

Dr Oliver Mathematics
GCSE Mathematics
2019 November Paper 3H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Expand and simplify (2)
 $(x + 5)(x - 9)$.

Solution

$$\begin{array}{r|rr} \times & x & +5 \\ \hline x & x^2 & +5x \\ -9 & -9x & -45 \\ \hline \end{array}$$

Hence,

$$(x + 5)(x - 9) = \underline{\underline{x^2 - 4x - 45}}.$$

- (b) Factorise fully (2)
 $9x^2 + 6x$.

Solution

$$9x^2 + 6x = \underline{\underline{3x(3x + 2)}}.$$

2. (a) Use your calculator to work out (2)

$$\frac{29^2 - 4.6}{\sqrt{35 - 1.9^3}}$$

Write down all the figures on your calculator display.

Solution

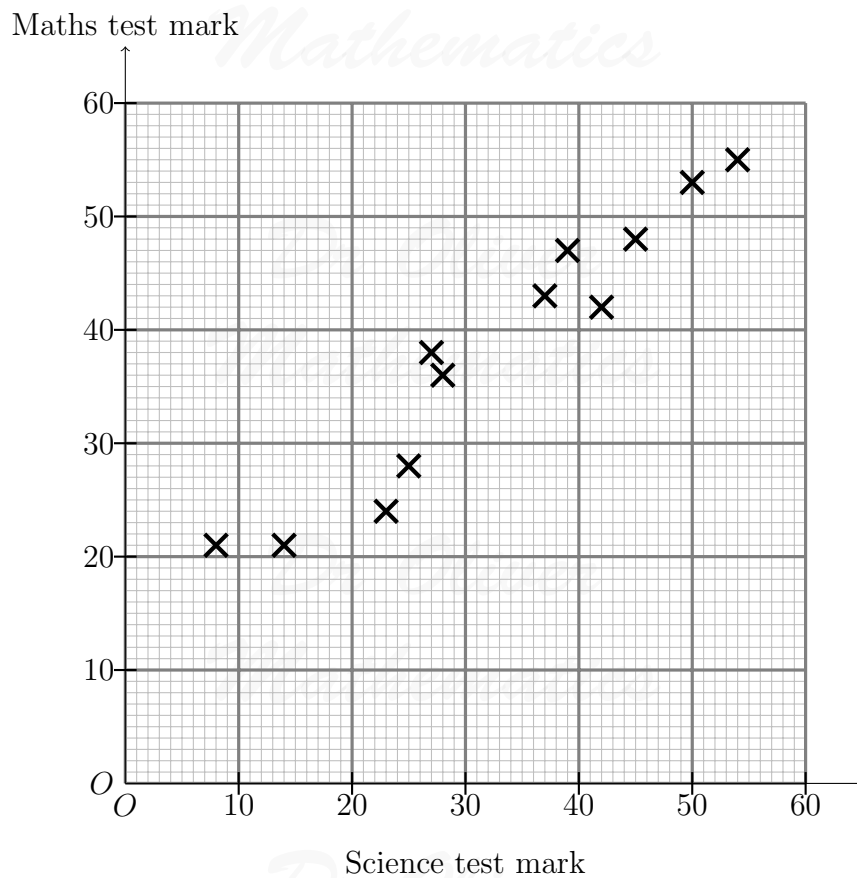
$$\frac{29^2 - 4.6}{\sqrt{35 - 1.9^3}} = \frac{836.4}{\sqrt{28.141}} \\ = \underline{\underline{157.668\ 255\ \text{(FCD)}}}.$$

(b) Write your answer to part (a) correct to 4 significant figures. (1)

Solution

$$157.668\ 255\ \text{(FCD)} = \underline{\underline{157.7\ \text{(4 sf)}}}.$$

3. The scatter graph shows information about the marks a group of students got in a Science test and in a Maths test. (2)

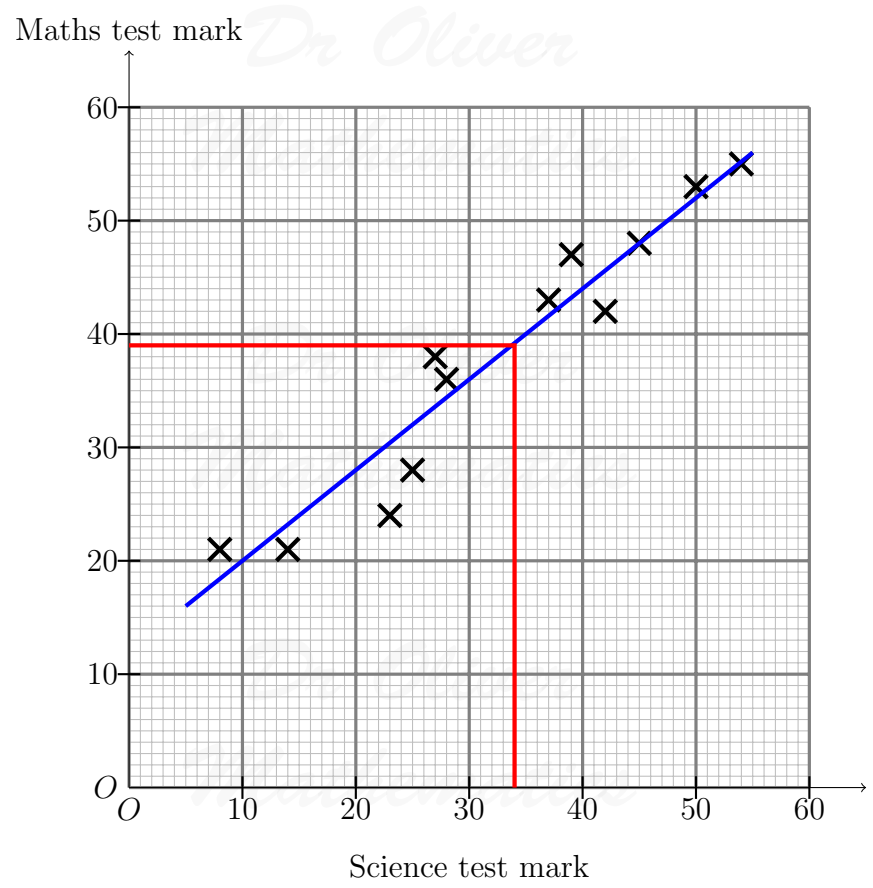


Jamie got a mark of 34 in the Science test.

Using the scatter graph, find an estimate for Jamie's mark in the Maths test.

Solution

Use a line of best fit:



Correct read-off: Jamie got 39 in the Maths test.

4. The table gives information about the times taken, in seconds, by 18 students to run a race. (3)

Time (t seconds)	Frequency
$5 < t \leq 10$	1
$10 < t \leq 15$	2
$15 < t \leq 20$	7
$20 < t \leq 25$	8

Work out an estimate for the mean time.
Give your answer correct to 3 significant figures.

Solution

Time (t seconds)	Frequency	Midpoint	Frequency \times Midpoint
$5 < t \leq 10$	1	7.5	$7.5 \times 1 = 7.5$
$10 < t \leq 15$	2	12.5	$12.5 \times 2 = 25$
$15 < t \leq 20$	7	17.5	$17.5 \times 7 = 122.5$
$20 < t \leq 25$	8	22.5	$22.5 \times 8 = 180$
	18		335

$$\begin{aligned} \text{Mean} &= \frac{\sum fx}{\sum f} \\ &\approx \frac{335}{18} \\ &= \underline{\underline{18\frac{11}{18} \text{ or } 18.6 \text{ s (1 dp)}}}. \end{aligned}$$

5. Write 37 cm^3 in mm^3 .

(1)

Solution

$$\begin{aligned} 37 \text{ cm}^3 &= 37 \times 1 \text{ cm}^3 \\ &= 37 \times (1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}) \\ &= 37 \times (10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm}) \\ &= 37 \times 1000 \text{ mm}^3 \\ &= \underline{\underline{37000 \text{ mm}^3}}. \end{aligned}$$

6. Nimer was driving to a hotel.
He looked at his Sat Nav at 1330.

(4)

Time	13 30
Distance to destination	65 miles

Nimer arrived at the hotel at 14 48.

Work out the average speed of the car from 13 30 to 14 48.
You must show all your working.

Solution

He drove 1 hr 18 minutes: in hours,

$$\begin{aligned} 1 \text{ hour } 18 \text{ minutes} &= 1 + \frac{18}{60} \\ &= 1.3 \text{ hours} \end{aligned}$$

and

$$\begin{aligned} \text{average speed} &= \frac{65}{1.3} \\ &= \underline{\underline{50 \text{ mph.}}} \end{aligned}$$

7. (a) Write

32 460 000

(1)

in standard form.

Solution

$$32\,460\,000 = \underline{\underline{3.246 \times 10^7}}.$$

(b) Write

4.96×10^{-3}

(1)

as an ordinary number.

Solution

$$4.96 \times 10^{-3} = \underline{\underline{0.00496}}.$$

Asma was asked to compare the following two numbers:

$$A = 6.212 \times 10^8 \text{ and } B = 4.73 \times 10^9.$$

She says, “6.212 is bigger than 4.73 so A is bigger than B .”

(c) Is Asma correct?

(1)

You must give a reason for your answer.

Solution

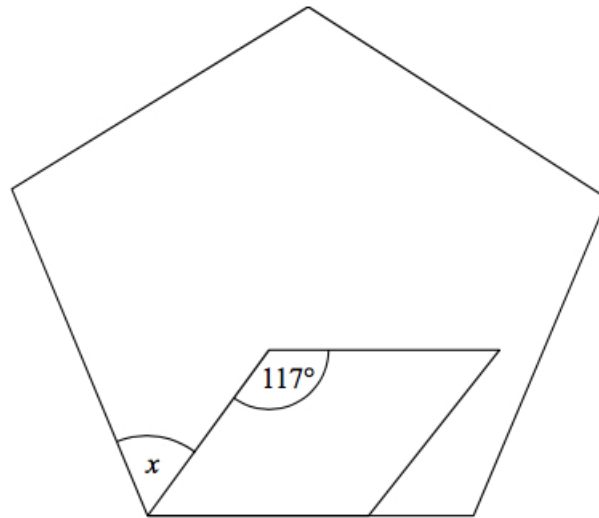
No:

$$4.73 \times 10^9 = 47.3 \times 10^8$$

so B is plainly bigger.

8. The diagram shows a regular pentagon and a parallelogram.

(4)



Work out the size of the angle marked x .

You must show all your working.

Solution

The internal angles are all 108° (why?). Now,

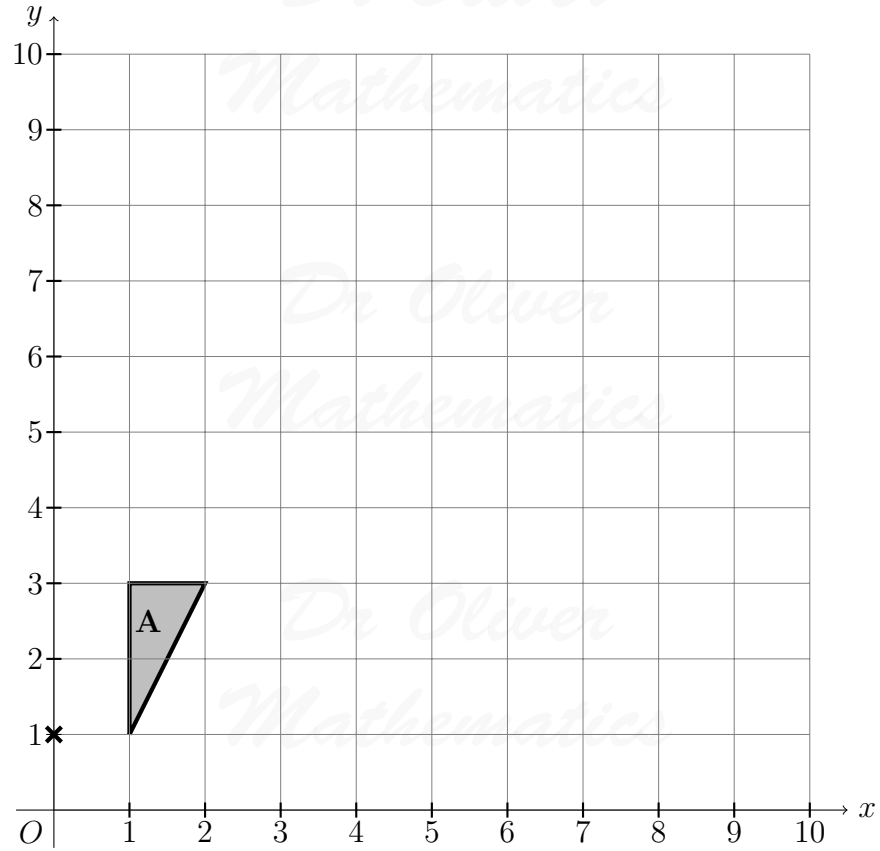
$$180 - 117 = 63^\circ$$

and

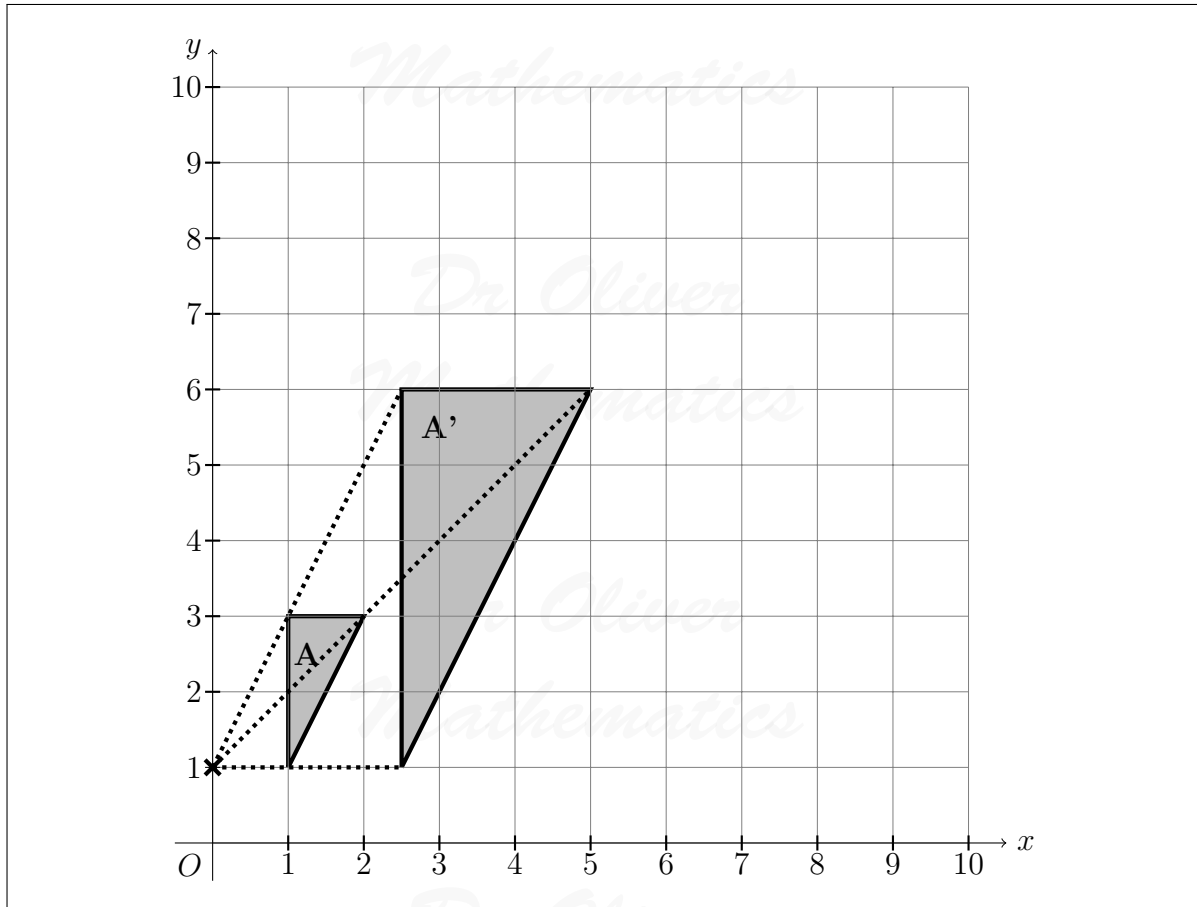
$$x = 108 - 63 = \underline{45^\circ}.$$

9. Enlarge triangle **A** by scale factor 2.5 with centre $(0, 1)$.

(2)



Solution



10. (a) Solve

$$\frac{9+x}{7} = 11-x.$$

(3)

Solution

$$\begin{aligned} \frac{9+x}{7} = 11-x &\Rightarrow 9+x = 7(11-x) \\ &\Rightarrow 9+x = 77-7x \\ &\Rightarrow 8x = 68 \\ &\Rightarrow \underline{\underline{x = 8\frac{1}{2}}}. \end{aligned}$$

(b) Simplify

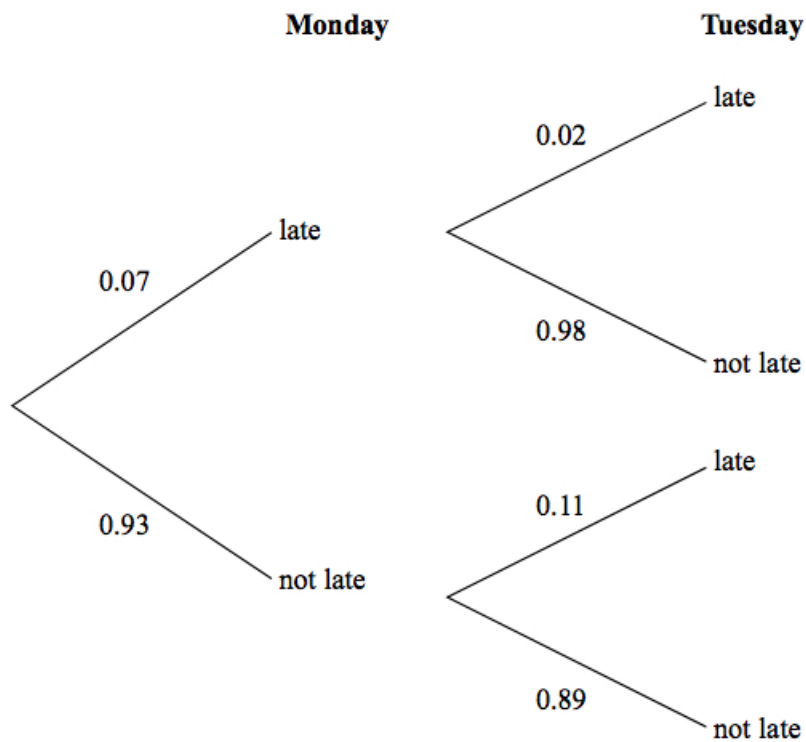
$$\frac{4(y+3)^3}{(y+3)^2}.$$

(1)

Solution

$$\frac{4(y + 3)^3}{(y + 3)^2} = 4(y + 3)$$
$$= \underline{\underline{4y + 12.}}$$

11. The probability tree diagram shows the probabilities that Bismah will be late for work on two days next week. (3)



Calculate the probability that Bismah will be late on exactly one of the two days.

Solution

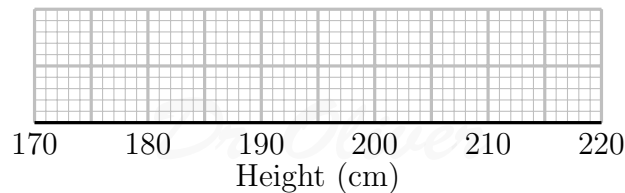
$$\begin{aligned}
 P(\text{exactly one of the two days}) &= P(L, NL) + P(NL, L) \\
 &= (0.07 \times 0.98) + (0.93 \times 0.11) \\
 &= 0.0686 + 0.1023 \\
 &= \underline{\underline{0.1709}}.
 \end{aligned}$$

12. The stem and leaf diagram shows information about the heights, in cm, of 23 sunflowers. (3)

17	3	4	9				
18	6	8	8				
19	0	0	1	4	6	7	8
20	1	4	7	7	9	9	
21	4	8	8	9			

Key: 17|3 represents 173 cm

On the grid, draw a box plot for this information.



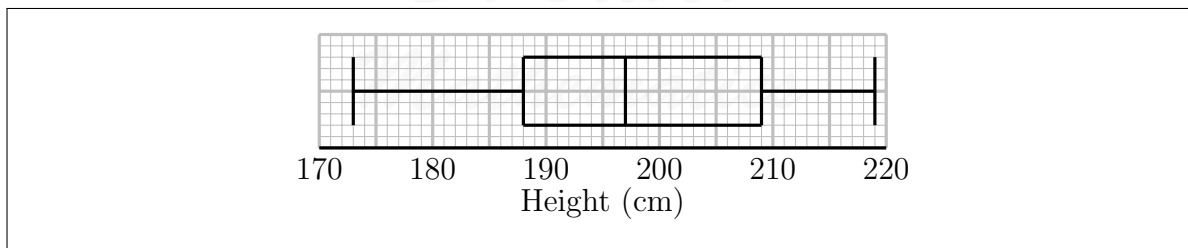
Solution

The quartiles are at

$$\left(\frac{23 + 1}{4}\right) = 6\text{th}, 12\text{th}, \text{ and } 18\text{th}$$

respectively.

Quantity	Value
Minimum	173
LQ	188
Median	197
UQ	209
Maximum	219



13. Liquid A and liquid B are mixed together in the ratio $2 : 13$ by volume to make liquid C . (4)

Liquid A has density 1.21 g/cm^3 .

Liquid B has density 1.02 g/cm^3 .

A cylindrical container is filled completely with liquid C .

The cylinder has radius 3 cm and height 25 cm .

Work out the mass of the liquid in the container.

Give your answer correct to 3 significant figures.

You must show all your working.

Solution

First,

$$\text{density} = \frac{\text{mass}}{\text{volume}}.$$

Liquid C has density

$$\begin{aligned} \frac{(2 \times 1.21) + (13 \times 1.02)}{2 + 13} &= \frac{2.42 + 13.26}{15} \\ &= \frac{15.68}{15} \\ &= \frac{392}{375} \text{ g/cm}^3. \end{aligned}$$

The

$$\begin{aligned} \text{volume} &= \pi \times 3^2 \times 25 \\ &= 225\pi \text{ cm}^3. \end{aligned}$$

Finally,

$$\begin{aligned} \text{mass} &= \text{density} \times \text{volume} \\ &= \frac{392}{375} \times 225\pi \\ &= 738.902\ 592\ 1 \text{ (FCD)} \\ &= \underline{\underline{739 \text{ g (3 sf)}}}. \end{aligned}$$

14. A group of people went to a restaurant.
Each person chose one starter and one main course.

(4)

Starter	Main Course
Soup	Lasagne
Prawns	Curry

The number of people who chose soup : the number of people who chose prawns = 2 : 3.

Of those who chose soup,

the number of people who chose lasagne : the number of people who chose curry = 5 : 3.

Of those who chose prawns,

the number of people who chose lasagne : the number of people who chose curry = 1 : 5.

What fraction of the people chose curry?

You must show how you get your answer.

Solution

$$2 + 3 = 5,$$

$$5 + 3 = 8, \text{ and}$$

$$1 + 5 = 6.$$

So consider

$$5 \times 8 \times 6 = 240$$

going for a meal (it is a common multiple of 5, 6, and 8). Now,

$$\text{chose soup} = \frac{2}{5} \times 240 = 96$$

and

$$\text{chose prawns} = \frac{3}{5} \times 240 = 144.$$

Starter	Main Course
Soup: 96	Lasagne:
Prawns: 144	Curry:

For those who chose soup,

$$\text{chose lasagne} = \frac{5}{8} \times 96 = 60$$

and

$$\text{chose curry} = \frac{3}{8} \times 96 = 36.$$

Starter	Main Course
Soup: 96	Lasagne: 60+
Prawns: 144	Curry: 36+

For those who chose prawns,

$$\text{chose lasagne} = \frac{1}{6} \times 144 = 24$$

and

$$\text{chose curry} = \frac{5}{6} \times 114 = 120.$$

Starter	Main Course
Soup: 96	Lasagne: 60 + 24 = 84
Prawns: 144	Curry: 36 + 120 = 156

Hence,

$$\text{chose curry} = \frac{156}{240} = \underline{\underline{0.65}}.$$

15. Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4. (3)

Solution

Let the two numbers be $2n$ and $2n + 2$, where $n \in \mathbb{N}$. Then

$$\begin{aligned}(2n)^2 + (2n + 2)^2 &= 4n^2 + (4n^2 + 8n + 4) \\ &= 8n^2 + 8n + 4 \\ &= 4 \times (2n^2 + 2n + 1).\end{aligned}$$

Hence, the sum of the squares of any two consecutive even numbers is always a multiple of 4.

16. y is inversely proportional to the square of x .

(3)

$y = 8$ when $x = 2.5$.

Find the negative value of x when $y = \frac{8}{9}$.

Solution

$$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2},$$

for some constant k . Now,

$$\begin{aligned} 8 &= \frac{k}{2.5^2} \Rightarrow 8 = \frac{k}{6.25} \\ &\Rightarrow k = 50; \end{aligned}$$

so,

$$y = \frac{50}{x^2}.$$

Finally,

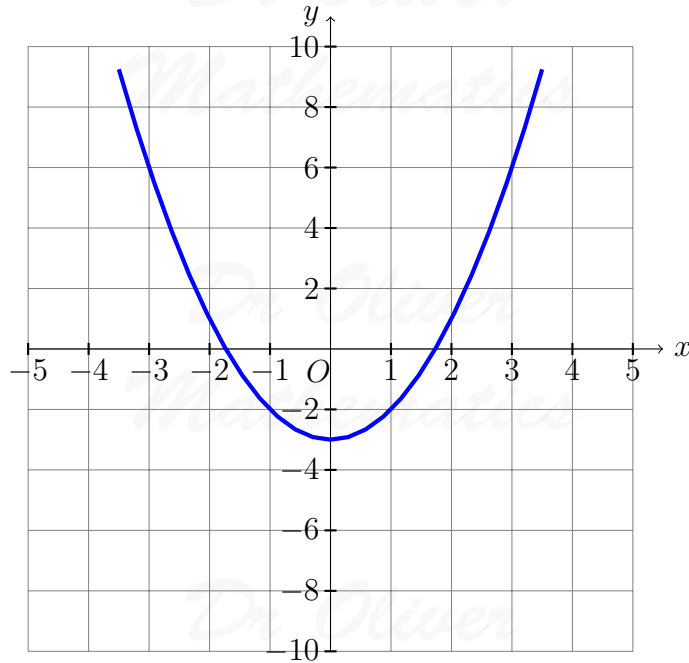
$$\begin{aligned} \frac{8}{9} &= \frac{50}{x^2} \Rightarrow x^2 = \frac{50}{\frac{8}{9}} \\ &\Rightarrow x^2 = 56.25 \\ &\Rightarrow \underline{\underline{x = -7.5}}, \end{aligned}$$

as we are asked for the 'negative value'.

17. Here is the graph of

$$y = x^2 - 3.$$

(4)



Use the graph to find estimates for the solutions to the equation

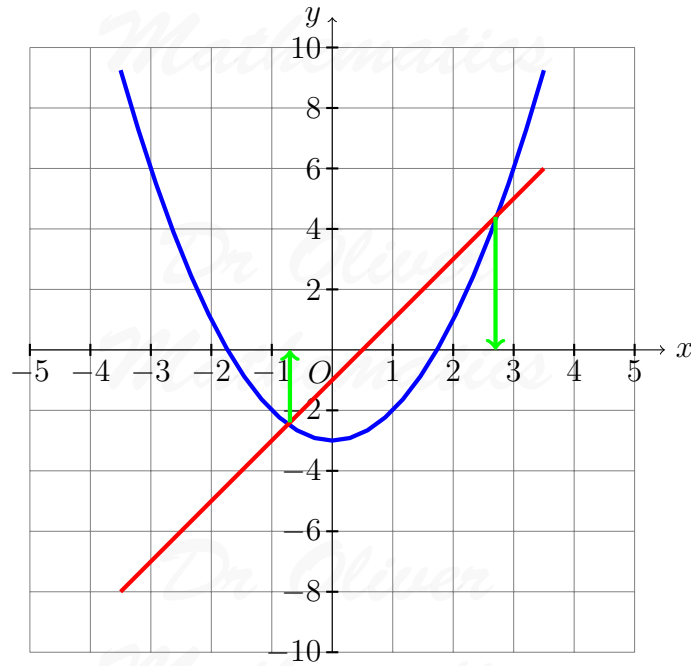
$$x^2 - 2x - 2 = 0.$$

You must show how you get your solutions.

Solution

$$x^2 - 2x - 2 = 0 \Rightarrow x^2 - 3 = 2x - 1$$

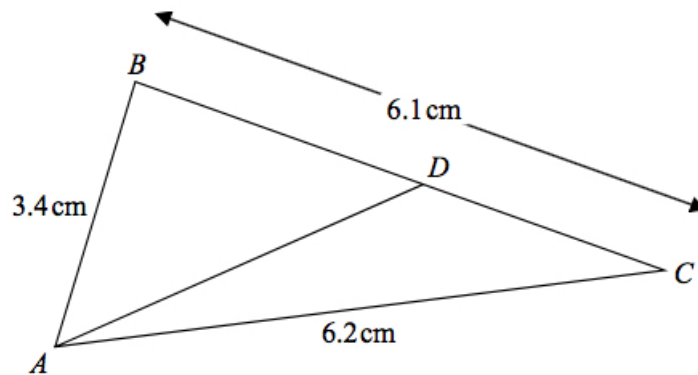
so we draw the line $y = 2x - 1$ on to the diagram.



Correct read-off: approximately $x = -0.7$ and $x = 2.7$.

18. The diagram shows triangle ABC .

(5)



$$AB = 3.4 \text{ cm.}$$

$$AC = 6.2 \text{ cm.}$$

$$BC = 6.1 \text{ cm.}$$

D is the point on BC such that

$$\text{size of angle } DAC = \frac{2}{5} \times \text{size of angle } BCA.$$

Calculate the length DC .

Give your answer correct to 3 significant figures.

You must show all your working.

Solution

We use the cosine rule:

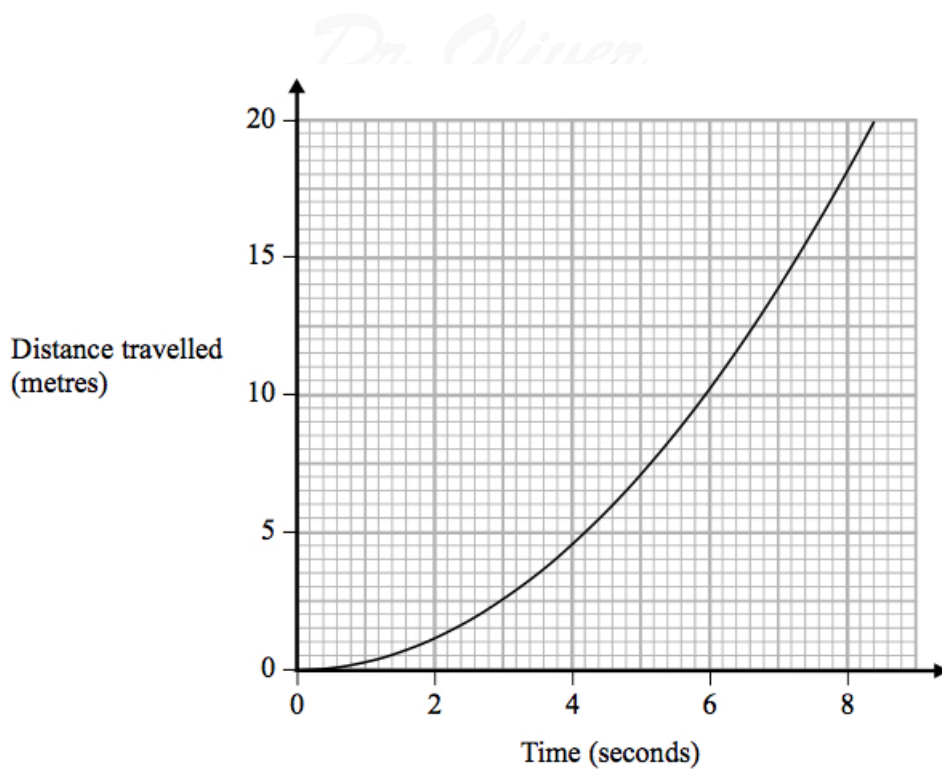
$$\begin{aligned}\cos BCA &= \frac{AC^2 + BC^2 - AB^2}{2 \cdot AC \cdot BC} \Rightarrow \cos BCA = \frac{6.2^2 + 6.1^2 - 3.4^2}{2 \cdot 6.2 \cdot 6.1} \\ &\Rightarrow \cos BCA = \frac{64.09}{75.64} \\ &\Rightarrow \angle BCA = 32.080\,469\,13 \text{ (FCD)} \\ &\Rightarrow \angle DAC = \frac{2}{5} \times 32.080\,469\,13 \text{ (FCD)} \\ &\Rightarrow \angle DAC = 12.832\,187\,65 \text{ (FCD)}.\end{aligned}$$

Finally, we use the sine rule:

$$\begin{aligned}\frac{DC}{\sin DAC} &= \frac{AC}{\sin ABC} \Rightarrow \frac{DC}{\sin 12.832\dots^\circ} = \frac{6.2}{\sin(180 - 32.080\dots - 12.832\dots)^\circ} \\ &\Rightarrow DC = \frac{6.2 \sin 12.832\dots^\circ}{\sin 135.087\dots^\circ} \\ &\Rightarrow DC = 1.950\,343\,185 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{DC = 1.95 \text{ cm (3 sf)}}}.\end{aligned}$$

19. The graph shows information about part of a cyclist's journey.

(3)



Work out an estimate of the speed, in m/s, of the cyclist at time 6 seconds.

Solution

Draw the graph's tangent at (6, 10): it goes through (4, 3.5) and (8, 17) and

$$\begin{aligned}
 \text{speed} &= \frac{17 - 3.5}{8 - 4} \\
 &= \frac{13.5}{4} \\
 &= \underline{\underline{3\frac{3}{8} \text{ m/s.}}}
 \end{aligned}$$

20. Here are the first five terms of a sequence:

(2)

-1 0 3 8 15.

Find an expression, in terms of n , for the n th term of this sequence.

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Solution

Clearly, it is not a linear sequence (why?). Let the

$$n\text{th term} = an^2 + bn + c.$$

$$\begin{array}{cccccc} -1 & 0 & 3 & 8 & 15 & \\ & 1 & 3 & 5 & 7 & \\ & & 2 & 2 & 2 & \end{array}$$

Compare this with the first three ($an^2 + bn + c$)s:

$$\begin{array}{ccc} a + b + c & 4a + 2b + c & 9a + 3b + c \\ 3a + b & 5a + b & \\ & 2a & \end{array}$$

We compare terms:

$$2a = 2 \Rightarrow a = 1,$$

$$\begin{aligned} 3a + b = -2 &\Rightarrow 3 \times 1 + b = 1 \\ &\Rightarrow b = -2, \end{aligned}$$

and

$$\begin{aligned} a + b + c = 9 &\Rightarrow 1 - 2 + c = -1 \\ &\Rightarrow c = 0; \end{aligned}$$

hence,

$$n\text{th term} = \underline{\underline{n^2 - 2n}}.$$

21. When a biased coin is thrown 4 times, the probability of getting 4 heads is $\frac{16}{81}$. (2)

Work out the probability of getting 4 tails when the coin is thrown 4 times.

Solution

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$$\begin{aligned}P(H) &= \sqrt[4]{\frac{16}{81}} \\ &= \frac{\sqrt[4]{16}}{\sqrt[4]{81}} \\ &= \frac{2}{3}\end{aligned}$$

and

$$P(T) = 1 - \frac{2}{3} = \frac{1}{3}.$$

Hence,

$$\begin{aligned}P(TTTT) &= \left(\frac{1}{3}\right)^4 \\ &= \frac{1}{81}.\end{aligned}$$

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22. Show that

$$\frac{7x - 14}{x^2 + 4x - 12} \div \frac{x - 6}{x^3 - 36x}$$

(4)

simplifies to ax where a is an integer.

Solution

$$\begin{array}{l} \text{add to:} \quad +4 \\ \text{multiply to:} \quad -12 \end{array} \left. \vphantom{\begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array}} \right\} + 6, -2$$

So

$$x^2 + 4x - 12 = (x + 6)(x - 2).$$

Now,

$$x^3 - 36x = x(x^2 - 36)$$

$$\begin{array}{l} \text{add to:} \quad 0 \\ \text{multiply to:} \quad -36 \end{array} \left. \vphantom{\begin{array}{l} \text{add to:} \\ \text{multiply to:} \end{array}} \right\} + 6, -6$$

$$= x(x + 6)(x - 6).$$

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Finally,

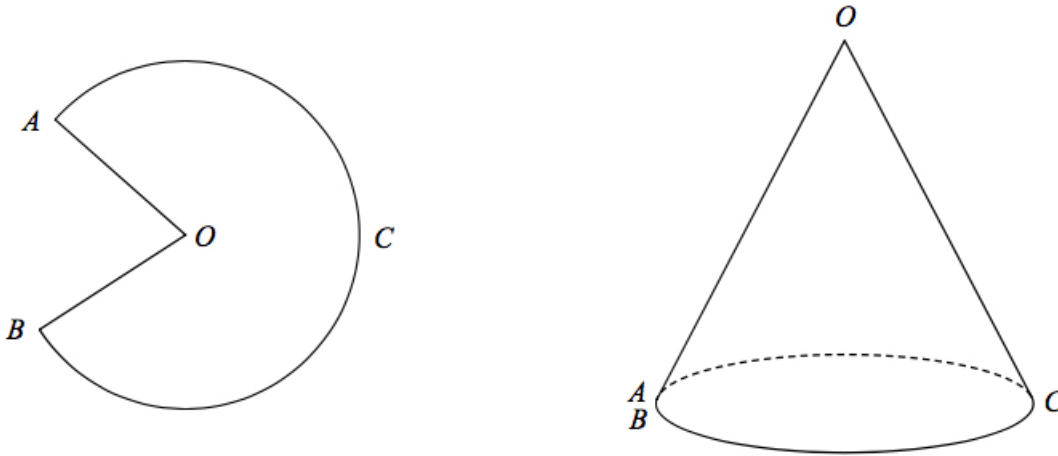
$$\begin{aligned} \frac{7x - 14}{x^2 + 4x - 12} \div \frac{x - 6}{x^3 - 36x} &= \frac{7x - 14}{x^2 + 4x - 12} \times \frac{x^3 - 36x}{x - 6} \\ &= \frac{7(x - 2)}{(x + 6)(x - 2)} \times \frac{x(x + 6)(x - 6)}{x - 6} \\ &= \frac{7\cancel{(x - 2)}}{(x + 6)\cancel{(x - 2)}} \times \frac{x\cancel{(x + 6)}\cancel{(x - 6)}}{\cancel{x - 6}} \\ &= \underline{7x}; \end{aligned}$$

hence, a = 7.

23. The diagram shows a sector $OACB$ of a circle with centre O .
The point C is the midpoint of the arc AB .

(5)

The diagram also shows a hollow cone with vertex O .
The cone is formed by joining OA and OB .



The cone has volume 56.8 cm^3 and height 3.6 cm .

Calculate the size of angle AOB of sector $OACB$.

Give your answer correct to 3 significant figures.

You must show all your working.

Solution

Well,

$$\text{volume} = \frac{1}{3}\pi r^2 h,$$

where $h = 3.6$ cm. Now,

$$\begin{aligned} 56.8 &= \frac{1}{3} \times \pi \times r^2 \times 3.6 \Rightarrow r^2 = \frac{170.4}{3.6\pi} \\ &\Rightarrow r = 3.881\ 580\ 599 \text{ cm (FCD)}. \end{aligned}$$

We apply Pythagoras:

$$\begin{aligned} l^2 &= r^2 + h^2 \Rightarrow l^2 = \frac{170.4}{3.6\pi} + 3.6^2 \\ &\Rightarrow l^2 = \frac{170.4}{3.6\pi} + 3.6^2 \\ &\Rightarrow l = 5.294\ 021\ 906 \text{ cm (FCD)}. \end{aligned}$$

Now,

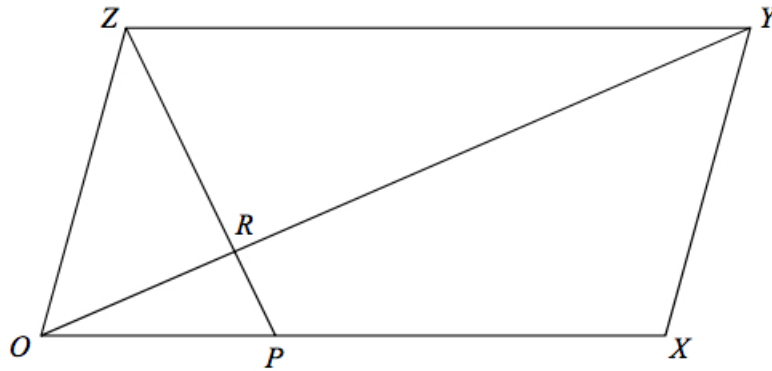
$$\text{curved surface area} = \pi r l$$

and

$$\begin{aligned} \pi \times 3.881 \dots \times 5.294 \dots &= \frac{\angle AOB}{360} \times \pi \times 5.294 \dots^2 \\ \Rightarrow 3.881 \dots &= \frac{\angle AOB}{360} \times 5.294 \dots \\ \Rightarrow \angle AOB &= \frac{3.881 \dots \times 360}{5.294 \dots} \\ \Rightarrow \angle AOB &= 263.952\ 254\ 1 \text{ (FCD)} \\ \Rightarrow \underline{\underline{\angle AOB}} &= \underline{\underline{264^\circ}} \text{ (3 sf)}. \end{aligned}$$

24. $OXYZ$ is a parallelogram.

(5)



$$\begin{aligned}\overrightarrow{OX} &= \mathbf{a}. \\ \overrightarrow{OY} &= \mathbf{b}.\end{aligned}$$

P is the point on OX such that $OP : PX = 1 : 2$.

R is the point on OY such that $OR : RY = 1 : 3$.

Work out, in its simplest form, the ratio $ZP : ZR$.

You must show all your working.

Solution

Well, $\overrightarrow{OP} = \frac{1}{3}\mathbf{a}$ and $\overrightarrow{OR} = \frac{1}{4}\mathbf{b}$. Now,

$$\begin{aligned}\overrightarrow{ZP} &= \overrightarrow{ZO} + \overrightarrow{OP} \\ &= \overrightarrow{YX} + \overrightarrow{OP} \\ &= (-\mathbf{b} + \mathbf{a}) + \frac{1}{3}\mathbf{a} \\ &= \frac{4}{3}\mathbf{a} - \mathbf{b} \\ &= 4\left(\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right)\end{aligned}$$

and

$$\begin{aligned}\overrightarrow{ZR} &= \overrightarrow{ZY} + \overrightarrow{YR} \\ &= \overrightarrow{OX} + \frac{3}{4}\overrightarrow{YO} \\ &= \mathbf{a} + \frac{3}{4}(-\mathbf{b}) \\ &= \mathbf{a} - \frac{3}{4}\mathbf{b} \\ &= 3\left(\frac{1}{3}\mathbf{a} - \frac{1}{4}\mathbf{b}\right).\end{aligned}$$

Hence,

$$ZP : ZR = \underline{\underline{4 : 3}}.$$