

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2011 June Paper 2 Variant 2: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. (a) Given that (1)

$$y = \sin 3x,$$

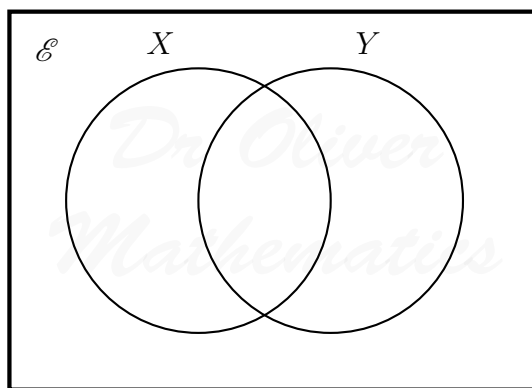
find $\frac{dy}{dx}$.

- (b) Hence, find the approximate increase in y as x increases from $\frac{1}{9}\pi$ to $\frac{1}{9}\pi + p$, where p is small (2)

2. (a) An outdoor club has three sections, walking, biking, and rock-climbing.

Using to denote \mathcal{E} the set of all members of the club and W , B , and R to denote the members of the walking, biking, and rock-climbing sections respectively, write each of the following statements using set notation.

- (i) There are 72 members in the club. (1)
(ii) Every member of the rock-climbing section is also a member of the walking section. (1)
- (b) (i) On the diagram shade the region which represents the set $X \cup Y'$. (1)



- (ii) Using set notation express the set $X \cup Y'$ in an alternative way. (1)

3. (a) Given that (3)

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -2 & 5 \end{pmatrix},$$

find the inverse of the matrix $\mathbf{A} + \mathbf{I}$, where \mathbf{I} is the identity matrix.

- (b) Hence, or otherwise, find the matrix \mathbf{X} such that (2)

$$\mathbf{AX} + \mathbf{X} = \mathbf{B},$$

where

$$\mathbf{B} = \begin{pmatrix} 14 \\ 4 \end{pmatrix}.$$

4. (a) Prove that (3)

$$\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} \equiv 2 \tan x \sec x.$$

An acute angle x is such that

$$\sin x = p.$$

- (b) Given that (3)

$$\sin 2x = 2 \sin x \cos x,$$

find an expression, in terms of p , for $\operatorname{cosec} 2x$.

5. (a) Given that (3)

$$y = x\sqrt{2x + 15},$$

show that

$$\frac{dy}{dx} = \frac{k(x + 5)}{\sqrt{2x + 15}},$$

where k is a constant to be found.

- (b) Hence find (3)

$$\int \frac{x + 5}{\sqrt{2x + 15}} dx,$$

and evaluate

$$\int_{-3}^5 \frac{x + 5}{\sqrt{2x + 15}} dx.$$

6. The line (7)

$$y = 3x - 9$$

intersects the curve

$$49x^2 - y^2 + 42x + 8y = 247$$

at the points A and B .

Find the length of the line AB .

7. A particle moves in a straight line so that, t s after passing through a fixed point O , its velocity, v ms^{-1} , is given by

$$v = \frac{60}{(3t + 4)^2}.$$

- (a) Find the velocity of the particle as it passes through O . (1)
 (b) Find the acceleration of the particle when $t = 2$. (3)
 (c) Find an expression for the displacement of the particle from O , t s after it has passed through O . (4)

8. (a) (i) Solve (2)

$$3^x = 200,$$

 giving your answer to 2 decimal places.

- (ii) Solve (4)

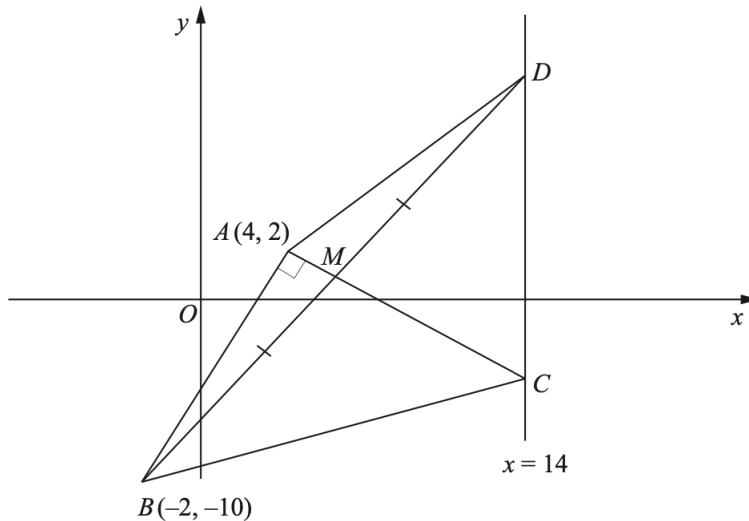
$$\log_5(5y + 40) - \log_5(y + 2) = 2.$$

- (b) Given that (3)

$$\frac{(24z^3)^2}{27 \times 12z} = 2^a 3^b z^c,$$

 evaluate a , b , and c .

9. **Solutions to this question by accurate drawing will not be accepted.** (9)
 The diagram shows the quadrilateral $ABCD$ in which A is the point $(4, 2)$ and B is the point $(-2, -10)$.



- The points C and D lie on the line $x = 14$.
- The diagonal AC is perpendicular to AB and passes through the mid-point, M , of the diagonal BD .

Find the area of the quadrilateral $ABCD$.

10. (a) (i) Express (3)

$$18 + 16x - 2x^2$$

in the form

$$a + b(x + c)^2,$$

where a , b , and c are integers.

A function f is defined by

$$f : x \mapsto 18 + 16x - 2x^2 \text{ for } x \in \mathbb{R}.$$

- (ii) Write down the coordinates of the stationary point on the graph of $y = f(x)$. (1)

- (iii) Sketch the graph of $y = f(x)$. (2)

- (b) A function g is defined by

$$g : x \mapsto (x + 3)^2 - 7 \text{ for } x > -3.$$

- (i) Find an expression for $g^{-1}(x)$. (2)

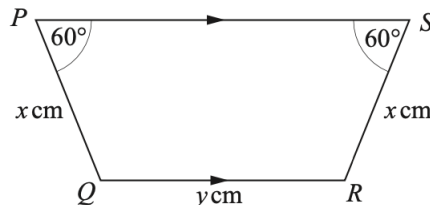
- (ii) Solve the equation (3)

$$g^{-1}(x) = g(0).$$

EITHER

11. (a) Using an equilateral triangle of side 2 units, find the exact value of $\sin 60^\circ$ and of $\cos 60^\circ$. (3)

- (b) Consider the diagram below.



- $PQRS$ is a trapezium in which $PQ = RS = x$ cm and $QR = y$ cm.

- Angle $QPS =$ angle $RSP = 60^\circ$ and QR is parallel to PS .

- (i) Given that the perimeter of the trapezium is 60 cm, express y in terms of x . (2)

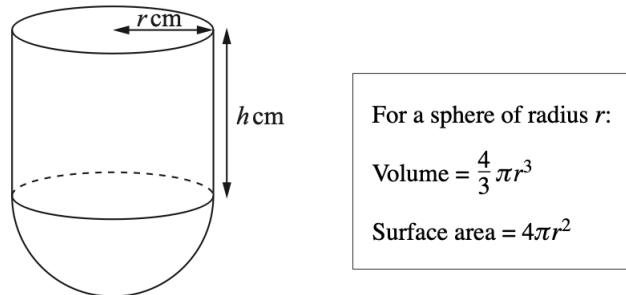
- (ii) Given that the area of the trapezium is A cm², show that (3)

$$A = \frac{\sqrt{3}(30x - x^2)}{2}.$$

- (iii) Given that x can vary, find the value of x for which A has a stationary value and determine the nature of this stationary value. (4)

OR

12. The diagram shows a solid object in the form of a cylinder of height h cm and radius r cm on top of a hemisphere of radius r cm.



Given that the volume of the object is 2880π cm³,

- (a) express h in terms of r , (2)

- (b) show that the external surface area, A cm², of the object is given by (3)

$$A = \frac{5}{3}\pi r^2 + \frac{5760\pi}{r}.$$

Given that r can vary,

- (c) find the value of r for which A has a stationary value, (4)

- (d) find this stationary value of A , leaving your answer in terms of π , (2)

- (e) determine the nature of this stationary value. (1)