Dr Oliver Mathematics
AQA GCSE Mathematics
2018 November Paper 1: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. Simplify (1)

$$(5^4)^2.$$

Circle your answer.

$$5^6 \quad 5^8 \quad 25^6 \quad 25^8$$

Solution

Well,

$$(5^4)^2 = 5^{4\times 2} = 5^8$$

so

$$5^6 \quad \underline{\underline{5^8}} \quad 25^6 \quad 25^8$$

2. Circle the volume, in cm^3, of a cylinder with radius 5 cm and height 8 cm. (1)

$$40\pi \quad 80\pi \quad 200\pi \quad 1\,600\pi$$

Solution

Now,

$$\begin{aligned} \text{volume} &= \pi \times 5^2 \times 8 \\ &= \pi \times 25 \times 8 \\ &= 200\pi \end{aligned}$$

so

$$40\pi \quad 80\pi \quad \underline{\underline{200\pi}} \quad 1\,600\pi$$

3. Simplify (1)

$$16a^2 \div a + 3a \times 2.$$

Circle your answer.

$$22a \quad 8a \quad 38a \quad 2a$$

Solution

Well,

$$\begin{aligned} 16a^2 \div a + 3a \times 2 &= (16a^2 \div a) + (3a \times 2) \\ &= 16a + 6a \\ &= 22a \end{aligned}$$

so

$$\underline{\underline{22a}} \quad 8a \quad 38a \quad 2a$$

4. Circle the value of $\cos 30°$. (1)

$$\tfrac{1}{2} \quad \tfrac{\sqrt{3}}{2} \quad 0 \quad 1$$

Solution

Who doesn't know their trig tables?

$$\tfrac{1}{2} \quad \underline{\underline{\tfrac{\sqrt{3}}{2}}} \quad 0 \quad 1$$

5. Work out (4)

$$8\tfrac{1}{2} \div 2\tfrac{2}{3}.$$

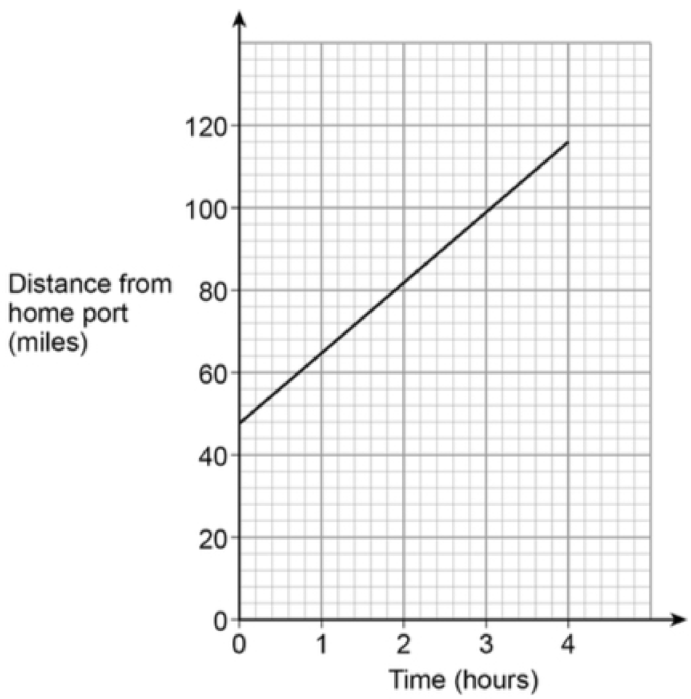
Give your answer as a mixed number.

Solution

$$
\begin{aligned}
8\tfrac{1}{2} \div 2\tfrac{2}{3} &= \tfrac{17}{2} \div \tfrac{8}{3} \\
&= \tfrac{17}{2} \times \tfrac{3}{8} \\
&= \tfrac{51}{16} \\
&= \underline{\underline{3\tfrac{3}{16}}}.
\end{aligned}
$$

6. A ship is sailing in a straight line from its home port. (3)

The distance-time graph shows 4 hours of the journey.



Work out the speed of the ship during these 4 hours.

Solution

$$
\begin{aligned}
\text{Speed} &= \frac{116 - 48}{4 - 0} \\
&= \frac{68}{4} \\
&= \underline{\underline{17 \text{ miles hour}^{-1}}}.
\end{aligned}
$$

7. The sum of the angles in any quadrilateral is 360°. (2)

For example, in a rectangle, $4 \times 90° = 360°$.

Zak writes, "$5 \times 90° = 450°$ so the sum of the angles in any pentagon must be 450°."

Is he correct?
Tick a box.

☐ Yes ☐ No

Show working to support your answer.

Solution

No: a pentagon has

$$(5 - 2) \times 180 = \underline{\underline{540°}}.$$

8. Kim works at an airport in the UK.
She records the number of planes landing between 10 am and 2 pm each day.
The table shows the data for the first 10 days in January.

Day	1	2	3	4	5	6	7	8	9	10
Number of planes	148	151	147	155	153	147	155	102	151	154

(a) The airport was affected by fog on one of the days. (1)
Which day do you think it was?
Give a reason for your answer.

Solution

Day 8 as fewer planes could take-off/ land/ it is an outlier.

Kim uses the data to predict how many planes will land at the airport in a year.

In her method, she

- uses an estimate of 150 planes in each 4-hour period throughout the day assumes the same and
- number of planes each day.

(b) Work out her prediction. (3)

Solution
Well,

$$\begin{aligned}\text{prediction} &= \frac{150 \times 24}{4} \times 365 \\ &= 150 \times 6 \times 365 \\ &= 900 \times 365 \\ &= \underline{\underline{328\,500}}.\end{aligned}$$

In fact,

- fewer planes land in winter than in summer and
- fewer planes land at night than during the day.

(c) What does this tell you about Kim's prediction? (2)
Tick **one** box.

☐ Her prediction is too low

☐ Her prediction is too high

☐ Her prediction could be too low or too high

Give a reason for your answer.

Solution
Ticks the box 'Her prediction could be too low or too high': e.g., fewer landings in winter would make it too low but fewer landings at night would make it too high.

9. (4)

$$\sqrt{6^2 + 8^2} = \sqrt[3]{125a^3}.$$

Work out the value of a.

Solution

$$\begin{aligned}\sqrt{6^2 + 8^2} = \sqrt[3]{125a^3} &\Rightarrow \sqrt{36 + 64} = \sqrt[3]{(5a)^3} \\ &\Rightarrow \sqrt{100} = 5a \\ &\Rightarrow 10 = 5a \\ &\Rightarrow \underline{\underline{a = 2}}.\end{aligned}$$

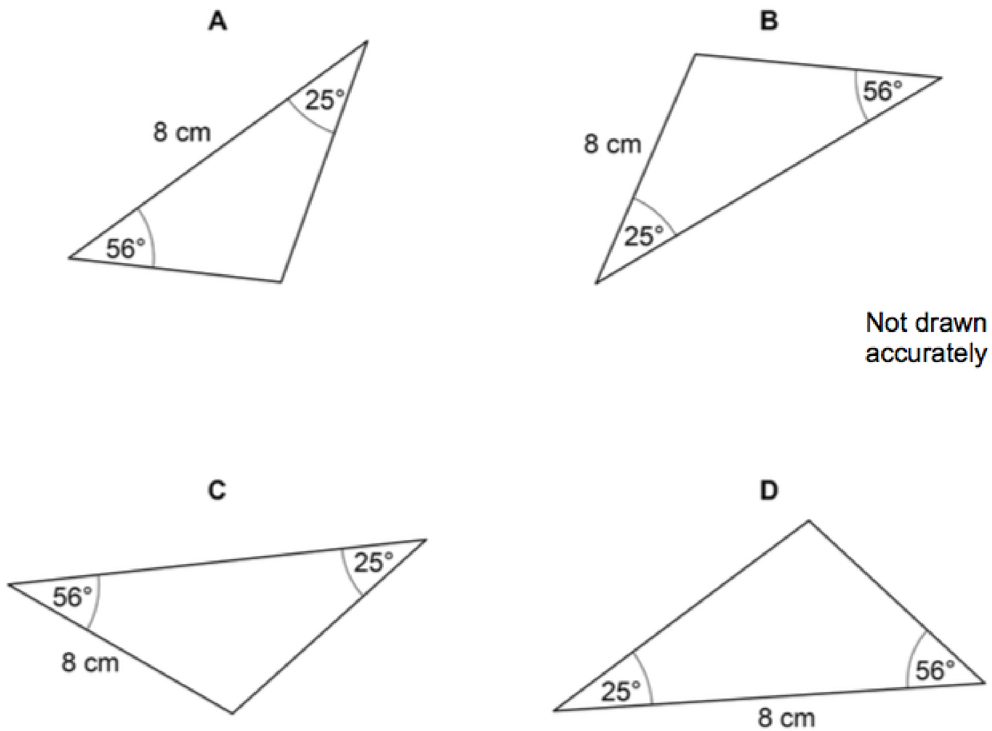
10. Work out the percentage increase from 80 to 280. (3)

Solution

Well,

$$\begin{aligned}\left(\frac{280 - 80}{80}\right) \times 100\% &= \frac{200}{80} \times 100\% \\ &= \tfrac{5}{2} \times 100\% \\ &= 5 \times 50\% \\ &= \underline{\underline{250\%}}.\end{aligned}$$

11. Here are four triangles. (1)



Which **two** triangles are congruent?
Circle **two** letters below.

$$A \quad B \quad C \quad D$$

Solution

$$\underline{\underline{A}} \quad B \quad C \quad \underline{\underline{D}}$$

12. Solve (3)

$$x^2 - x - 12 = 0.$$

Solution

$$\left.\begin{array}{ll} \text{add to:} & -1 \\ \text{multiply to:} & -12 \end{array}\right\} -4,\ +3$$

Well,

$$\begin{aligned} x^2 - x - 12 = 0 &\Rightarrow (x-4)(x+3) = 0 \\ &\Rightarrow x - 4 = 0 \text{ or } x + 3 = 0 \\ &\Rightarrow \underline{\underline{x = 4 \text{ or } x = -3}}. \end{aligned}$$

13. (3)

$$e : f = 2 : 3 \text{ and } f : g = 5 : 4.$$

Work out

$$e : g.$$

Give your answer in its simplest form.

Solution

Now,

$$e : f = 2 : 3 = 10 : 15$$

and

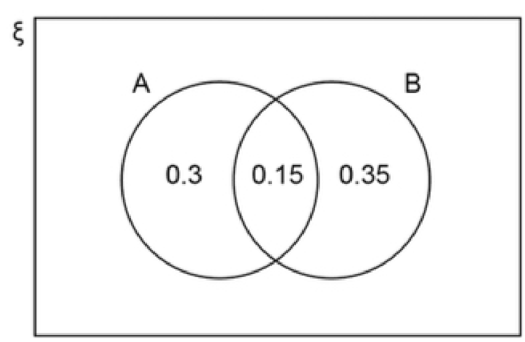
$$f : g = 5 : 4 = 15 : 12.$$

So,

$$\begin{aligned} e : g &= 10 : 12 \\ &= \underline{\underline{5 : 6}}. \end{aligned}$$

14. A and B are two events. (2)

Some probabilities are shown on the Venn diagram.



Work out

$$P(A' \cup B).$$

Solution

Now,

$$\begin{aligned} P(A \cup B) &= 1 - (0.3 + 0.15 + 0.35) \\ &= 1 - 0.8 \\ &= 0.2 \end{aligned}$$

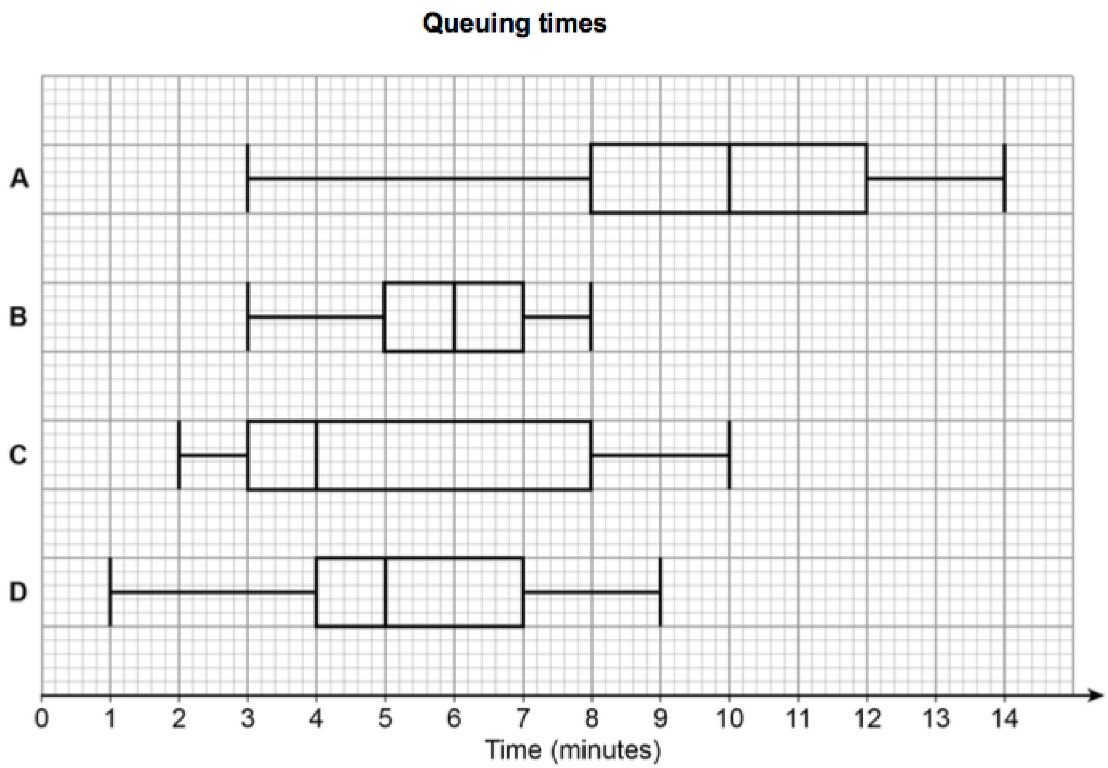
and

$$\begin{aligned} P(A' \cup B) &= 0.15 + 0.35 + 0.2 \\ &= \underline{\underline{0.7}}. \end{aligned}$$

15. In a survey, queuing times at supermarket checkouts were recorded.

One morning, samples of 50 customers were taken at supermarkets A, B, C, and D.

The box plots represent the results.



(a) On average, which supermarket had the lowest queuing times? (2)
Give a reason for your answer.

Solution
Well,

	Median	IQR	Range
A	10	$12 - 8 = 4$	$14 - 3 = 11$
B	6	$7 - 5 = 2$	$8 - 3 = 5$
C	4	$8 - 4 = 4$	$10 - 2 = 8$
D	5	$7 - 4 = 3$	$9 - 1 = 8$

As judged by the median, supermarket $\underline{\underline{C}}$ has $\underline{\underline{\text{lowest queuing times}}}$.

(b) At which supermarket were the queuing times most consistent? (2)
Give a reason for your answer.

Solution

As judged by the IQR and the range, the queuing times $\underline{\underline{B}}$ were $\underline{\underline{\text{most consistent}}}$.

16. Circle the number that is closest to the value of (1)

$$29^3.$$

27 000   90   2 700   9 000

Solution

Well, 29^3 is bigger than $30^3 = 27\,000$ and so

$\underline{\underline{27\,000}}$ 90   2 700   9 000

17. Work out the exact value of (2)

$$\left(\tfrac{3}{4}\right)^{-3}.$$

Solution

$$\begin{aligned}\left(\tfrac{3}{4}\right)^{-3} &= \left(\tfrac{4}{3}\right)^{3} \\ &= \frac{4^3}{3^3} \\ &= \underline{\underline{\tfrac{64}{27} \text{ or } 2\tfrac{10}{27}}}.\end{aligned}$$

18. Beth and Mia translate documents from Spanish into English. (4)

A set of documents that would take Beth 8 days would take Mia 10 days.

Beth starts to translate the documents.

After 2 days Beth and Mia both work on translating the documents.

How many **more** days will it take to complete the work?

You **must** show your working.

Solution

After two days, Beth has completed

$$\tfrac{2}{8} = \tfrac{1}{4}$$

of the work, leaving $\frac{3}{4}$ of the work to be done between the two of them. Now,

$$\begin{aligned}\tfrac{1}{8} + \tfrac{1}{10} &= \tfrac{5}{40} + \tfrac{4}{40} \\ &= \tfrac{9}{40}\end{aligned}$$

and

$$\begin{aligned}\frac{\frac{3}{4}}{\frac{9}{40}} &= \tfrac{3}{4} \times \tfrac{40}{9} \\ &= \tfrac{1}{1} \times \tfrac{10}{3} \\ &= \underline{\underline{3\tfrac{1}{3} \text{ more days}}}.\end{aligned}$$

19. In a chess club, there are x boys and y girls.

(a) If 5 more boys and 8 more girls join, there would be half as many boys as girls. (2)

Show that

$$y = 2x + 2.$$

Solution

Well,

$$\begin{aligned}2(x+5) = y + 8 \Rightarrow 2x + 10 &= y + 8 \\ \Rightarrow \underline{\underline{y = 2x + 2}},\end{aligned}$$

as required.

(b) If instead, 10 more boys and 1 more girl join, there would be the same number of boys and girls. (3)

Work out x and y.

Solution

Now

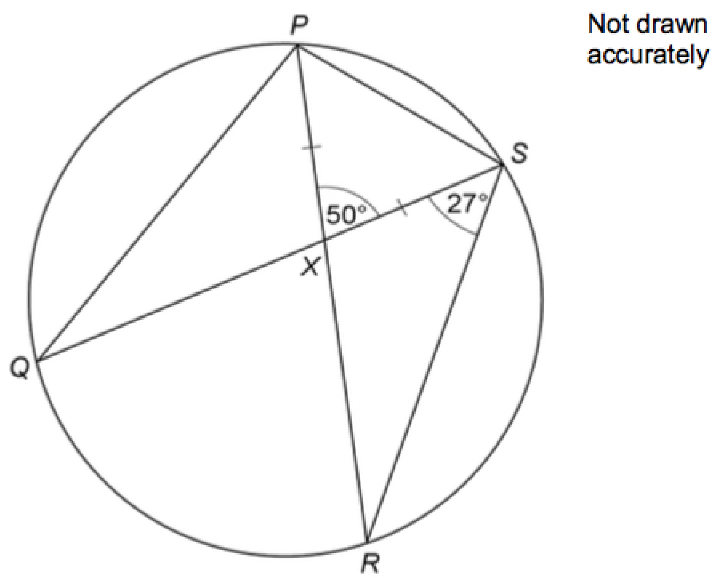
$$x + 10 = y + 1 \Rightarrow y = x + 9$$

and now subtract the two equations:

$$2x + 2 = x + 9 \Rightarrow \underline{\underline{x = 7}}$$
$$\Rightarrow y = 2(7) + 2$$
$$\Rightarrow \underline{\underline{y = 16}}.$$

20. • P, Q, R, and S are points on a circle. (4)

- PXR and QXS are straight lines.
- $PX = SX$.



Prove that QS is not a diameter of the circle.

Solution

Well, if QS **was** the diameter of the circle:

$\angle SXR = 180 - 50 = 130^\circ$ (supplementary angles)

$\angle XRS = 180 - (130 + 27) = 180 - 157 = 23°$ (completing the triangle)

Now, $\angle PXQ = 180 - 50 = 130°$ (supplementary angles)
$\angle XSP = \angle XPS = \frac{1}{2}(180 - 50) = 65°$ (base angles)
$\angle SPQ = 90°$ (right angle)
$\angle PQS = 180 - (65 + 90) = 25°$ (completing the triangle)

Now, we see that $\angle XRS \neq \angle PQS$, a contradiction, as the angles in same segment are not equal.

Hence, QS is not a diameter of the circle.

21. Here are the first four terms of a quadratic sequence: (3)

$$11 \quad 26 \quad 45 \quad 68.$$

Work out an expression for the nth term.

Solution

Let the

$$n\text{th term} = an^2 + bn + c.$$

Then

11		26		45		68
	15		19		23	
		4		4		

and

$a + b + c$		$4a + 2b + c$		$9a + 3b + c$		$16a + 4b + c$
	$3a + b$		$5a + b$		$7a + b$	
		$2a$		$2a$		

We compare terms:

$$2a = 4 \Rightarrow a = 2,$$

$$3a + b = 15 \Rightarrow 3 \times 2 + b = 15$$
$$\Rightarrow b = 9,$$

and

$$a + b + c = 11 \Rightarrow 2 + 9 + c = 11$$
$$\Rightarrow c = 0;$$

hence,

$$n\text{th term} = \underline{\underline{2n^2 + 9n}}.$$

22. Solve (4)

$$\frac{x}{x+4} + \frac{7}{x-2} = 1.$$

You **must** show your working.

Solution

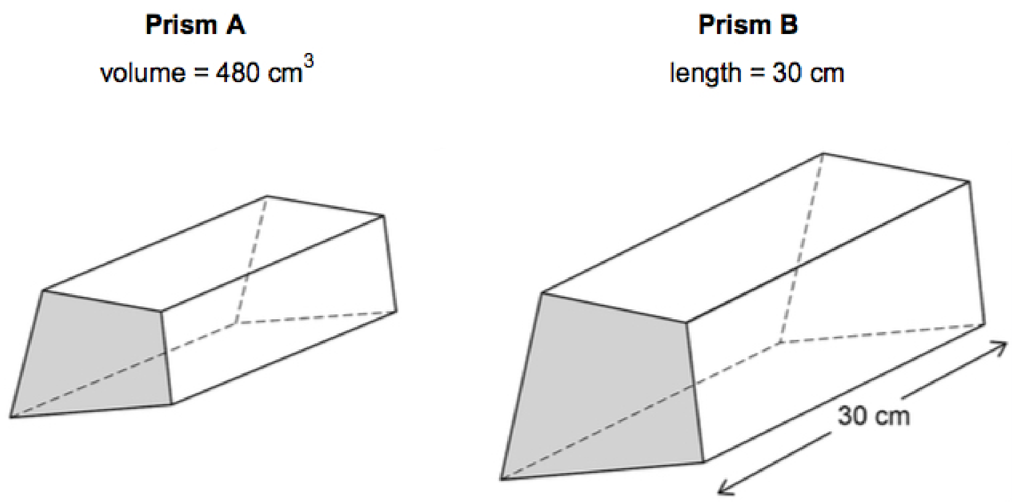
Now,

$\times$	x	$+4$
x	x^2	$+4x$
-2	$-2x$	-8

and multiply by $(x+4)(x-2)$:

$$\frac{x}{x+4} + \frac{7}{x-2} = 1$$
$$\Rightarrow \left[(x+4)(x-2) \times \frac{x}{x+4}\right] + \left[(x+4)(x-2) \times \frac{7}{x-2}\right] = (x+4)(x-2)$$
$$\Rightarrow x(x-2) + 7(x+4) = (x+4)(x-2)$$
$$\Rightarrow x^2 - 2x + 7x + 28 = x^2 + 2x - 8$$
$$\Rightarrow 3x = -36$$
$$\Rightarrow \underline{\underline{x = -12}}.$$

23. Prisms A and B are similar. (5)
The cross sections are shaded.



Area of the cross-section of A : area of the cross-section of $B = 4 : 9$.
Work out the area of the cross section of B.

Solution

The Area Scale Factor (ASF) is

$$4 : 9 = 2^2 : 3^2,$$

and the Length Scale Factor (LSF) is

$$2 : 3,$$

and the Volume Scale Factor (VSF) is

$$2^3 : 3^3 = 8 : 27.$$

Finally,

$$\begin{aligned}\text{volume of } B &= 480 \times \tfrac{27}{8} \\ &= 60 \times 27 \\ &= 1\,620\end{aligned}$$

and

$$\begin{aligned}\text{area of the cross section of } B &= \frac{1\,620}{30} \\ &= \underline{\underline{54 \text{ cm}^2}}.\end{aligned}$$

24. Show that

$$\frac{2\sqrt{6}}{\sqrt{5}} - \frac{\sqrt{3}}{\sqrt{10}}$$

can be written in the form

$$\frac{c\sqrt{d}}{10},$$

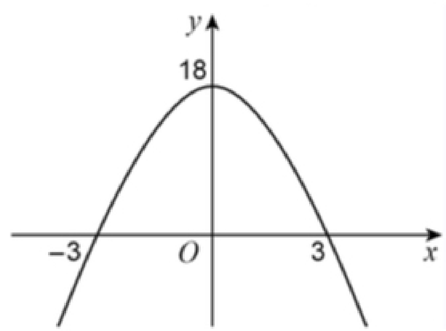
where c and d are integers. (3)

Solution

$$\begin{aligned}\frac{2\sqrt{6}}{\sqrt{5}} - \frac{\sqrt{3}}{\sqrt{10}} &= \frac{2\sqrt{6}}{\sqrt{5}} \times \frac{\sqrt{2}}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{10}} \\ &= \frac{2\sqrt{6} \times \sqrt{2}}{\sqrt{5} \times \sqrt{2}} - \frac{\sqrt{3}}{\sqrt{10}} \\ &= \frac{2\sqrt{12}}{\sqrt{10}} - \frac{\sqrt{3}}{\sqrt{10}} \\ &= \frac{2(2\sqrt{3})}{\sqrt{10}} - \frac{\sqrt{3}}{\sqrt{10}} \\ &= \frac{4\sqrt{3}}{\sqrt{10}} - \frac{\sqrt{3}}{\sqrt{10}} \\ &= \frac{3\sqrt{3}}{\sqrt{10}} \\ &= \frac{3\sqrt{3}}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} \\ &= \underline{\underline{\frac{3\sqrt{30}}{10}}};\end{aligned}$$

hence, $\underline{\underline{c = 3}}$ and $\underline{\underline{d = 30}}$.

25. A quadratic curve intersects the axes at $(-3, 0)$, $(3, 0)$, and $(0, 18)$. (3)



Not drawn accurately

Work out the equation of the curve.

Solution

Suppose the equation of the curve is

$$y = ax^2 + bx + c,$$

for some constants a, b, and c. Now,

$$x = 3, y = 0 \Rightarrow 9a + 3b + c = 0 \quad (1)$$
$$x = -3, y = 0 \Rightarrow 9a - 3b + c = 0 \quad (2).$$

Next, add (1) − (2):

$$6b = 0 \Rightarrow b = 0.$$

Subtract (1) + (2):

$$18a + 2c = 0 \Rightarrow c = -9a.$$

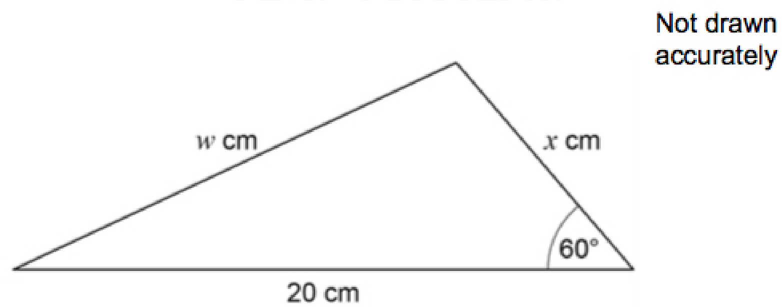
Next,

$$\begin{aligned} x = 0, y = 18 &\Rightarrow 18 = 0 + c \\ &\Rightarrow c = 18 \\ &\Rightarrow a = \frac{18}{-9} \\ &\Rightarrow a = -2. \end{aligned}$$

Hence, the equation of the curve is

$$\underline{\underline{y = -2x^2 + 18}}.$$

26. The area of this triangle is $25\sqrt{3}$ cm^2. (5)



Work out the value of w.
Give your answer in the form $a\sqrt{b}$, where a and b are integers greater than 1.

Solution

Well,

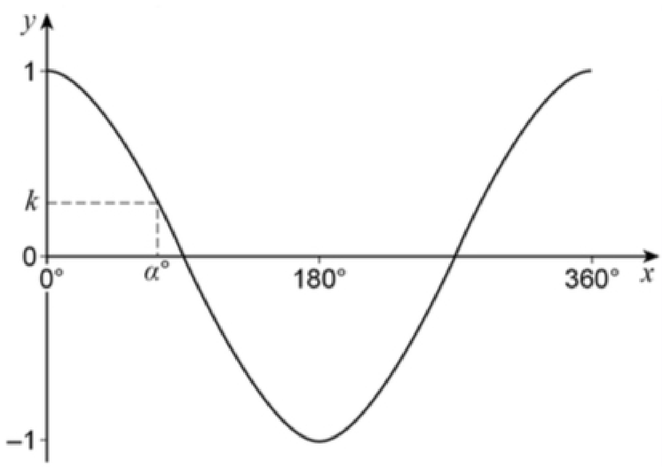
$$\begin{aligned} & \tfrac{1}{2} \times 20 \times x \times \sin 60^\circ = 25\sqrt{3} \\ \Rightarrow \quad & x = \frac{25\sqrt{3}}{\frac{1}{2} \times 20 \times \sin 60^\circ} \\ \Rightarrow \quad & x = \frac{25\sqrt{3}}{10 \times \frac{\sqrt{3}}{2}} \\ \Rightarrow \quad & x = \frac{25}{10 \times \frac{1}{2}} \\ \Rightarrow \quad & x = \frac{25}{5} \\ \Rightarrow \quad & x = 5. \end{aligned}$$

Finally, we use the cosine rule:

$$\begin{aligned} & w^2 = 20^2 + 5^2 - 2 \times 20 \times 5 \times \cos 60^\circ \\ \Rightarrow \quad & w^2 = 400 + 25 - (200 \times \tfrac{1}{2}) \\ \Rightarrow \quad & w^2 = 425 - 100 \\ \Rightarrow \quad & w^2 = 325 \\ \Rightarrow \quad & w = \sqrt{325} \\ \Rightarrow \quad & w = \sqrt{25 \times 13} \\ \Rightarrow \quad & w = \sqrt{25} \times \sqrt{13} \\ \Rightarrow \quad & \underline{\underline{w = 5\sqrt{13}}}; \end{aligned}$$

hence, $\underline{\underline{a = 5}}$ and $\underline{\underline{b = 13}}$.

27. Here is a sketch of $y = \cos x$ for values of x from 0° to 360°.



α° is an acute angle.
$\cos \alpha^\circ = k$.

(a) Circle the value of $\cos(180^\circ - \alpha^\circ)$. (1)

$$1 - k \quad k \quad -k \quad -1 - k$$

Solution

$$1 - k \quad k \quad \underline{\underline{-k}} \quad -1 - k$$

(b) Circle the value of $\cos(360^\circ + \alpha^\circ)$. (1)

$$k - 1 \quad k + 1 \quad -k \quad k$$

Solution

$$k - 1 \quad k + 1 \quad -k \quad \underline{\underline{k}}$$