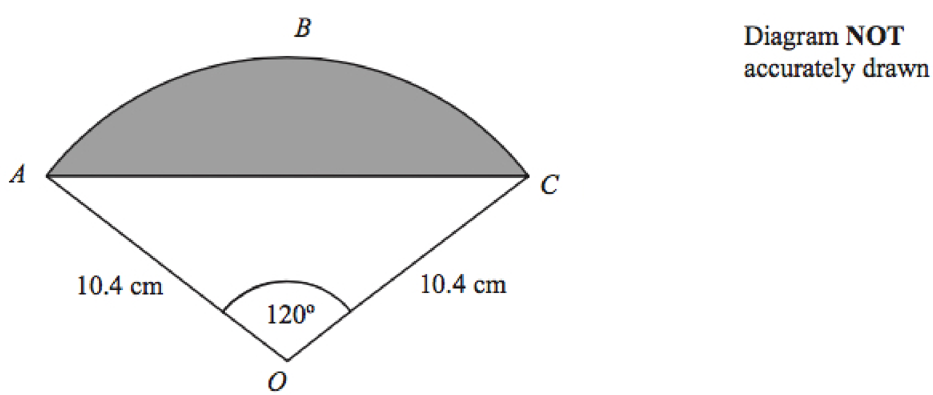
Give your answers correct to 3 significant figures.

Solution

$a = 1$, $b = -5$, and $c = -8$:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\ &= \frac{5 \pm \sqrt{(-5)^2 - 4 \times 1 \times (-8)}}{2 \times 1} \\ &= \frac{5 \pm \sqrt{57}}{2} \\ &= -1.274\,917\,218 \text{ or } 6.274\,917\,218 \text{ (FCD)} \\ &= \underline{\underline{-1.27 \text{ or } 6.27 \text{ (3 sf)}}}.\end{aligned}$$

22. The diagram shows a sector $OABC$ of a circle with centre O.



$OA = OC = 10.4$ cm.
Angle $AOC = 120°$.

(a) Calculate the length of the arc ABC of the sector. (3)
Give your answer correct to 3 significant figures.

Solution

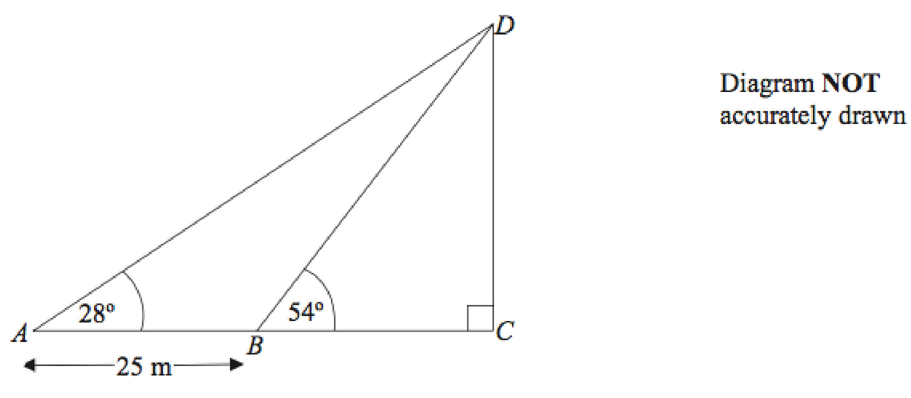
$$\begin{aligned}\text{Arc length} &= \frac{120}{360} \times 2 \times \pi \times 10.4 \\ &= 21.781\,709\,06 \text{ (FCD)} \\ &= \underline{\underline{21.8 \text{ cm (3 sf)}}}.\end{aligned}$$

(b) Calculate the area of the shaded segment ABC. (4)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Area} &= \text{sector} - \text{triangle} \\ &= \left(\tfrac{120}{360} \times \pi \times 10.4^2\right) - \left(\tfrac{1}{2} \times 10.4 \times 10.4 \times \sin 120°\right) \\ &= 66.430\,233\,3 \text{ (FCD)} \\ &= \underline{\underline{66.4 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

23. The diagram shows a vertical tower DC on horizontal ground ABC. (5)



ABC is a straight line.
The angle of elevation of D from A is 28°.

The angle of elevation of D from B is 54°.
$AB = 25$ m.
Calculate the height of the tower.
Give your answer correct to 3 significant figures.

Solution

Angle $ABD = 180 - 54 = 126^\circ$, angle $ADB = 180 - 28 - 126 = 26^\circ$ and now use the sine rule:

$$\frac{BD}{\sin 28^\circ} = \frac{25}{\sin 26^\circ} \Rightarrow BD = \frac{25 \sin 28^\circ}{\sin 26^\circ}.$$

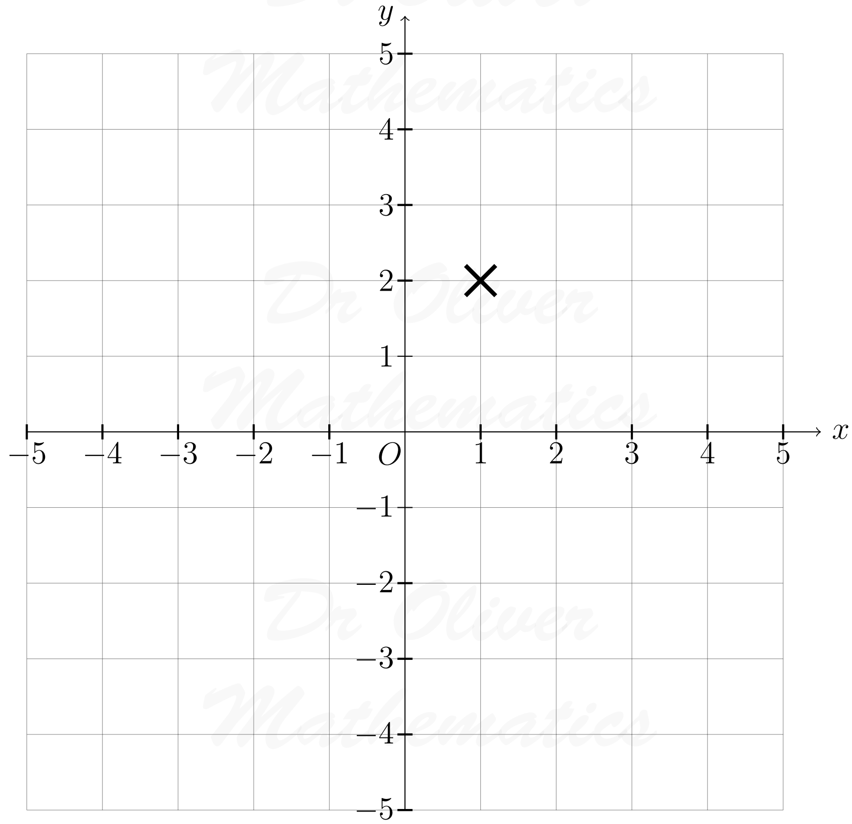
We now use a bit SOHCAHTOA:

$$\begin{aligned}\text{opp} = \text{hyp} \times \sin \Rightarrow BC &= \frac{25 \sin 28^\circ \sin 54^\circ}{\sin 26^\circ} \\ \Rightarrow BC &= 21.660\,325\,7 \text{ (FCD)} \\ \Rightarrow \underline{\underline{BC = 21.7 \text{ m (3 sf)}}}.\end{aligned}$$

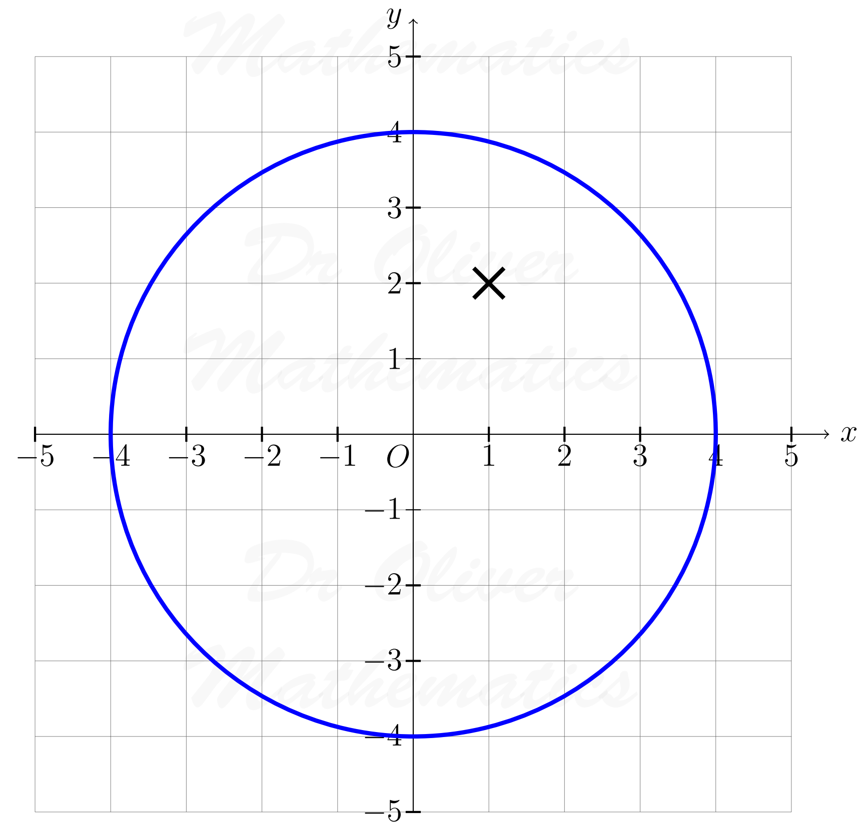
24. Show that any straight line that passes through the point $(1, 2)$ must intersect the curve with equation (3)

$$x^2 + y^2 = 16$$

at two points.



Solution



Because the point $(1, 2)$ is inside the circle and <u>any straight line that passes through the point $(1, 2)$ must intersect the curve twice</u>.