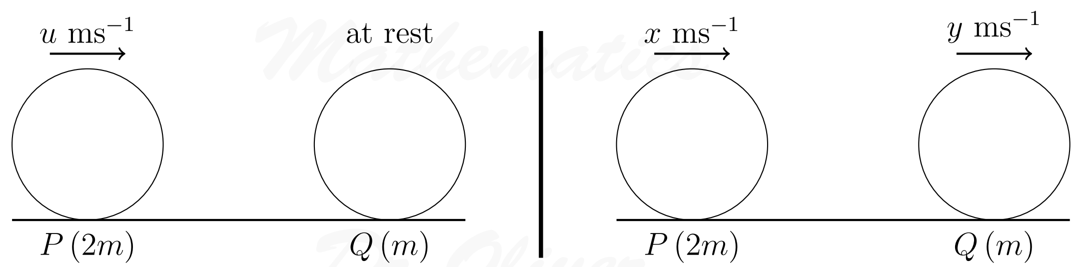
Dr Oliver Mathematics
Further Mathematics
Collisions
Past Examination Questions

This booklet consists of 27 questions across a variety of examination topics.
The total number of marks available is 351.

1. A smooth sphere P of mass $2m$ is moving in a straight line with speed u on a smooth horizontal table. Another smooth sphere Q of mass m is at rest on the table. The sphere P collides directly with Q. The coefficient of restitution between P and Q is $\frac{1}{3}$. The spheres are modelled as particles.

   (a) Show that, immediately after the collision, the speeds of P and Q are $\frac{5}{9}u$ and $\frac{8}{9}u$ respectively. (7)

Solution



$$\text{Conservation of momentum:} \quad (2m)u + 0 = (2m)x + my$$
$$\text{Newton's Law of Restitution:} \quad \frac{y-x}{u} = \tfrac{1}{3}.$$

Now,

$$2mu = 2mx + my \Rightarrow 2u = 2x + y \Rightarrow y = 2u - 2x$$

and

$$\frac{y-x}{u} = \tfrac{1}{3} \Rightarrow y - x = \tfrac{1}{3}u \Rightarrow y = x + \tfrac{1}{3}u.$$

For P,

$$2u - 2x = x + \tfrac{1}{3}u \Rightarrow 3x = \tfrac{5}{3}u \Rightarrow \underline{\underline{x = \tfrac{5}{9}u}}$$

and, for Q,

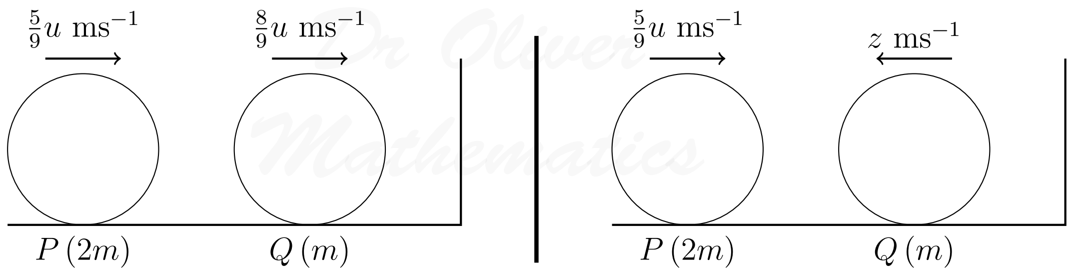
$$y = x + \tfrac{1}{3}u = \tfrac{5}{9}u + \tfrac{1}{3}u = \underline{\underline{\tfrac{8}{9}u}},$$

as required.

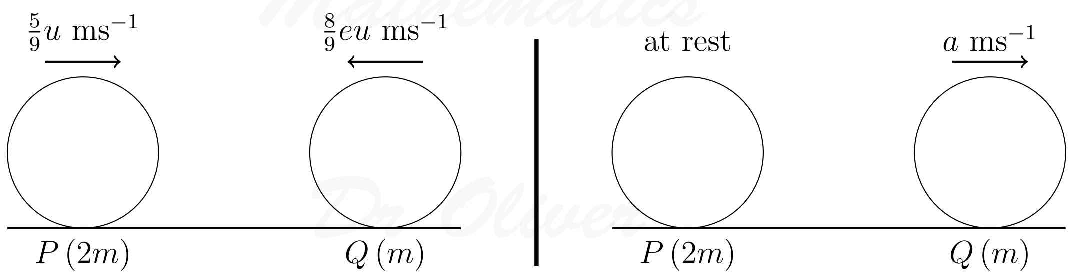
After the collision, Q strikes a fixed vertical wall which is perpendicular to the direction of motion of P and Q. The coefficient of restitution between Q and the wall is e. When P and Q collide again, P is brought to rest.

(b) Find the value of e. (7)

Solution



$$\text{Newton's Law of Restitution:} \quad \frac{z}{\frac{8}{9}u} = e \Rightarrow z = \tfrac{8}{9}eu.$$



$$\text{Conservation of momentum:} \quad (2m)(\tfrac{5}{9}mu) - \tfrac{8}{9}meu = 0 + ma$$

$$\text{Newton's Law of Restitution:} \quad \frac{a}{\frac{5}{9}u + \frac{8}{9}eu} = \tfrac{1}{3}.$$

Now,

$$\tfrac{10}{9}mu - \tfrac{8}{9}meu = ma \Rightarrow u(\tfrac{10}{9} - \tfrac{8}{9}e) = a$$

and

$$\frac{a}{\frac{5}{9}u + \frac{8}{9}eu} = \tfrac{1}{3} \Rightarrow a = \tfrac{1}{3}u(\tfrac{5}{9} + \tfrac{8}{9}e) \Rightarrow a = u(\tfrac{5}{27} + \tfrac{8}{27}e).$$

Now,

$$\tfrac{10}{9} - \tfrac{8}{9}e = \tfrac{5}{27} + \tfrac{8}{27}e \Rightarrow \tfrac{25}{27} = \tfrac{32}{27}e$$
$$\Rightarrow \underline{\underline{e = \tfrac{25}{32}}}.$$

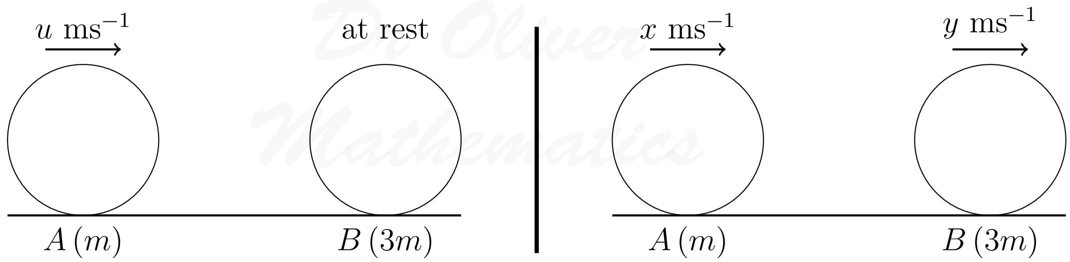
(c) Explain why there must be a third collision between P and Q. (1)

> **Solution**
> Q still has velocity $\frac{5}{12}u$ ms^{-1} and will bounce back from wall colliding with stationary P; therefore, there will be a third collision.

2. A smooth sphere A of mass m is moving with speed u on a smooth horizontal table when it collides directly with another smooth sphere B of mass $3m$, which is at rest on the table. The coefficient of restitution between A and B is e. The spheres have the same radius and are modelled as particles.

(a) Show that the speed of B immediately after the collision is $\frac{1}{4}(1+e)u$. (5)

Solution



$$\text{Conservation of momentum:} \quad mu + 0 = mx + 3my$$
$$\text{Newton's Law of Restitution:} \quad \frac{y-x}{u} = e.$$

Now,
$$mu + 0 = mx + 3myz \Rightarrow u = x + 3y \Rightarrow x = u - 3y$$
and
$$\frac{y-x}{u} = e \Rightarrow y - x = eu \Rightarrow x = y - ue$$
and eliminate:
$$u - 3y = y - ue \Rightarrow 4y = u(1+e) \Rightarrow y = \tfrac{1}{4}u(1+e).$$

(b) Find the speed of A immediately after the collision. (2)

Solution

$$\begin{aligned} x &= u - 3y \\ &= u - \tfrac{3}{4}u(1+e) \\ &= \tfrac{1}{4}u[4 - 3(1+e)] \\ &= \tfrac{1}{4}u(4 - 3 - 3e) \\ &= \tfrac{1}{4}u(1 - 3e), \end{aligned}$$

and hence the speed is $\underline{\underline{\left|\tfrac{1}{4}u(1-3e)\right|}}$.

Immediately after the collision the total kinetic energy of the spheres is $\frac{1}{6}mu^2$.

(c) Find the value of e. (6)

Solution

$$\begin{aligned} & \tfrac{1}{2}m\left[\tfrac{1}{4}u(1-3e)\right]^2 + \tfrac{1}{2}(3m)\left[\tfrac{1}{4}u(1+e)\right]^2 = \tfrac{1}{6}mu^2 \\ \Rightarrow\ & \tfrac{1}{32}(1-3e)^2 + \tfrac{3}{32}(1+e)^2 = \tfrac{1}{6} \\ \Rightarrow\ & (1-3e)^2 + 3(1+e)^2 = \tfrac{32}{6} \\ \Rightarrow\ & 6(1-3e)^2 + 18(1+e)^2 = 32 \\ \Rightarrow\ & 6(1-6e+9e^2) + 18(1+2e+e^2) = 32 \\ \Rightarrow\ & (6-36e+54e^2) + (18+36e+18e^2) = 32 \\ \Rightarrow\ & 24 + 72e^2 = 32 \\ \Rightarrow\ & 72e^2 = 8 \\ \Rightarrow\ & e^2 = \tfrac{1}{9} \\ \Rightarrow\ & \underline{\underline{e = \tfrac{1}{3}}}. \end{aligned}$$

(d) Hence show that A is at rest after the collision. (1)

Solution

$$e = \tfrac{1}{3} \Rightarrow \tfrac{1}{4}u(1 - 3 \times \tfrac{1}{3}) = 0;$$

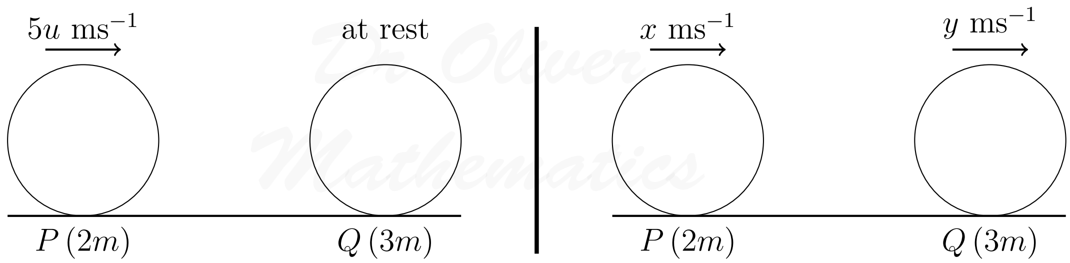
A $\underline{\underline{\text{is at rest}}}$.

3. Two small smooth spheres, P and Q, of equal radius, have masses $2m$ and $3m$ respectively. The sphere P is moving with speed $5u$ on a smooth horizontal table when it

collides directly with Q, which is at rest on the table. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $2(1+e)u$. (5)

Solution



$$\text{Conservation of momentum:} \quad (2m)(5u) + 0 = (2m)x + (3m)y$$
$$\text{Newton's Law of Restitution:} \quad \frac{y-x}{5u} = e.$$

Now,

$$10mu = 2mx + 3my \Rightarrow 10u = 2x + 3y \Rightarrow x = 5u - \tfrac{3}{2}y$$

and

$$\frac{y-x}{5u} = e \Rightarrow y - x = 5eu \Rightarrow x = y - 5ue$$

and eliminate x:

$$5u - \tfrac{3}{2}y = y - 5ue \Rightarrow \tfrac{5}{2}y = 5u(1+e) \Rightarrow \underline{\underline{y = 2u(1+e)}}.$$

After the collision, Q hits a smooth vertical wall which is at the edge of the table and perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f, $0 < f \leqslant 1$.

(b) Show that, when $e = 0.4$, there is a second collision between P and Q. (3)

Solution

$$x = 5u - \tfrac{3}{2} \times 2u(1 + 0.4) = 5u - 4.2u = 0.8u$$

and Q hits the wall and the speed back is

$$y = 2u(1 + 0.4)f = 2.8fu;$$

there is, indeed, <u>a second collision</u>.

Given that $e = 0.8$ and there is a second collision between P and Q,

(c) find the range of possible values of f. (3)

Solution

$$x = 5u - \tfrac{3}{2} \times 2u(1 + 0.8) = 5u - 5.4u = -0.4u$$

and Q hits the wall and the speed back is

$$y = 2u(1 + 0.8)f = 3.6fu;$$

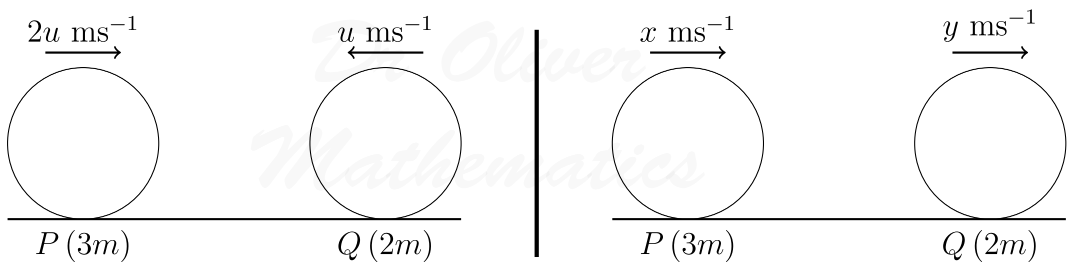
there is a second collision when

$$3.6fu > 0.4u \Rightarrow \underline{\underline{f > \tfrac{1}{9}}}.$$

4. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal table. The particle P collides with a particle Q of mass $2m$ moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q after the collision is $\frac{1}{5}u(9e + 4)$. (5)

Solution



$$\text{Conservation of momentum:} \quad (3m)(2u) - (2m)u = (3m)x + (2m)y$$

$$\text{Newton's Law of Restitution:} \quad \frac{y - x}{3u} = e.$$

Now,

$$6mu - 2mu = 3mx + 2my \Rightarrow 4u = 3x + 2y \Rightarrow x = \tfrac{4}{3}u - \tfrac{2}{3}y$$

and

$$\frac{y - x}{3u} = e \Rightarrow y - x = 3eu \Rightarrow x = y - 3ue$$

and eliminate x:

$$\tfrac{4}{3}u - \tfrac{2}{3}y = y - 3ue \Rightarrow \tfrac{5}{3}y = \tfrac{1}{3}u(4+9e) \Rightarrow \underline{\underline{y = \tfrac{1}{5}u(9e+4)}}.$$

As a result of the collision, the direction of motion of P is reversed.

(b) Find the range of possible values of e. (5)

Solution

$$\begin{aligned} x < 0 &\Rightarrow \tfrac{1}{5}u(9e+4) - 3ue < 0 \\ &\Rightarrow \tfrac{1}{5}(9e+4) < 3e \\ &\Rightarrow 9e + 4 < 15e \\ &\Rightarrow 4 < 6e \\ &\Rightarrow \tfrac{2}{3} < e, \end{aligned}$$

and hence

$$\underline{\underline{\tfrac{2}{3} < e \leqslant 1}}.$$

Given that the magnitude of the impulse of P on Q is $\frac{32}{5}mu$,

(c) find the value of e. (4)

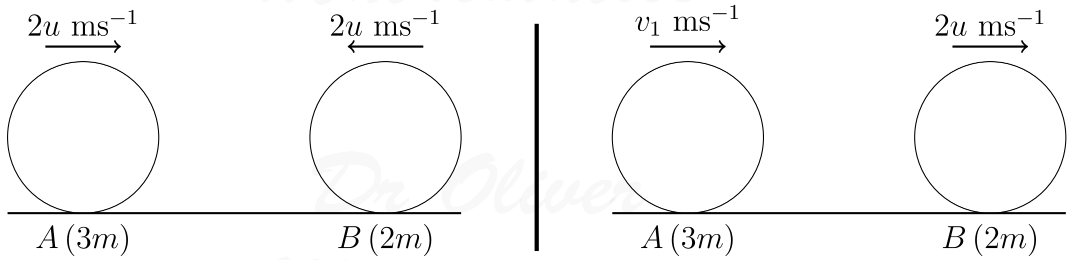
Solution

$$\begin{aligned} 2m[\tfrac{1}{5}u(9e+4) - (-u)] = \tfrac{32}{5}mu &\Rightarrow \tfrac{2}{5}(9e+4) + 2 = \tfrac{32}{5} \\ &\Rightarrow \tfrac{2}{5}(9e+4) = \tfrac{22}{5} \\ &\Rightarrow 9e + 4 = 11 \\ &\Rightarrow 9e = 7 \\ &\Rightarrow \underline{\underline{e = \tfrac{7}{9}}}. \end{aligned}$$

5. Two small spheres A and B have mass $3m$ and $2m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane, both with speed $2u$, when they collide directly. As a result of the collision, the direction of motion of B is reversed and its speed is unchanged.

(a) Find the coefficient of restitution between the spheres. (7)

Solution



Conservation of momentum: $(3m)(2u) - (2m)(2u) = (3m)v_1 + (2m)(2u)$

Newton's Law of Restitution: $\dfrac{2u - v_1}{4u} = e.$

Now,

$$2mu = 3mv_1 + 4mu \Rightarrow -2u = 3v_1 \Rightarrow v_1 = -\tfrac{2}{3}u$$

and

$$\frac{2u - v_1}{4u} = e \Rightarrow 2u - v_1 = 4eu \Rightarrow v_1 = 2u - 4eu$$

and eliminate v_1:

$$\begin{aligned} -\tfrac{2}{3}u = 2u - 4eu &\Rightarrow -\tfrac{2}{3} = 2 - 4e \\ &\Rightarrow 4e = \tfrac{8}{3} \\ &\Rightarrow \underline{\underline{e = \tfrac{2}{3}}}. \end{aligned}$$

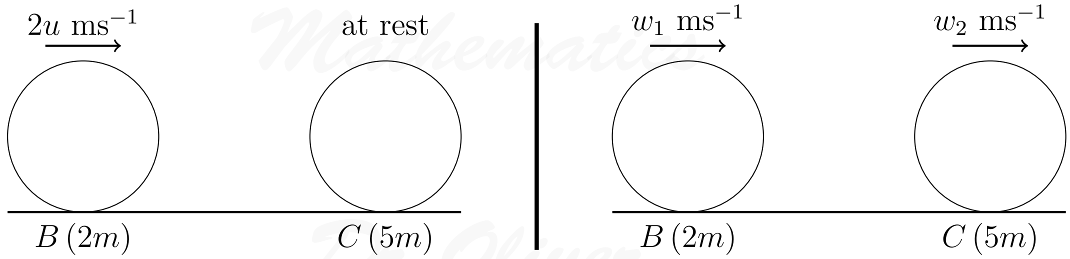
Subsequently, B collides directly with another small sphere C of mass $5m$ which is at rest. The coefficient of restitution between B and C is $\frac{3}{5}$.

(b) Show that, after B collides with C, there will be no further collisions between the spheres. (7)

Solution

We need v_1:

$$v_1 = 2u - 4u \times \tfrac{2}{3} = -\tfrac{2}{3}u.$$



$$\text{Conservation of momentum:} \quad (2m)(2u) + 0 = (2m)w_1 + (5m)w_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{w_2 - w_1}{2u} = \tfrac{3}{5}.$$

Now,

$$4mu = 2mw_1 + 5mw_2 \Rightarrow 4u = 2w_1 + 5w_2 \Rightarrow w_1 = 2u - \tfrac{5}{2}w_2$$

and

$$\frac{w_2 - w_1}{2u} = \tfrac{3}{5} \Rightarrow w_2 - w_1 = \tfrac{6}{5}u \Rightarrow w_1 = w_2 - \tfrac{6}{5}u$$

and we eliminate w_1:

$$2u - \tfrac{5}{2}w_2 = w_2 - \tfrac{6}{5}u \Rightarrow \tfrac{7}{2}w_2 = \tfrac{16}{5}u \Rightarrow w_2 = \tfrac{32}{35}u$$

and

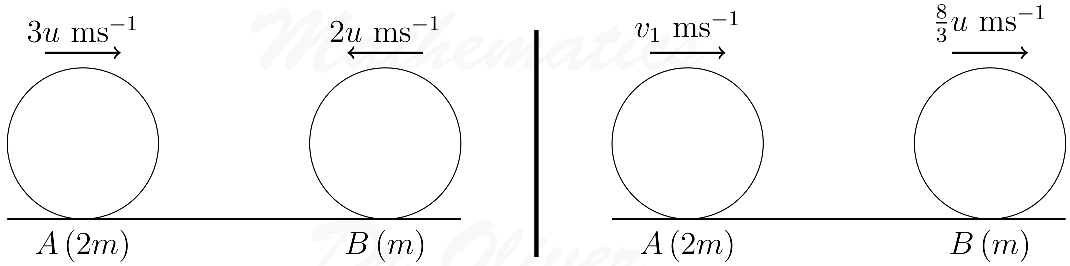
$$w_1 = 2u - \tfrac{5}{2} \times \tfrac{32}{35}u = -\tfrac{2}{7}u.$$

So, A and B are going backwards (A is going faster than B is) and C is going forward; hence, there are no more collisions.

6. A particle A of mass $2m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle collides directly with a particle B of mass m moving with speed $2u$ in the opposite direction to A. Immediately after the collision, the speed of B is $\frac{8}{3}u$ and the direction of motion of B is reversed.

   (a) Calculate the coefficient of restitution between A and B. (6)

Solution



Conservation of momentum: $(2m)(3u) - m(2u) = (2m)v_1 + m(\frac{8}{3}u)$

Newton's Law of Restitution: $\dfrac{\frac{8}{3}u - v_1}{5u} = e.$

Now,

$$4u = 2v_1 + \tfrac{8}{3}u \Rightarrow \tfrac{4}{3}u = 2v_1 \Rightarrow v_1 = \tfrac{2}{3}u$$

and

$$\frac{\frac{8}{3}u - v_1}{5u} = e \Rightarrow \tfrac{8}{3}u - v_1 = 5eu \Rightarrow v_1 = \tfrac{8}{3}u - 5eu$$

and eliminate v_1:

$$\tfrac{2}{3}u = \tfrac{8}{3}u - 5eu \Rightarrow 5e = 2 \Rightarrow \underline{\underline{e = \tfrac{2}{5}}}.$$

(b) Show that the kinetic energy lost in the collision is $7mu^2$. (3)

Solution

$$\begin{aligned}\text{Initial KE} &= \tfrac{1}{2}(2m)(3u)^2 + \tfrac{1}{2}m(2u)^2 \\ &= 9mu^2 + 2mu^2 \\ &= 11mu^2\end{aligned}$$

and

$$\begin{aligned}\text{final KE} &= \tfrac{1}{2}(2m)(\tfrac{2}{3}u)^2 + \tfrac{1}{2}m(\tfrac{8}{3}u)^2 \\ &= \tfrac{4}{9}mu^2 + \tfrac{32}{9}mu^2 \\ &= 4mu^2;\end{aligned}$$

hence, there is $\underline{\underline{7mu^2}}$ lost in the collision.

After the collision B strikes a fixed vertical wall that is perpendicular to the direction of motion of B. The magnitude of the impulse of the wall on B is $\frac{14}{3}mu$.

(c) Calculate the coefficient of restitution between B and the wall. (4)

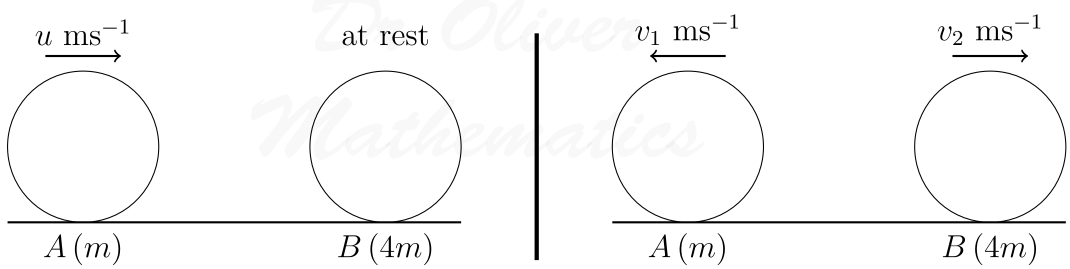
Solution

$$\begin{aligned} mv - (-\tfrac{8}{3}mu) = \tfrac{14}{3}mu &\Rightarrow v + \tfrac{8}{3}u = \tfrac{14}{3}u \\ &\Rightarrow v = 2u \\ &\Rightarrow e = \frac{2u}{\frac{8}{3}u} \\ &\Rightarrow \underline{\underline{e = \tfrac{3}{4}}}. \end{aligned}$$

7. Two particles A and B move on a smooth horizontal table. The mass of A is m, and the mass of B is $4m$. Initially A is moving with speed u when it collides directly with B, which is at rest on the table. As a result of the collision, the direction of motion of A is reversed. The coefficient of restitution between the particles is e.

(a) Find expressions for the speed of $A$and the speed of B immediately after the collision. (7)

Solution



$$\text{Conservation of momentum:} \quad mu + 0 = -mv_1 + 4mv_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 + v_1}{u} = e.$$

Now,

$$mu = -mv_1 + 4mv_2 \Rightarrow u = -v_1 + 4v_2 \Rightarrow v_1 = 4v_2 - u$$

and

$$\frac{v_2 + v_1}{u} = e \Rightarrow v_2 + v_1 = eu \Rightarrow v_1 = eu - v_2$$

and eliminate v_1:

$$4v_2 - u = eu - v_2 \Rightarrow 5v_2 = u(1+e) \Rightarrow \underline{\underline{v_2 = \tfrac{1}{5}u(1+e)}}$$

and

$$v_1 = eu - v_2 = eu - \tfrac{1}{5}u(1+e) = \tfrac{4}{5}eu - \tfrac{1}{5}u = \underline{\underline{\tfrac{1}{5}u(4e-1)}}.$$

In the subsequent motion, B strikes a smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{4}{5}$. Given that there is a second collision between A and B,

(b) show that $\frac{1}{4} < e < \frac{9}{16}$. (5)

Solution

B's speed as it leaves the wall is

$$\tfrac{1}{5}u(1+e) \times \tfrac{4}{5} = \tfrac{4}{25}u(1+e)$$

and, as there is a second collision between A and B,

$$\begin{aligned}\tfrac{4}{25}u(1+e) > \tfrac{1}{5}u(4e-1) &\Rightarrow 4(1+e) > 5(4e-1)\\ &\Rightarrow 4 + 4e > 20e - 5\\ &\Rightarrow 9 > 16e\\ &\Rightarrow \tfrac{9}{16} > e\end{aligned}$$

and

$$\begin{aligned}v_1 > 0 &\Rightarrow \tfrac{1}{5}u(4e-1) > 0\\ &\Rightarrow 4e - 1 > 0\\ &\Rightarrow 4e > 1\\ &\Rightarrow e > \tfrac{1}{4};\end{aligned}$$

hence,

$$\underline{\underline{\tfrac{1}{4} < e < \tfrac{9}{16}}}.$$

Given that $e = \frac{1}{2}$,

(c) find the total kinetic energy lost in the first collision between A and B. (3)

Solution

$$\text{Initial KE} = \tfrac{1}{2}mu^2 + 0 = \tfrac{1}{2}mu^2$$

and

$$\begin{aligned}\text{final KE} &= \tfrac{1}{2}m(\tfrac{1}{5}u)^2 + \tfrac{1}{2}(4m)(\tfrac{3}{10}u)^2 \\ &= \tfrac{1}{50}mu^2 + \tfrac{9}{50}mu^2 \\ &= \tfrac{1}{5}mu^2;\end{aligned}$$

hence,

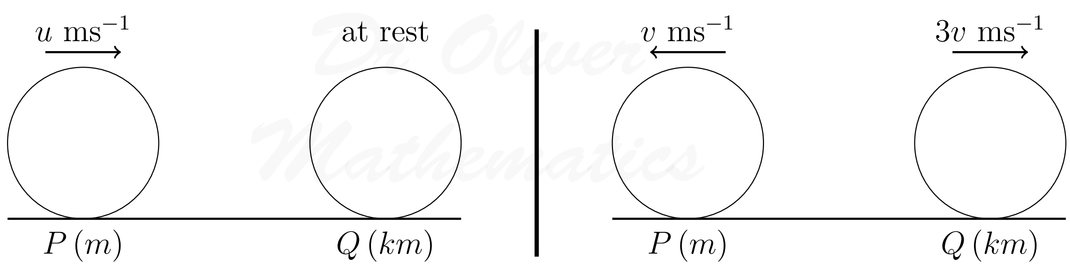
$$\tfrac{1}{2}mu^2 - \tfrac{1}{5}mu^2 = \underline{\underline{\tfrac{3}{10}mu^2}}$$

was lost.

8. A particle P of mass m is moving in a straight line on a smooth horizontal table. Another particle Q of mass km is at rest on the table. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. After the collision, the speed of P is v and the speed of Q is $3v$. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Find, in terms of v only, the speed of P before the collision. (3)

Solution



Conservation of momentum: $mu + 0 = -mv + 3kmv$

Newton's Law of Restitution: $\dfrac{4v}{u} = \tfrac{1}{2}$.

Now,

$$\frac{4v}{u} = \tfrac{1}{2} \Rightarrow 4v = \tfrac{1}{2}u \Rightarrow \underline{\underline{u = 8v}}.$$

(b) Find the value of k. (3)

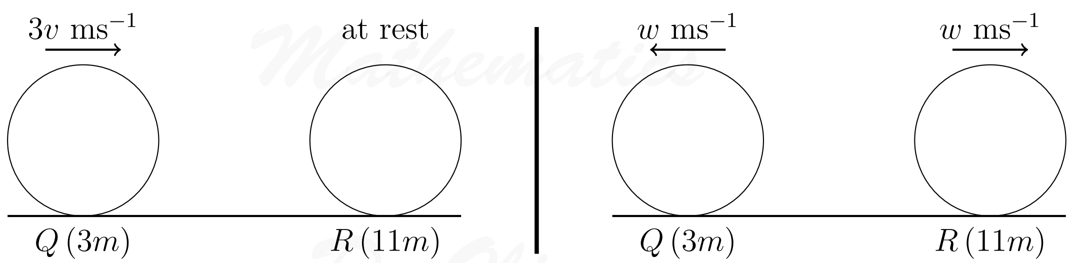
Solution

$$
\begin{aligned}
mu = -mv + 3kmv &\Rightarrow 8mv = -mv + 3kmv \\
&\Rightarrow 8 = -1 + 3k \\
&\Rightarrow 3k = 9 \\
&\Rightarrow \underline{\underline{k = 3}}.
\end{aligned}
$$

After being struck by P, the particle Q collides directly with a particle R of mass $11m$ which is at rest on the table. After this second collision, Q and R have the same speed and are moving in opposite directions. Show that

(c) the coefficient of restitution between Q and R is $\frac{3}{4}$, (4)

Solution



Conservation of momentum: $(3m)(3v) + 0 = -(3m)w + (11m)w$

Newton's Law of Restitution: $\dfrac{2w}{3v} = e.$

Now,

$$9v = -3w + 11w \Rightarrow 8w = 9v \Rightarrow w = \tfrac{9}{8}v$$

and

$$\frac{2w}{3v} = e \Rightarrow 2w = 3ev \Rightarrow w = \tfrac{3}{2}ev$$

and eliminate w:

$$\tfrac{9}{8}v = \tfrac{3}{2}ev \Rightarrow \underline{\underline{e = \tfrac{3}{4}}}.$$

(d) there will be a further collision between P and Q. (2)

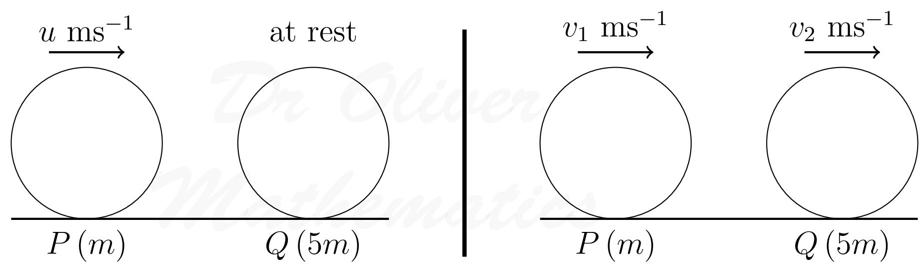
Solution

P is going backwards at v ms^{-1} and Q is going backwards at $\frac{9}{8}v$ ms^{-1}; hence, there will be a further collision between P and Q.

9. Two small spheres P and Q of equal radius have masses m and $5m$ respectively. They lie on a smooth horizontal table. Sphere P is moving with speed u when it collides directly with sphere Q which is at rest. The coefficient of restitution between the spheres is e, where $e > \frac{1}{5}$.

(a) (i) Show that the speed of P immediately after the collision is $\frac{1}{6}u(5e - 1)$. (6)

Solution



$$\text{Conservation of momentum:} \quad mu + 0 = mv_1 + (5m)v_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{u} = e.$$

Now,

$$u = v_1 + 5v_2 \Rightarrow 5v_2 = u - v_1 \Rightarrow v_2 = \tfrac{1}{5}u - \tfrac{1}{5}v_1$$

and

$$\frac{v_2 - v_1}{u} = e \Rightarrow v_2 - v_1 = eu \Rightarrow v_2 = v_1 + eu$$

and eliminate v_2:

$$\tfrac{1}{5}u - \tfrac{1}{5}v_1 = v_1 + eu \Rightarrow \tfrac{6}{5}v_1 = \tfrac{1}{5}u(1 - 5e) \Rightarrow v_1 = \tfrac{1}{6}u(1 - 5e);$$

hence, as $e > \frac{1}{5}$, P's speed is $\frac{1}{6}u(5e - 1)$ backwards.

(ii) Find an expression for the speed of Q immediately after the collision, giving your answer in the form λu, where λ is in terms of e.

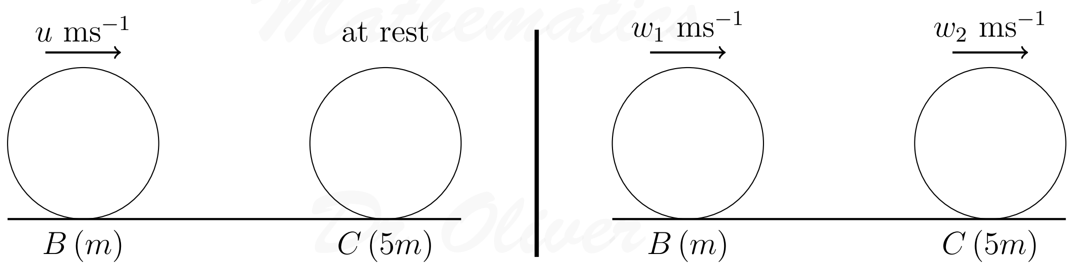
Solution

$$\begin{aligned} v_2 &= \tfrac{1}{6}u(1-5e) + eu \\ &= \tfrac{1}{6}u - \tfrac{5}{6}eu + eu \\ &= \tfrac{1}{6}u + \tfrac{1}{6}eu \\ &= \underline{\underline{\tfrac{1}{6}u(1+e)}}. \end{aligned}$$

Three small spheres A, B, and C of equal radius lie at rest in a straight line on a smooth horizontal table, with B between A and C. The spheres A and C each have mass $5m$, and the mass of B is m. Sphere B is projected towards C with speed u. The coefficient of restitution between each pair of spheres is $\frac{4}{5}$.

(b) Show that, after B and C have collided, there is a collision between B and A. (3)

Solution



$$\begin{aligned} \text{Conservation of momentum:} &\quad mu + 0 = mw_1 + 5mw_2 \\ \text{Newton's Law of Restitution:} &\quad \frac{w_2 - w_1}{u} = \tfrac{4}{5}. \end{aligned}$$

Now,

$$u = w_1 + 5w_2 \Rightarrow 5w_2 = u - w_1 \Rightarrow w_2 = \tfrac{1}{5}u - \tfrac{1}{5}w_1$$

and

$$\frac{w_2 - w_1}{u} = \tfrac{4}{5} \Rightarrow w_2 - w_1 = \tfrac{4}{5}u \Rightarrow w_2 = \tfrac{4}{5}u + w_1$$

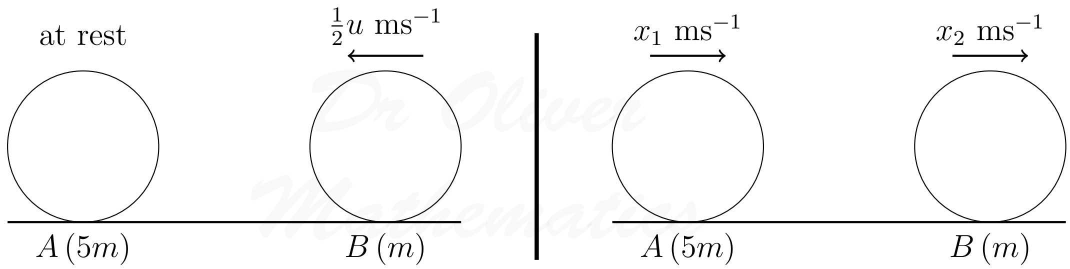
and eliminate w_2:

$$\tfrac{1}{5}u - \tfrac{1}{5}w_1 = w_1 + \tfrac{4}{5}u \Rightarrow \tfrac{6}{5}w_1 = -\tfrac{3}{5}u \Rightarrow w_1 = -\tfrac{1}{2}u;$$

hence, B's speed is backwards and $\underline{\underline{\text{there is a collision between } B \text{ and } A}}$.

(c) Determine whether, after B and A have collided, there is a further collision between B and C. (4)

Solution



$$\text{Conservation of momentum:}\quad 0 - \tfrac{1}{2}mu = 5mx_1 + mx_2$$
$$\text{Newton's Law of Restitution:}\quad \frac{x_2 - x_1}{\frac{1}{2}u} = \tfrac{4}{5}.$$

Now,

$$-\tfrac{1}{2}u = 5x_1 + x_2 \Rightarrow 5x_1 = -\tfrac{1}{2}u - x_2 \Rightarrow x_1 = -\tfrac{1}{10}u - \tfrac{1}{5}x_2$$

and

$$\frac{x_2 - x_1}{\frac{1}{2}u} = \tfrac{4}{5} \Rightarrow x_2 - x_1 = \tfrac{2}{5}u \Rightarrow x_1 = x_2 - \tfrac{2}{5}u$$

and eliminate x_1:

$$-\tfrac{1}{10}u - \tfrac{1}{5}x_2 = x_2 - \tfrac{2}{5}u \Rightarrow \tfrac{6}{5}x_2 = \tfrac{3}{10}u \Rightarrow x_2 = \tfrac{1}{4}u.$$

From part (b),

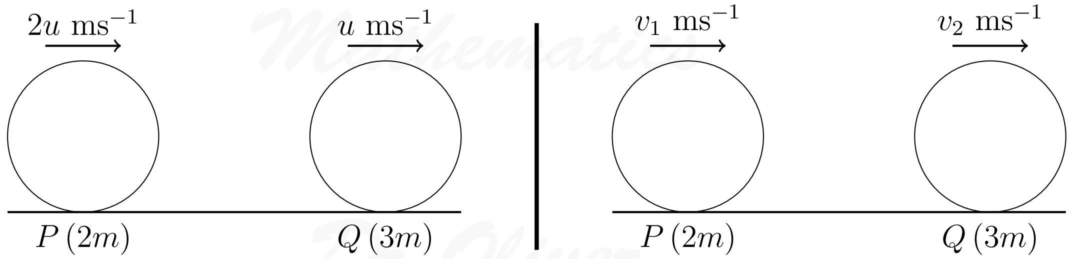
$$w_2 = \tfrac{1}{5}u - \tfrac{1}{5}w_1 = \tfrac{1}{5}u - \tfrac{1}{5} \times (-\tfrac{1}{2}u) = \tfrac{3}{10}u;$$

hence, there is no further collision between B and C.

10. A particle P of mass $2m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. A particle Q of mass $3m$ is moving with speed u in the same direction as P. The particles collide directly. The coefficient of restitution between P and Q is $\frac{1}{2}$.

(a) Show that the speed of Q immediately after the collision is $\frac{8}{5}u$. (5)

Solution



$$\text{Conservation of momentum:} \quad (2m)(2u) + (3m)u = (2m)v_1 + (3m)v_2$$

$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{u} = \tfrac{1}{2}.$$

Now,

$$7u = 2v_1 + 3v_2 \Rightarrow 2v_1 = 7u - 3v_2 \Rightarrow v_1 = \tfrac{7}{2}u - \tfrac{3}{2}v_2$$

and

$$\frac{v_2 - v_1}{u} = \tfrac{1}{2} \Rightarrow v_2 - v_1 = \tfrac{1}{2}u \Rightarrow v_1 = v_2 - \tfrac{1}{2}u$$

and eliminate v_1:

$$\tfrac{7}{2}u - \tfrac{3}{2}v_2 = v_2 - \tfrac{1}{2}u \Rightarrow \tfrac{5}{2}v_2 = 4u \Rightarrow \underline{\underline{e = \tfrac{8}{5}u}}.$$

(b) Find the total kinetic energy lost in the collision. (5)

Solution

$$v_1 = v_2 - \tfrac{1}{2}u = \tfrac{8}{5}u - \tfrac{1}{2}u = \tfrac{11}{10}u.$$

Now,

$$\begin{aligned} \text{initial KE} &= \tfrac{1}{2}(2m)(2u)^2 + \tfrac{1}{2}(3m)u^2 \\ &= 4mu^2 + \tfrac{3}{2}mu^2 \\ &= \tfrac{11}{2}mu^2 \end{aligned}$$

and

$$\begin{aligned} \text{final KE} &= \tfrac{1}{2}(2m)(\tfrac{11}{10}u)^2 + \tfrac{1}{2}(3m)(\tfrac{8}{5}u)^2 \\ &= \tfrac{121}{100}mu^2 + \tfrac{96}{25}mu^2 \\ &= \tfrac{101}{20}mu^2; \end{aligned}$$

hence,

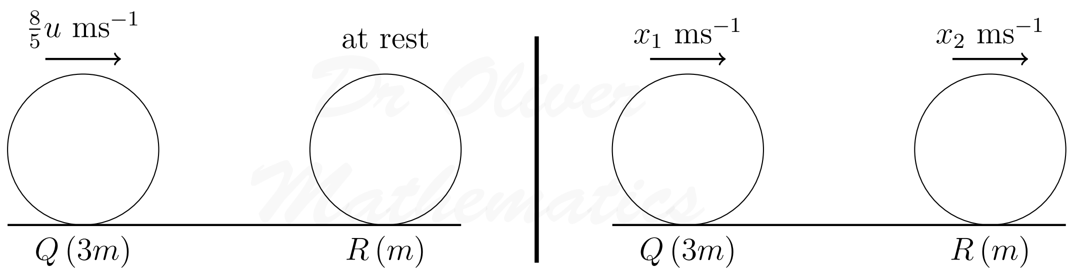
$$\tfrac{11}{2}mu^2 - \tfrac{101}{20}mu^2 = \underline{\underline{\tfrac{9}{20}mu^2}}$$

was lost.

After the collision between P and Q, the particle Q collides directly with a particle R of mass m which is at rest on the plane. The coefficient of restitution between Q and R is e.

(c) Calculate the range of values of e for which there will be a second collision between P and Q. (7)

Solution



$$\text{Conservation of momentum:} \quad (3m)(\tfrac{8}{5}u) + 0 = 3mx_1 + mx_2$$

$$\text{Newton's Law of Restitution:} \quad \frac{x_2 - x_1}{\frac{8}{5}u} = e.$$

Now,

$$\tfrac{24}{5}u = 3x_1 + x_2 \Rightarrow x_2 = \tfrac{24}{5}u - 3x_1$$

and

$$\frac{x_2 - x_1}{\frac{8}{5}u} = e \Rightarrow x_2 - x_1 = \tfrac{8}{5}eu \Rightarrow x_2 = x_1 + \tfrac{8}{5}eu$$

and eliminate x_2:

$$\tfrac{24}{5}u - 3x_1 = x_1 + \tfrac{8}{5}eu \Rightarrow 4x_1 = \tfrac{8}{5}u(3-e) \Rightarrow x_1 = \tfrac{2}{5}u(3-e).$$

Finally,

$$\begin{aligned}\tfrac{11}{10}u > \tfrac{2}{5}u(3-e) &\Rightarrow 11 > 4(3-e)\\ &\Rightarrow 11 > 12 - 4e\\ &\Rightarrow 4e > 1\\ &\Rightarrow e > \tfrac{1}{4};\end{aligned}$$

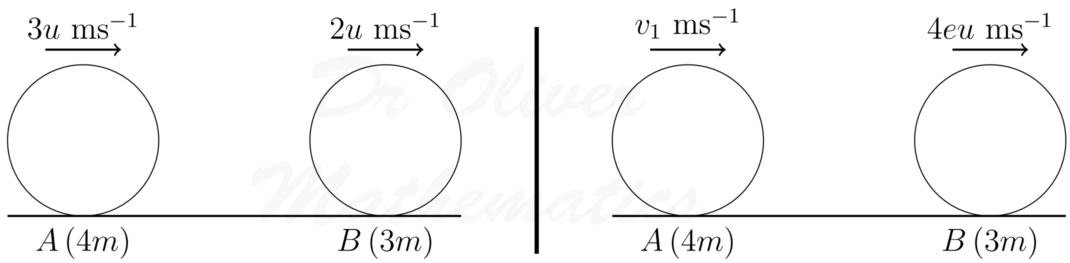
hence,

$$\underline{\underline{\tfrac{1}{4} < e \leqslant 1}}.$$

11. A particle A of mass $4m$ is moving with speed $3u$ in a straight line on a smooth horizontal table. The particle A collides directly with a particle B of mass $3m$ moving with speed $2u$ in the same direction as A. The coefficient of restitution between A and B is e. Immediately after the collision the speed of B is $4eu$.

    (a) Show that $e = \frac{3}{4}$. (5)

Solution



$$\text{Conservation of momentum:} \quad (4m)(3u) + (3m)(2u) = 4mv_1 + (3m)(4eu)$$

$$\text{Newton's Law of Restitution:} \quad \frac{4eu - v_1}{u} = e.$$

Now,

$$18u = 4v_1 + 12eu \Rightarrow 4v_1 = 18u - 12eu \Rightarrow v_1 = \tfrac{9}{2}u - 3eu$$

and

$$\frac{4eu - v_1}{u} = e \Rightarrow 4eu - v_1 = eu \Rightarrow v_1 = 3eu$$

and eliminate v_1:

$$\tfrac{9}{2}u - 3eu = 3eu \Rightarrow \tfrac{9}{2} = 6e \Rightarrow \underline{\underline{e = \tfrac{3}{4}}}.$$

    (b) Find the total kinetic energy lost in the collision. (4)

Solution

A's speed is

$$v_1 = 3 \times \tfrac{3}{4} \times u = \tfrac{9}{4}u$$

and B's speed is

$$v_2 = 4 \times \tfrac{3}{4} \times u = 3u.$$

Now,

$$\begin{aligned} \text{initial KE} &= \tfrac{1}{2}(4m)(3u)^2 + \tfrac{1}{2}(3m)(2u)^2 \\ &= 18mu^2 + 6mu^2 \\ &= 24mu^2 \end{aligned}$$

and

$$\begin{aligned} \text{final KE} &= \tfrac{1}{2}(4m)(\tfrac{9}{4}u)^2 + \tfrac{1}{2}(3m)(3u)^2 \\ &= \tfrac{81}{8}mu^2 + \tfrac{27}{2}mu^2 \\ &= \tfrac{189}{8}mu^2; \end{aligned}$$

hence,

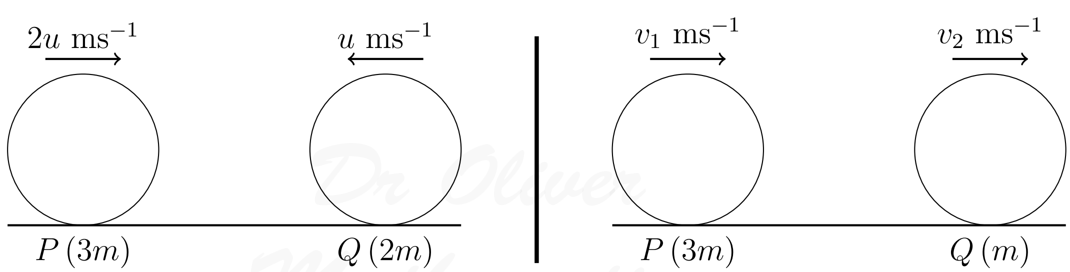
$$24mu^2 - \tfrac{189}{8}mu^2 = \underline{\underline{\tfrac{3}{8}mu^2}}$$

was lost.

12. A particle P of mass $3m$ is moving in a straight line with speed $2u$ on a smooth horizontal table. It collides directly with another particle Q of mass $2m$ which is moving with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}(9e + 4)u$. (5)

Solution



Conservation of momentum: $(3m)(2u) - (2m)u = 3mv_1 + 2mv_2$

Newton's Law of Restitution: $\dfrac{v_2 - v_1}{u} = e.$

Now,

$$4u = 3v_1 + 2v_2 \Rightarrow 3v_1 = 4u - 2v_2 \Rightarrow v_1 = \tfrac{4}{3}u - \tfrac{2}{3}v_2$$

and

$$\frac{v_2 - v_1}{u} = e \Rightarrow v_2 - v_1 = eu \Rightarrow v_1 = v_2 - eu$$

and eliminate v_1:

$$\tfrac{4}{3}u - \tfrac{2}{3}v_2 = v_2 - eu \Rightarrow \tfrac{5}{3}v_2 = \tfrac{1}{3}(9e+4)u \Rightarrow \underline{\underline{v_2 = \tfrac{1}{5}(9e+4)u}}.$$

The speed of P immediately after the collision is $\frac{1}{2}u$.

(b) Show that $e = \frac{1}{4}$. (4)

Solution

$$\begin{aligned} v_1 &= \tfrac{4}{3}u - \tfrac{2}{3}[\tfrac{1}{5}(9e+4)u] \\ &= \tfrac{4}{3}u - \tfrac{2}{15}(9e+4)u \\ &= \tfrac{4}{3}u - \tfrac{6}{5}eu - \tfrac{8}{15}u \\ &= \tfrac{4}{5}u - \tfrac{6}{5}eu \\ &= \tfrac{2}{5}(2-3e)u \end{aligned}$$

and

$$\begin{aligned} \tfrac{1}{2}u = \tfrac{2}{5}(2-3e)u &\Rightarrow \tfrac{1}{2} = \tfrac{2}{5}(2-3e) \\ &\Rightarrow \tfrac{1}{2} = \tfrac{4}{5} - \tfrac{6}{5}e \\ &\Rightarrow \tfrac{6}{5}e = \tfrac{3}{10} \\ &\Rightarrow \underline{\underline{e = \tfrac{1}{4}}}. \end{aligned}$$

The collision between P and Q takes place at the point A. After the collision Q hits a smooth fixed vertical wall which is at right-angles to the direction of motion of Q. The distance from A to the wall is d.

(c) Show that P is a distance $\frac{3}{5}d$ from the wall at the instant when Q hits the wall. (4)

Solution

Q's speed is

$$\tfrac{1}{5}(9 \times \tfrac{1}{4} + 4)u = \tfrac{5}{4}u$$

and the time taken is

$$\frac{d}{\frac{5}{4}u} = \tfrac{4d}{5u}.$$

P moves

$$\tfrac{1}{2}u \times \tfrac{4d}{5u} = \tfrac{2d}{5}$$

and so

$$d - \tfrac{2d}{5} = \underline{\underline{\tfrac{3d}{5}}}$$

remains.

Particle Q rebounds from the wall and moves so as to collide directly with particle P at the point B. Given that the coefficient of restitution between Q and the wall is $\frac{1}{5}$,

(d) find, in terms of d, the distance of the point B from the wall. (4)

Solution

Q's new speed is

$$\tfrac{5}{4}u \times \tfrac{1}{5} = \tfrac{1}{4}u$$

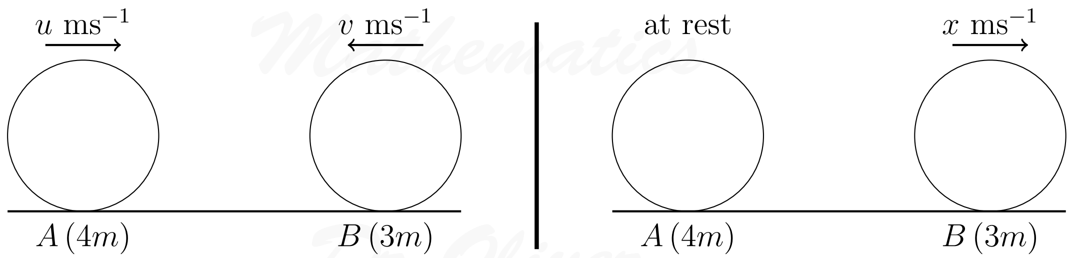
and

$$\begin{aligned} t_{AB} = t_{WB} &\Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u} \\ &\Rightarrow \tfrac{3d}{5} - x = 2x \\ &\Rightarrow 3x = \tfrac{3d}{5} \\ &\Rightarrow \underline{\underline{x = \tfrac{d}{5}}}. \end{aligned}$$

13. Particles A, B, and C of masses $4m$, $3m$, and m respectively, lie at rest in a straight line on a smooth horizontal plane with B between A and C. Particles A and B are projected towards each other with speeds u ms^{-1} and v ms^{-1} respectively, and collide directly. As a result of the collision, A is brought to rest and B rebounds with speed kv ms^{-1}. The coefficient of restitution between A and B is $\frac{3}{4}$.

(a) Show that $u = 3v$. (6)

Solution



$$\text{Conservation of momentum:} \quad 4mu - 3mv = 0 + 3mx$$
$$\text{Newton's Law of Restitution:} \quad \frac{x}{u+v} = \tfrac{3}{4}.$$

Now,

$$4u - 3v = 3x \Rightarrow x = \tfrac{4}{3}u - v$$

and

$$\frac{x}{u+v} = \tfrac{3}{4} \Rightarrow x = \tfrac{3}{4}u + \tfrac{3}{4}v$$

and eliminate x:

$$\tfrac{4}{3}u - v = \tfrac{3}{4}u + \tfrac{3}{4}v \Rightarrow \tfrac{7}{12}u = \tfrac{7}{4}v \Rightarrow \underline{\underline{u = 3v}}.$$

(b) Find the value of k. (2)

Solution

$$x = \tfrac{3}{4}u + \tfrac{3}{4}v = \tfrac{3}{4} \times 3v + \tfrac{3}{4}v = 3v$$

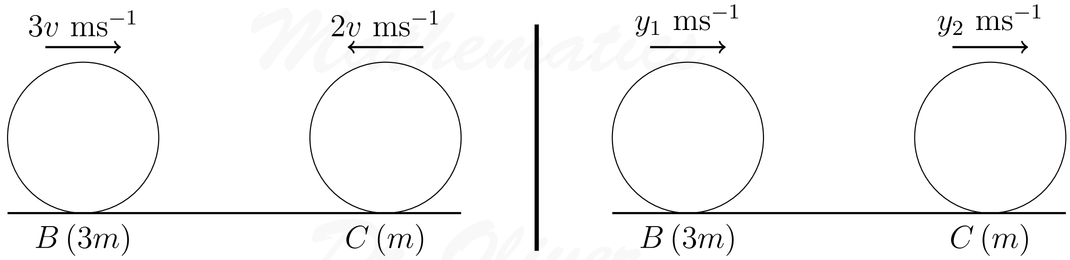
hence,

$$\underline{\underline{k = 3}}.$$

Immediately after the collision between A and B, particle C is projected with speed $2v$ ms^{-1} towards B so that B and C collide directly.

(c) Show that there is no further collision between A and B. (4)

Solution



Conservation of momentum: $(3m)(3v) - m(2v) = 3my_1 + my_2$

Newton's Law of Restitution: $\dfrac{y_2 - y_1}{5v} = \frac{3}{4}$.

Now,

$$7v = 3y_1 + y_2 \Rightarrow y_2 = 7v - 3y_1$$

and

$$\frac{y_2 - y_1}{5v} = \tfrac{3}{4} \Rightarrow y_2 - y_1 = \tfrac{15}{4}v \Rightarrow y_2 = \tfrac{15}{4}v + y_1$$

and eliminate y_2:

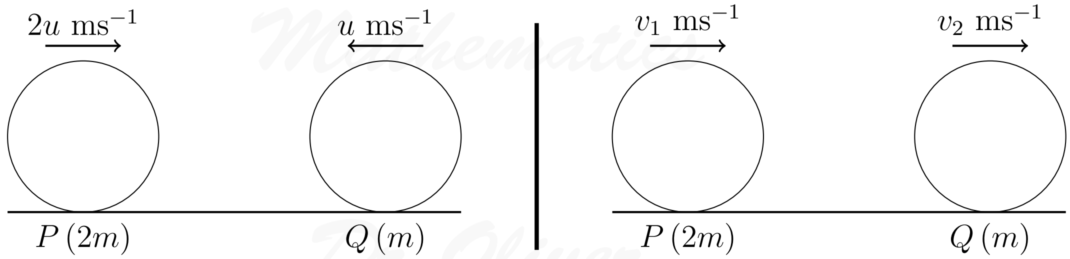
$$7v - 3y_1 = \tfrac{15}{4}v + y_1 \Rightarrow \tfrac{13}{4}v = 4y_1 \Rightarrow y_1 = \tfrac{13}{16}v;$$

hence, B carries on to the right with speed $\frac{13}{16}v$: there is no further collision between A and B.

14. Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e, (7)

(a) the speed of P immediately after the collision,

Solution



Conservation of momentum: $(2m)(2u) - mu = 2mv_1 + mv_2$

Newton's Law of Restitution: $\dfrac{v_2 - v_1}{3u} = e.$

Now,

$$3u = 2v_1 + v_2 \Rightarrow v_2 = 3u - 2v_1$$

and

$$\frac{v_2 - v_1}{3v} = e \Rightarrow v_2 - v_1 = 3eu \Rightarrow v_2 = v_1 + 3eu$$

and eliminate v_2:

$$3u - 2v_1 = v_1 + 3eu \Rightarrow 3u(1-e) = 3v_1 \Rightarrow \underline{\underline{v_1 = u(1-e)}}.$$

(b) the speed of Q immediately after the collision.

Solution

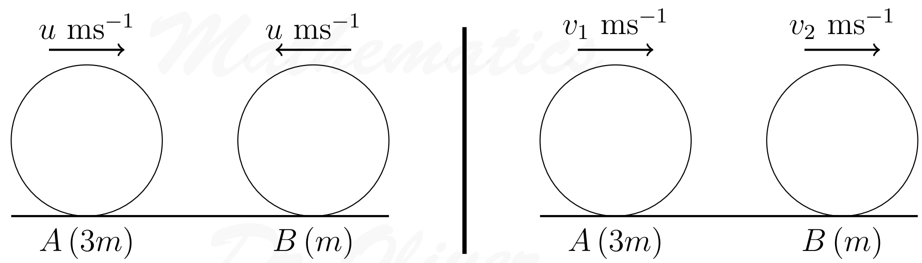
$$v_2 = v_1 + 3eu = u(1-e) + 3eu = u[(1-e) + 3e] = \underline{\underline{u(1+2e)}}.$$

15. A small ball A of mass $3m$ is moving with speed u in a straight line on a smooth horizontal table. The ball collides directly with another small ball B of mass m moving with speed u towards A along the same straight line. The coefficient of restitution between A and B is $\frac{1}{2}$. The balls have the same radius and can be modelled as particles.

(a) Find (7)

(i) the speed of A immediately after the collision,

Solution



$$\text{Conservation of momentum:} \quad (3m)u - mu = 3mv_1 + mv_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{2u} = \tfrac{1}{2}.$$

Now,

$$2u = 3v_1 + v_2 \Rightarrow v_2 = 2u - 3v_1$$

and

$$\frac{v_2 - v_1}{3v} = \tfrac{1}{2} \Rightarrow v_2 - v_1 = u \Rightarrow v_2 = v_1 + u$$

and eliminate v_2:

$$2u - 3v_1 = v_1 + u \Rightarrow u = 4v_1 \Rightarrow \underline{\underline{v_1 = \tfrac{1}{4}u}}.$$

(ii) the speed of B immediately after the collision.

Solution

$$v_2 = 2u - 3v_1 = 2u - \tfrac{3}{4}u = \underline{\underline{\tfrac{5}{4}u}}.$$

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

(b) Find the speed of B immediately after hitting the wall. (2)

Solution

B's speed after the rebound is

$$\tfrac{5}{4}u \times \tfrac{2}{5} = \underline{\underline{\tfrac{1}{2}u \text{ ms}^{-1}}}.$$

The first collision between A and B occurred at a distance $4a$ from the wall. The balls collide again T seconds after the first collision.

(c) Show that $T = \dfrac{112a}{15u}$. (6)

Solution

The time taken for B to hit the wall is

$$\frac{4a}{\frac{5}{4}u} = \tfrac{16a}{5u} \text{ s}$$

and, in this time, A moves

$$\tfrac{1}{4}u \times \tfrac{16a}{5u} = \tfrac{4a}{5} \text{ m.}$$

In the remaining time, A travels $\frac{1}{4}ut$ and B travels $\frac{1}{2}ut$. They collide when $4a - \frac{4a}{5} = \frac{16a}{5}$ and this takes

$$\frac{\frac{16a}{5}}{\frac{1}{4}u + \frac{1}{2}u} = \frac{\frac{16a}{5}}{\frac{3}{4}u} = \frac{64a}{15u} \text{ s.}$$

So,

$$\text{total time} = \frac{16a}{5u} + \frac{64a}{15u} = \underline{\underline{\frac{112a}{15u}}} \text{ s,}$$

as required.

16. A particle P of mass m kg is moving with speed 6 ms^{-1} in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds. The kinetic energy lost in the impact is 64 J. The coefficient of restitution between P and the wall is $\frac{1}{3}$.

(a) Show that $m = 4$. (6)

Solution

Newton's Law of Restitution:

$$\frac{v}{6} = \tfrac{1}{3} \Rightarrow v = 2$$

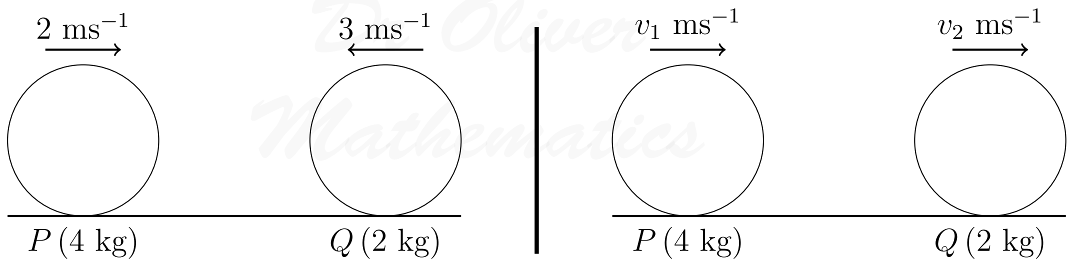
and

$$\begin{aligned}\tfrac{1}{2}(m)(6^2) - \tfrac{1}{2}(m)(2^2) = 64 &\Rightarrow 36m - 4m = 128 \\ &\Rightarrow 32m = 128 \\ &\Rightarrow \underline{\underline{m = 4}}.\end{aligned}$$

After rebounding from the wall, P collides directly with a particle Q which is moving towards P with speed 3 ms^{-1}. The mass of Q is 2 kg and the coefficient of restitution between P and Q is $\frac{1}{3}$.

(b) Show that there will be a second collision between P and the wall. (7)

Solution



$$\text{Conservation of momentum:} \quad (4)(2) - (2)(3) = 4v_1 + 2v_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{5} = \tfrac{1}{3}.$$

Now,

$$2 = 4v_1 + 2v_2 \Rightarrow 2v_2 = 2 - 4v_1 \Rightarrow v_2 = 1 - 2v_1$$

and

$$\frac{v_2 - v_1}{5} = \tfrac{1}{3} \Rightarrow v_2 - v_1 = \tfrac{5}{3} \Rightarrow v_2 = v_1 + \tfrac{5}{3}$$

and eliminate v_2:

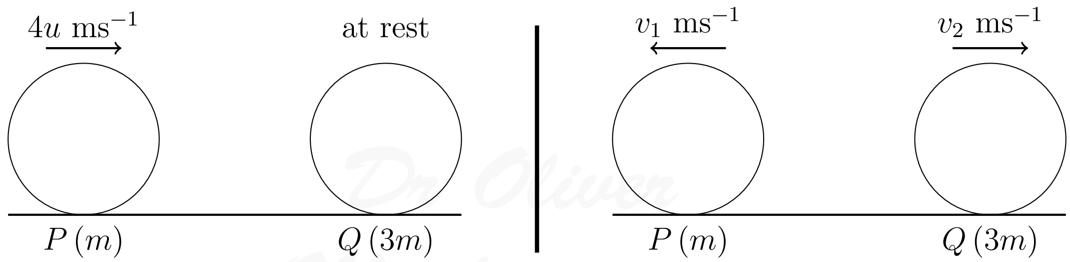
$$1 - 2v_1 = v_1 + \tfrac{5}{3} \Rightarrow -\tfrac{2}{3} = 3v_1 \Rightarrow v_1 = -\tfrac{2}{9};$$

hence, P is projected backwards and there will be a second collision between P and the wall.

17. A particle P of mass m is moving in a straight line on a smooth horizontal surface with speed $4u$. The particle P collides directly with a particle Q of mass $3m$ which is at rest on the surface. The coefficient of restitution between P and Q is e. The direction of motion of P is reversed by the collision. (8)

Show that $e > \frac{1}{3}$.

Solution



$$\text{Conservation of momentum:} \quad m(4u) + 0 = -mv_1 + 3mv_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 + v_1}{4u} = e.$$

Now,

$$4u = -v_1 + 3v_2 \Rightarrow 3v_2 = 4u + v_1 \Rightarrow v_2 = \tfrac{4}{3}u + \tfrac{1}{3}v_1$$

and

$$\frac{v_2 + v_1}{4u} = e \Rightarrow v_2 + v_1 = 4eu \Rightarrow v_2 = 4eu - v_1$$

and eliminate v_2:

$$\tfrac{4}{3}u + \tfrac{1}{3}v_1 = 4eu - v_1 \Rightarrow \tfrac{4}{3}v_1 = \tfrac{4}{3}u(3e-1) \Rightarrow v_1 = u(3e-1).$$

Finally,

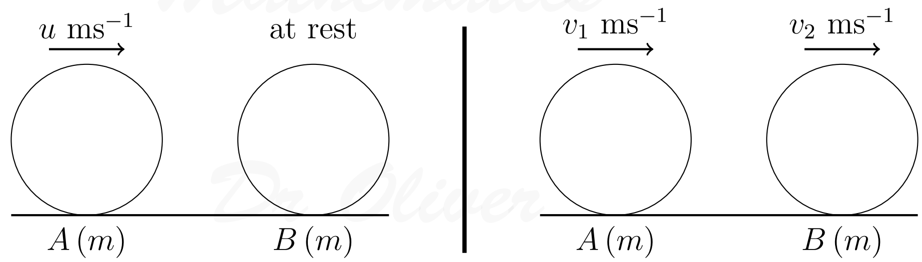
$$\begin{aligned} v_1 > 0 &\Rightarrow u(3e-1) > 0 \\ &\Rightarrow 3e - 1 > 0 \\ &\Rightarrow 3e > 1 \\ &\Rightarrow \underline{\underline{e > \tfrac{1}{3}}}. \end{aligned}$$

18. Three identical particles, A, B, and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

    (a) Find, in terms of u, (7)

    (i) the speed of A after this collision,

Solution



$$\text{Conservation of momentum:} \quad mu + 0 = mv_1 + mv_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{u} = \tfrac{2}{3}.$$

Now,

$$u = v_1 + v_2 \Rightarrow v_2 = u - v_1$$

and

$$\frac{v_2 - v_1}{u} = \tfrac{2}{3} \Rightarrow v_2 - v_1 = \tfrac{2}{3}u \Rightarrow v_2 = v_1 + \tfrac{2}{3}u$$

and eliminate v_2:

$$u - v_1 = v_1 + \tfrac{2}{3}u \Rightarrow \tfrac{1}{3}u = 2v_1 \Rightarrow \underline{\underline{v_1 = \tfrac{1}{6}u}}.$$

(ii) the speed of B after this collision

Solution

$$v_2 = u - v_1 = u - \tfrac{1}{6}u = \underline{\underline{\tfrac{5}{6}u}}.$$

(b) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$. (4)

Solution

$$\text{Initial KE} = \tfrac{1}{2}mu^2$$

and

$$\begin{aligned}\text{final KE} &= \tfrac{1}{2}m(\tfrac{1}{6}u)^2 + \tfrac{1}{2}m(\tfrac{5}{6}u)^2 \\ &= \tfrac{1}{72}mu^2 + \tfrac{25}{72}mu^2 \\ &= \tfrac{13}{36}mu^2;\end{aligned}$$

hence,

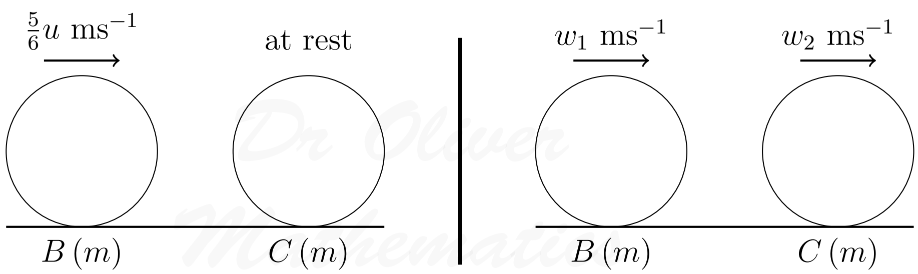
$$\tfrac{1}{2}mu^2 - \tfrac{13}{36}mu^2 = \underline{\underline{\tfrac{5}{36}mu^2}}$$

was lost.

After the collision between A and B, particle B collides directly with C.

(c) Find, in terms of u, the speed of C immediately after this collision between B and C. (4)

Solution



Conservation of momentum: $\tfrac{5}{6}mu + 0 = mw_1 + mw_2$

Newton's Law of Restitution: $\dfrac{w_2 - w_1}{\tfrac{5}{6}u} = \tfrac{2}{3}$.

Now,

$$\tfrac{5}{6}u = w_1 + w_2 \Rightarrow w_1 = \tfrac{5}{6}u - w_2$$

and

$$\frac{w_2 - w_1}{\tfrac{5}{6}u} = \tfrac{2}{3} \Rightarrow w_2 - w_1 = \tfrac{5}{9}u \Rightarrow w_1 = w_2 - \tfrac{5}{9}u$$

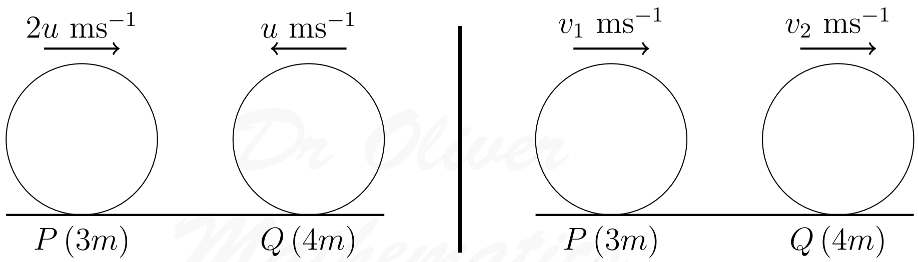
and eliminate w_1:

$$\tfrac{5}{6}u - w_2 = w_2 - \tfrac{5}{9}u \Rightarrow \tfrac{25}{18}u = 2w_2 \Rightarrow \underline{\underline{w_2 = \tfrac{25}{36}u}}.$$

19. A particle P of mass $3m$ is moving with speed $2u$ in a straight line on a smooth horizontal plane. The particle P collides directly with a particle Q of mass $4m$ moving on the plane with speed u in the opposite direction to P. The coefficient of restitution between P and Q is e.

(a) Find the speed of Q immediately after the collision. (6)

Solution



$$\text{Conservation of momentum:} \quad (3m)(2u) - 4mu = 3mv_1 + 4mv_2$$

$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{3u} = e.$$

Now,

$$2u = 3v_1 + 4v_2 \Rightarrow 3v_1 = 2u - 4v_2 \Rightarrow v_1 = \tfrac{2}{3}u - \tfrac{4}{3}v_2$$

and

$$\frac{v_2 - v_1}{3u} = e \Rightarrow v_2 - v_1 = 3eu \Rightarrow v_1 = v_2 - 3eu$$

and eliminate v_1:

$$\tfrac{2}{3}u - \tfrac{4}{3}v_2 = v_2 - 3eu \Rightarrow \tfrac{7}{3}v_2 = \tfrac{1}{3}u(2 + 9e) \Rightarrow \underline{\underline{v_2 = \tfrac{1}{7}u(2 + 9e)}}.$$

Given that the direction of motion of P is reversed by the collision,

(b) find the range of possible values of e. (5)

Solution

$$\begin{aligned} v_1 &= \tfrac{2}{3}u - \tfrac{4}{3}[\tfrac{1}{7}u(2 + 9e)] \\ &= \tfrac{2}{3}u - \tfrac{4}{21}u(2 + 9e) \\ &= \tfrac{2}{3}u - \tfrac{8}{21}u - \tfrac{12}{7}eu \\ &= \tfrac{2}{7}u - \tfrac{12}{7}eu \\ &= \tfrac{2}{7}u(1 - 6e) \end{aligned}$$

and

$$\begin{aligned} v_1 < 0 &\Rightarrow \tfrac{2}{7}u(1-6e) < 0 \\ &\Rightarrow 1 - 6e < 0 \\ &\Rightarrow 1 < 6e \\ &\Rightarrow \tfrac{1}{6} < e; \end{aligned}$$

hence,

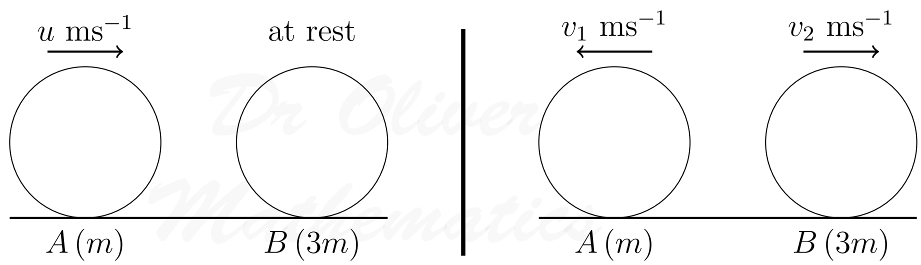
$$\underline{\underline{\tfrac{1}{6} < e \leqslant 1}}.$$

20. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

 (a) Find, in terms of e and u, (7)

 (i) the speed of A immediately after the collision,

Solution



$$\begin{aligned} \text{Conservation of momentum:} \quad & mu + 0 = -mv_1 + 3mv_2 \\ \text{Newton's Law of Restitution:} \quad & \frac{v_2 + v_1}{u} = e. \end{aligned}$$

Now,

$$u = -v_1 + 3v_2 \Rightarrow 3v_2 = u + v_1 \Rightarrow v_2 = \tfrac{1}{3}u + \tfrac{1}{3}v_1$$

and

$$\frac{v_2 + v_1}{u} = e \Rightarrow v_2 + v_1 = eu \Rightarrow v_2 = eu - v_1$$

and eliminate v_1:

$$\tfrac{1}{3}u + \tfrac{1}{3}v_1 = eu - v_1 \Rightarrow \tfrac{4}{3}v_1 = \tfrac{1}{3}u(3e-1) \Rightarrow \underline{\underline{v_1 = \tfrac{1}{4}u(3e-1)}}.$$

(ii) the speed of B immediately after the collision.

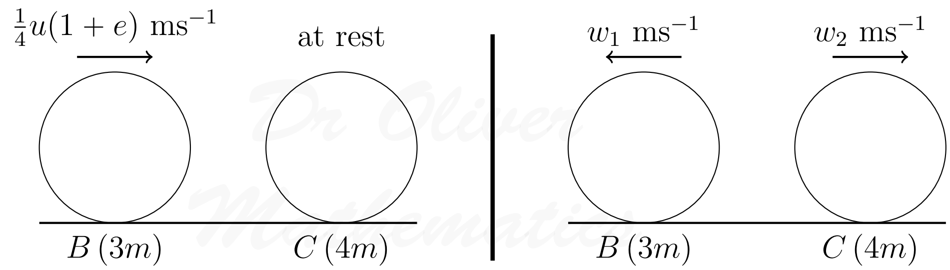
Solution

$$v_2 = eu - \tfrac{1}{4}u(3e - 1) = \underline{\underline{\tfrac{1}{4}u(1 + e)}}.$$

After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(b) find the range of possible values of e, (6)

Solution



Conservation of momentum: $(3m)[\tfrac{1}{4}u(1 + e)] + 0 = -3mw_1 + 4mw_2$

Newton's Law of Restitution: $\dfrac{w_2 + w_1}{\frac{1}{4}u(1 + e)} = 2e.$

Now,

$$\begin{aligned}\tfrac{3}{4}u(1 + e) = -3w_1 + 4w_2 \Rightarrow 4w_2 &= 3w_1 + \tfrac{3}{4}u(1 + e)\\ \Rightarrow w_2 &= \tfrac{3}{4}w_1 + \tfrac{3}{16}u(1 + e)\end{aligned}$$

and

$$\begin{aligned}\frac{w_2 + w_1}{\frac{1}{4}u(1 + e)} = 2e \Rightarrow w_2 + w_1 &= \tfrac{1}{2}eu(1 + e)\\ \Rightarrow w_2 &= \tfrac{1}{2}eu(1 + e) - w_1\end{aligned}$$

and eliminate w_2:

$$\begin{aligned}& \tfrac{3}{4}w_1 + \tfrac{3}{16}u(1 + e) = \tfrac{1}{2}eu(1 + e) - w_1\\ \Rightarrow\quad & \tfrac{7}{4}w_1 = \tfrac{1}{2}eu(1 + e) - \tfrac{3}{16}u(1 + e)\\ \Rightarrow\quad & \tfrac{7}{4}w_1 = \tfrac{1}{16}u(1 + e)(8e - 3)\\ \Rightarrow\quad & w_1 = \tfrac{1}{28}u(1 + e)(8e - 3).\end{aligned}$$

Finally,

$$\begin{aligned} w_1 > 0 &\Rightarrow \tfrac{1}{28}u(1+e)(8e-3) > 0 \\ &\Rightarrow 8e - 3 > 0 \\ &\Rightarrow 8e > 3 \\ &\Rightarrow e > \tfrac{3}{8}. \end{aligned}$$

Consider w_2:

$$\begin{aligned} w_2 &= \tfrac{1}{2}eu(1+e) - w_1 \\ &= \tfrac{1}{2}eu(1+e) - \tfrac{1}{28}u(1+e)(8e-3) \\ &= \tfrac{1}{14}u(1+e)[2e - (8e-3)] \\ &= \tfrac{1}{14}u(1+e)(3-6e) \\ &= \tfrac{1}{7}u(1+e)(1-2e) \end{aligned}$$

and

$$\begin{aligned} w_2 \leqslant 0 &\Rightarrow \tfrac{1}{7}u(1+e)(1-2e) \leqslant 0 \\ &\Rightarrow 1 - 2e \leqslant 0 \\ &\Rightarrow 1 \leqslant 2e \\ &\Rightarrow \tfrac{1}{2} \leqslant e. \end{aligned}$$

Hence,

$$\underline{\underline{\tfrac{3}{8} < e \leqslant \tfrac{1}{2}}}.$$

(c) determine whether there will be a second collision between A and B. (3)

Solution

There will be a second collision between A and B if

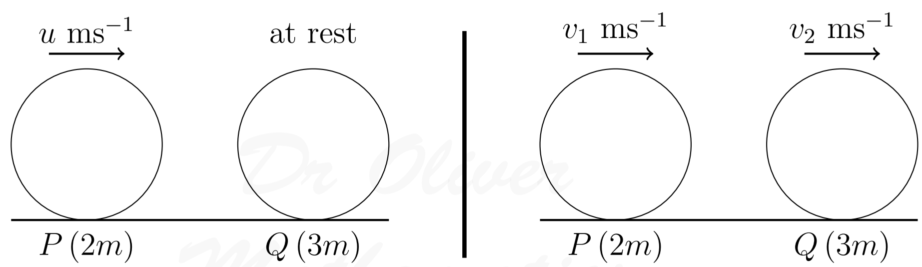
$$\begin{aligned} & \tfrac{1}{4}u(3e-1) < \tfrac{1}{28}u(1+e)(8e-3) \\ \Rightarrow \quad & 7(3e-1) < (1+e)(8e-3) \\ \Rightarrow \quad & 21e - 7 < 8e^2 + 5e - 3 \\ \Rightarrow \quad & 8e^2 - 16e + 4 > 0 \\ \Rightarrow \quad & 2e^2 - 4e + 1 > 0 \\ \Rightarrow \quad & e < \frac{4-\sqrt{8}}{2} \text{ or } e > \frac{4+\sqrt{8}}{2} \\ \Rightarrow \quad & e < 0.292\ldots \text{ or } e > 1.707\ldots; \end{aligned}$$

hence, there will not be a second collision between A and B.

21. Three particles P, Q, and R lie at rest in a straight line on a smooth horizontal table with Q between P and R. The particles P, Q, and R have masses $2m$, $3m$, and $4m$ respectively. Particle P is projected towards Q with speed u and collides directly with it. The coefficient of restitution between each pair of particles is e.

    (a) Show that the speed of Q immediately after the collision with P is $\frac{2}{5}(1+e)u$. (6)

Solution



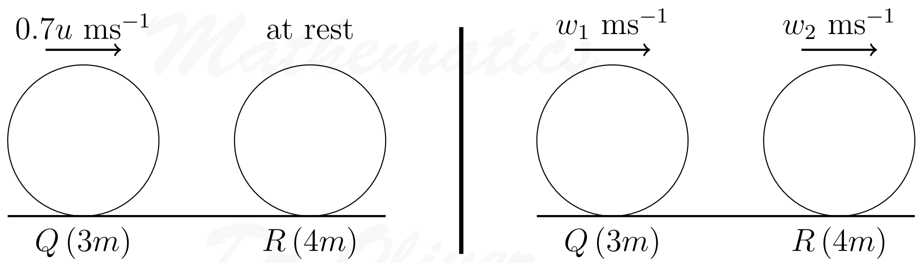
$$\text{Conservation of momentum:} \quad 2mu + 0 = 2mv_1 + 3mv_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{u} = e.$$

Now,
$$2u = 2v_1 + 3v_2 \Rightarrow 2v_1 = 2u - 3v_2 \Rightarrow v_1 = u - \tfrac{3}{2}v_2$$
and
$$\frac{v_2 - v_1}{u} = e \Rightarrow v_2 - v_1 = eu \Rightarrow v_1 = v_2 - eu$$
and eliminate v_1:
$$u - \tfrac{3}{2}v_2 = v_2 - eu \Rightarrow \tfrac{5}{2}v_2 = (1+e)u \Rightarrow \underline{\underline{v_2 = \tfrac{2}{5}(1+e)u.}}$$

After the collision between P and Q there is a direct collision between Q and R. Given that $e = \frac{3}{4}$, find

(b) (i) the speed of Q after this collision, (6)

Solution



Conservation of momentum: $(3m)(0.7u) + 0 = 3mw_1 + 4mw_2$

Newton's Law of Restitution: $\dfrac{w_2 - w_1}{0.7u} = \tfrac{3}{4}$.

Now,

$$2.1u = 3w_1 + 4w_2 \Rightarrow 4w_2 = 2.1u - 3w_1 \Rightarrow w_2 = \tfrac{21}{40}u - \tfrac{3}{4}w_1$$

and

$$\frac{w_2 - w_1}{0.7u} = \tfrac{3}{4} \Rightarrow w_2 - w_1 = \tfrac{21}{40}u \Rightarrow w_2 = w_1 + \tfrac{21}{40}u$$

and eliminate w_2:

$$\tfrac{21}{40}u - \tfrac{3}{4}w_1 = w_1 + \tfrac{21}{40}u \Rightarrow \tfrac{7}{4}w_1 = 0 \Rightarrow \underline{\underline{w_1 = 0}}.$$

(ii) the speed of R after this collision.

Solution

$$w_2 = w_1 + \tfrac{21}{40}u = \underline{\underline{\tfrac{21}{40}u}}.$$

Immediately after the collision between Q and R, the rate of increase of the distance between P and R is V.

(c) Find V in terms of u. (3)

Solution

$$v_1 = v_2 - eu = 0.7u - \tfrac{3}{4}u = -\tfrac{1}{20}u$$

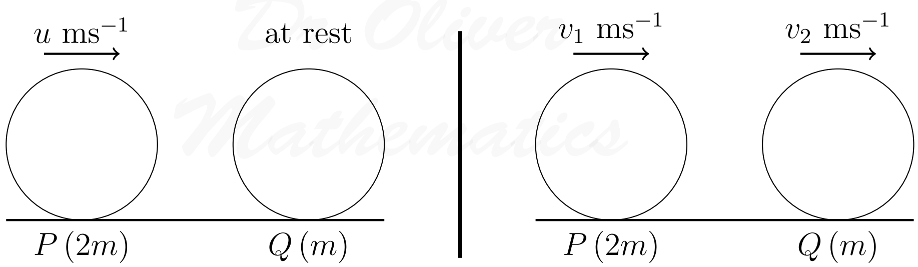
and

$$\text{speed of separation} = \tfrac{1}{20}u + \tfrac{21}{40}u = \underline{\underline{\tfrac{23}{40}u}}.$$

22. Two particles P and Q, of masses $2m$ and m respectively, are on a smooth horizontal table. Particle Q is at rest and particle P collides directly with it when moving with speed u. After the collision the total kinetic energy of the two particles is $\frac{3}{4}mu^2$. Find

(a) the speed of Q immediately after the collision, (10)

Solution



$$\text{Conservation of momentum:} \quad 2mu + 0 = 2mv_1 + mv_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{u} = e.$$

Now,

$$2u = 2v_1 + v_2 \Rightarrow 2v_1 = 2u - v_2 \Rightarrow v_1 = u - \tfrac{1}{2}v_2$$

and

$$\tfrac{1}{2}(2m)v_1^2 + \tfrac{1}{2}mv_2^2 = \tfrac{3}{4}mu^2 \Rightarrow 4v_1^2 + 2v_2^2 = 3u^2.$$

Substitute:

$$\begin{aligned} 4(u - \tfrac{1}{2}v_2)^2 + 2v_2^2 = 3u^2 &\Rightarrow 4u^2 - 4uv_2 + v_2^2 + 2v_2^2 = 3u^2 \\ &\Rightarrow u^2 - 4uv_2 + 3v_2^2 = 0 \\ &\Rightarrow (u - v_2)(u - 3v_2) = 0 \\ &\Rightarrow v_2 = u \text{ or } v_2 = \tfrac{1}{3}u \\ &\Rightarrow v_1 = \tfrac{1}{2}u \text{ or } v_1 = \tfrac{5}{6}u. \end{aligned}$$

Now, $v_1 \neq \frac{5}{6}u$ (why?) and so we have $\underline{\underline{v_2 = u}}$.

(b) the coefficient of restitution between the particles. (3)

Solution

$$e = \frac{u - \frac{1}{2}u}{u} = \underline{\underline{\tfrac{1}{2}}}.$$

23. A particle of mass m kg lies on a smooth horizontal surface. Initially the particle is at rest at a point O midway between a pair of fixed parallel vertical walls. The walls are 2 m apart. At time $t = 0$ the particle is projected from O with speed u ms^{-1} in a direction perpendicular to the walls. The coefficient of restitution between the particle and each wall is $\frac{2}{3}$. The magnitude of the impulse on the particle due to the first impact with a wall is λmu Ns.

(a) Find the value of λ. (3)

Solution

$$\begin{aligned}\text{Impulse} &= mv - mu \\ &= m[\tfrac{2}{3}u - (-u)] \\ &= \underline{\underline{\tfrac{5}{3}u}}.\end{aligned}$$

The particle returns to O, having bounced off each wall once, at time $t = 3$ seconds.

(b) Find the value of u. (6)

Solution

$$\text{Speed after the second collision} = (\tfrac{2}{3})^2 u = \tfrac{4}{9}u$$

and

$$\begin{aligned}\text{total time taken} &= \frac{1}{u} + \frac{2}{\frac{2}{3}u} + \frac{1}{\frac{4}{9}u} \\ &= \tfrac{1}{u} + \tfrac{3}{u} + \tfrac{9}{4u} \\ &= \tfrac{4+12+9}{4u} \\ &= \tfrac{25}{4u}.\end{aligned}$$

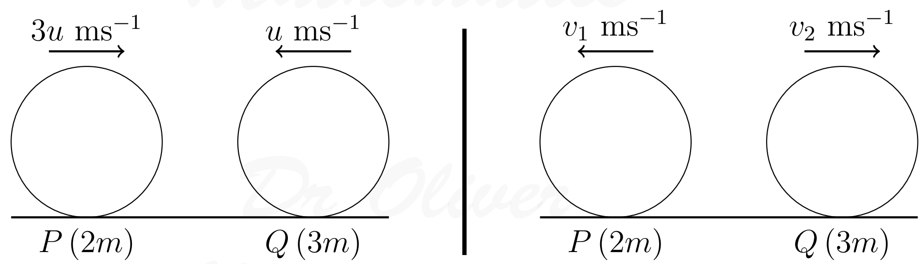
Now,

$$t = 3 \Rightarrow \tfrac{25}{4u} = 3 \Rightarrow \underline{\underline{u = \tfrac{25}{12}}}.$$

24. A particle P of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal table. A second particle Q of mass $3m$ is moving in the opposite direction to P along the same straight line with speed u. The particle P collides directly with Q. The direction of motion of P is reversed by the collision. The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{1}{5}u(8e + 3)$. (6)

Solution



Conservation of momentum: $(2m)(3u) - 3mu = -2mv_1 + 3mv_2$

Newton's Law of Restitution: $\dfrac{v_2 + v_1}{4u} = e.$

Now,

$$3u = -2v_1 + 3v_2 \Rightarrow 2v_1 = 3v_2 - 3u \Rightarrow v_1 = \tfrac{3}{2}v_2 - \tfrac{3}{2}u$$

and

$$\frac{v_2 + v_1}{4u} = e \Rightarrow v_2 + v_1 = 4eu \Rightarrow v_1 = 4eu - v_2$$

and eliminate v_1:

$$\tfrac{3}{2}v_2 - \tfrac{3}{2}u = 4eu - v_2 \Rightarrow \tfrac{5}{2}v_2 = \tfrac{1}{2}u(8e+3) \Rightarrow \underline{\underline{v_2 = \tfrac{1}{5}u(8e+3)}}.$$

(b) Find the range of possible values of e. (4)

Solution

$$v_1 = 4eu - \tfrac{1}{5}u(8e+3) = \tfrac{1}{5}u(20e - 8e - 3) = \tfrac{3}{5}u(4e-1)$$

and

$$\begin{aligned} v_1 > 0 &\Rightarrow \tfrac{3}{5}u(4e-1) > 0 \\ &\Rightarrow 4e - 1 > 0 \\ &\Rightarrow 4e > 1 \\ &\Rightarrow e > \tfrac{1}{4}; \end{aligned}$$

hence,

$$\underline{\underline{\tfrac{1}{4} < e \leqslant 1}}.$$

The total kinetic energy of the particles before the collision is T. The total kinetic energy of the particles after the collision is kT. Given that $e = \frac{1}{2}$,

(c) find the value of k. (4)

Solution

$$\begin{aligned} \text{initial KE} &= \tfrac{1}{2}(2m)(3u)^2 + \tfrac{1}{2}(3m)(u)^2 \\ &= 9mu^2 + \tfrac{3}{2}mu^2 \\ &= \tfrac{21}{2}mu^2 \end{aligned}$$

and

$$\begin{aligned} \text{final KE} &= \tfrac{1}{2}(2m)(\tfrac{3}{5}u)^2 + \tfrac{1}{2}(3m)(\tfrac{7}{5}u)^2 \\ &= \tfrac{9}{25}mu^2 + \tfrac{147}{50}mu^2 \\ &= \tfrac{33}{10}mu^2; \end{aligned}$$

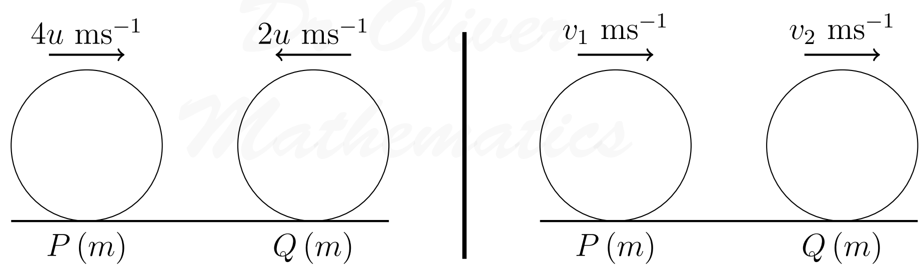
hence,

$$k = \frac{\frac{33}{10}mu^2}{\frac{21}{2}mu^2} = \underline{\underline{\tfrac{11}{35}}}.$$

25. Three identical particles P, Q, and R, each of mass m, lie in a straight line on a smooth horizontal plane with Q between P and R. Particles P and Q are projected directly towards each other with speeds $4u$ and $2u$ respectively, and at the same time particle R is projected along the line away from Q with speed $3u$. The coefficient of restitution between each pair of particles is e. After the collision between P and Q there is a collision between Q and R.

(a) Show that $e > \frac{2}{3}$. (7)

Solution



$$\text{Conservation of momentum:} \quad 4mu - 2mu = mv_1 + mv_2$$
$$\text{Newton's Law of Restitution:} \quad \frac{v_2 - v_1}{6u} = e.$$

Now,

$$2u = v_1 + v_2 \Rightarrow v_1 = 2u - v_2$$

and

$$\frac{v_2 - v_1}{6u} = e \Rightarrow v_2 - v_1 = 6eu \Rightarrow v_1 = v_2 - 6eu$$

and eliminate v_1:

$$2u - v_2 = v_2 - 6eu \Rightarrow 2v_2 = 2u(1 + 3e) \Rightarrow v_2 = u(1 + 3e)$$

and

$$v_1 = 2u - u(1 + 3e) = u(1 - 3e).$$

Finally,

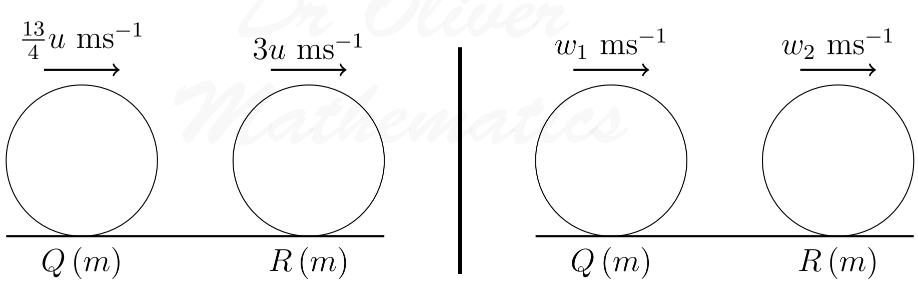
$$\begin{aligned} u(1 + 3e) > 3u &\Rightarrow 1 + 3e > 3 \\ &\Rightarrow 3e > 2 \\ &\Rightarrow \underline{\underline{e > \tfrac{2}{3}}}. \end{aligned}$$

It is given that $e = \frac{3}{4}$.

(b) Show that there will not be a further collision between P and Q. (6)

Solution

$$v_1 = u(1 - 3 \times \tfrac{3}{4}) = -\tfrac{5}{4}u \text{ and } v_2 = u(1 + 3 \times \tfrac{3}{4}) = \tfrac{13}{4}u.$$



$$\text{Conservation of momentum:}\quad \tfrac{13}{4}mu + 3mu = mw_1 + mw_2$$
$$\text{Newton's Law of Restitution:}\quad \frac{w_2 - w_1}{\frac{1}{4}u} = \tfrac{3}{4}.$$

Now,

$$\tfrac{25}{4}u = w_1 + w_2 \Rightarrow w_2 = \tfrac{25}{4}u - w_1$$

and

$$\frac{w_2 - w_1}{\frac{1}{4}u} = \tfrac{3}{4} \Rightarrow w_2 - w_1 = \tfrac{3}{16}u \Rightarrow w_2 = w_1 + \tfrac{3}{16}u$$

and eliminate w_2:

$$\tfrac{25}{4}u - w_1 = w_1 + \tfrac{3}{16}u \Rightarrow 2w_1 = \tfrac{97}{16} \Rightarrow w_1 = \tfrac{97}{32};$$

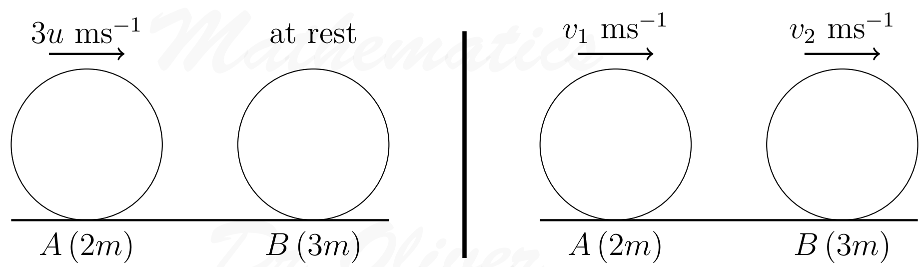
P and Q moving away from each other, so they do not collide.

26. Two particles A and B, of mass $2m$ and $3m$ respectively, are initially at rest on a smooth horizontal surface. Particle A is projected with speed $3u$ towards B. Particle A collides directly with particle B. The coefficient of restitution between A and B is $\frac{3}{4}$.

   (a) Find (7)

      (i) the speed of A immediately after the collision,

Solution



$$\text{Conservation of momentum:}\quad (2m)(3u) + 0 = 2mv_1 + 3mv_2$$
$$\text{Newton's Law of Restitution:}\quad \frac{v_2 - v_1}{3u} = \tfrac{3}{4}.$$

Now,

$$6u = 2v_1 + 3v_2 \Rightarrow 3v_2 = 6u - 2v_1 \Rightarrow v_2 = 2u - \tfrac{2}{3}v_1$$

and
$$\frac{v_2 - v_1}{3u} = \tfrac{3}{4} \Rightarrow v_2 - v_1 = \tfrac{9}{4}u \Rightarrow v_2 = v_1 + \tfrac{9}{4}u$$
and eliminate v_2:
$$2u - \tfrac{2}{3}v_1 = v_1 + \tfrac{9}{4}u \Rightarrow \tfrac{5}{3}v_1 = -\tfrac{1}{4}u \Rightarrow v_1 = -\tfrac{3}{20}u;$$
the speed is $\underline{\underline{\tfrac{3}{20}u \text{ ms}^{-1}}}$ backwards.

(ii) the speed of B immediately after the collision.

Solution
$$v_2 = 2u + \tfrac{2}{3}(-\tfrac{3}{20}u) = \underline{\underline{\tfrac{21}{10}u \text{ ms}^{-1}}}.$$

After the collision B hits a fixed smooth vertical wall and rebounds. The wall is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is e. The magnitude of the impulse received by B when it hits the wall is $\frac{27}{4}mu$.

(b) Find the value of e. (3)

Solution
$$\begin{aligned} \tfrac{27}{4}mu = \tfrac{21}{10}(3m)eu - (-\tfrac{21}{10}(3m)u) &\Rightarrow \tfrac{27}{4} = \tfrac{63}{10}(e+1) \\ &\Rightarrow e + 1 = \tfrac{15}{14} \\ &\Rightarrow \underline{\underline{e = \tfrac{1}{14}}}. \end{aligned}$$

(c) Determine whether there is a further collision between A and B after B rebounds from the wall. (2)

Solution
The new speed of B is
$$\tfrac{1}{14} \times \tfrac{21}{10}u = \tfrac{3}{20}u$$
which is the same speed as A. Hence, there is no second collision.

27. Two particles A and B, of masses $3m$ and $4m$ respectively, lie at rest on a smooth horizontal surface. Particle B lies between A and a smooth vertical wall which is perpendicular to the line joining A and B. Particle B is projected with speed $5u$ in a direction perpendicular to the wall and collides with the wall. The coefficient of restitution between B and the wall is $\frac{3}{5}$.

(a) Find the magnitude of the impulse received by B in the collision with the wall. (3)

Solution

Newton's Law of Restitution:

$$\frac{v}{5u} = \tfrac{3}{5} \Rightarrow v = 3u$$

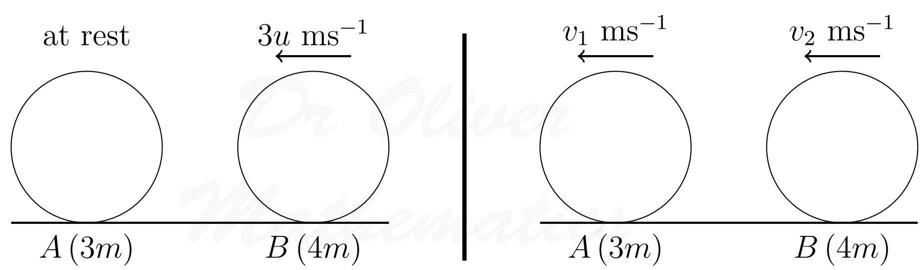
and

$$\begin{aligned}\text{Impulse} &= 4m[5u - (-3u)] \\ &= 4m \times 8u \\ &= \underline{\underline{32mu \text{ Ns}}}.\end{aligned}$$

After the collision with the wall, B rebounds from the wall and collides directly with A. The coefficient of restitution between A and B is e.

(b) Show that, immediately after they collide, A and B are both moving in the same direction. (7)

Solution



$$\text{Conservation of momentum:} \quad 0 - (4m)(3u) = -3mv_1 + 4mv_2$$

$$\text{Newton's Law of Restitution:} \quad \frac{v_2 + v_1}{3u} = e.$$

Now,

$$-12u = -3v_1 - 4v_2 \Rightarrow 3v_1 = 12u - 4v_2 \Rightarrow v_1 = 4u - \tfrac{4}{3}v_2$$

and

$$\frac{v_1 - v_2}{3u} = e \Rightarrow v_1 - v_2 = 3eu \Rightarrow v_1 = 3eu + v_2$$

and eliminate v_1:

$$4u - \tfrac{4}{3}v_2 = 3eu + v_2 \Rightarrow \tfrac{7}{3}v_2 = u(4 - 3u) \Rightarrow v_2 = \tfrac{3}{7}u(4 - 3e)$$

and

$$v_1 = eu + \tfrac{3}{7}u(4 - 3e) = \tfrac{2}{7}u(6 - e).$$

As $0 \leqslant e \leqslant 1$, $\underline{\underline{A \text{ and } B \text{ are both moving in the same direction}}}$.

The kinetic energy of B immediately after it collides with A is one-quarter of the kinetic energy of B immediately before it collides with A.

(c) Find the value of e. (4)

Solution

$$\begin{aligned}\text{initial KE} &= 0 + \tfrac{1}{2}(4m)(3u)^2 \\ &= 18mu^2\end{aligned}$$

and

$$\begin{aligned}\frac{\text{final KE}}{18mu^2} = \frac{1}{4} &\Rightarrow \tfrac{1}{2}(4m)[\tfrac{3}{7}u(4 - 3e)]^2 = \tfrac{9}{2}mu^2 \\ &\Rightarrow [\tfrac{3}{7}(4 - 3e)]^2 = \tfrac{9}{4} \\ &\Rightarrow \tfrac{3}{7}(4 - 3e) = \tfrac{3}{2} \\ &\Rightarrow 4 - 3e = \tfrac{7}{2} \\ &\Rightarrow 3e = \tfrac{1}{2} \\ &\Rightarrow \underline{\underline{e = \tfrac{1}{6}}}.\end{aligned}$$