Dr Oliver Mathematics
AQA GCSE Mathematics
2013 June Paper 2: Calculator
2 hours

The total number of marks available is 105.
You must write down all the stages in your working.

1. This formula is used for working out the cost, £C, of repairing a car:

$$C = nL + 1.2P,$$

where

- n is the number of hours worked,
- L is the labour rate (£), and
- P is the cost of parts (£).

(a) Work out the cost of repairing a car when $n = 3$, $L = 18$, and $P = 110$. (2)

Solution

$$\begin{aligned} C &= nL + 1.2P \\ &= (3 \times 18) + (1.2 \times 110) \\ &= 54 + 132 \\ &= \underline{186}. \end{aligned}$$

(b) Complete this table for another repair. (3)

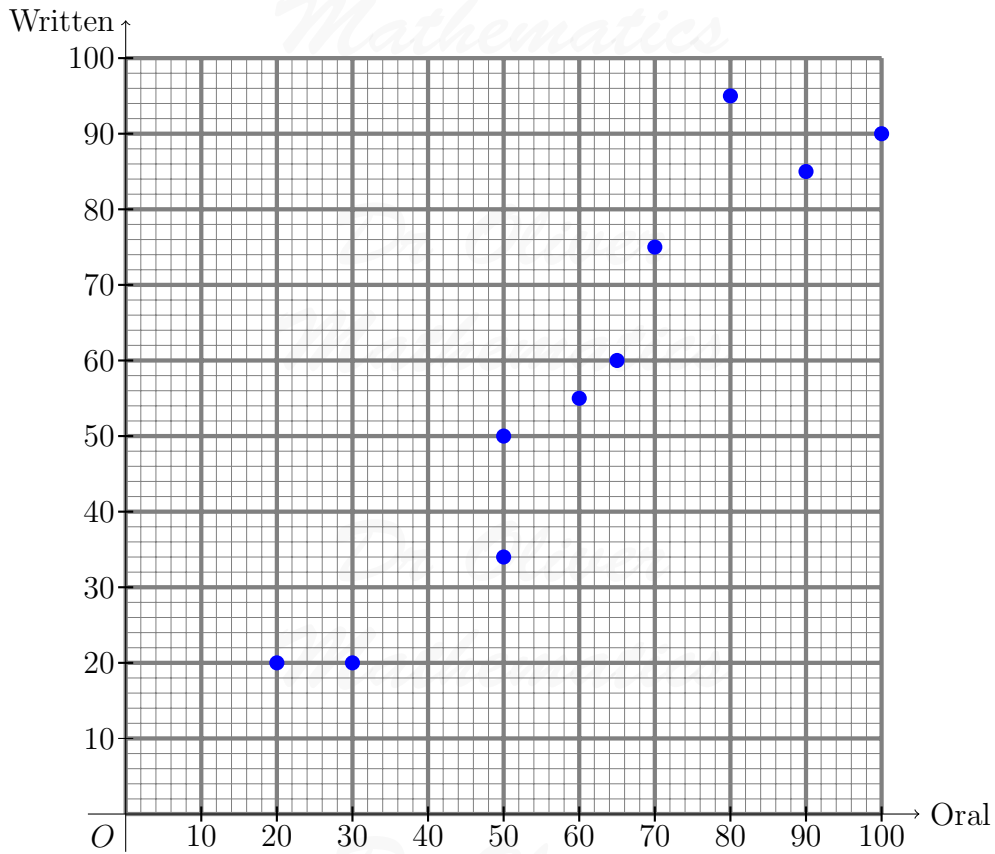| C	n	L	P
£235		£22	£150

Solution

$$\begin{aligned} C = nL + 1.2P \Rightarrow 235 &= 22n + (1.2 \times 150) \\ \Rightarrow 235 &= 22n + 180 \\ \Rightarrow 22n &= 55 \\ \Rightarrow \underline{\underline{n = 2\tfrac{1}{2}}}. \end{aligned}$$

Hence,

C	n	L	P
£235	$2\frac{1}{2}$	£22	£150

2. The scatter diagram shows the scores of 10 students in their Oral and Written tests.

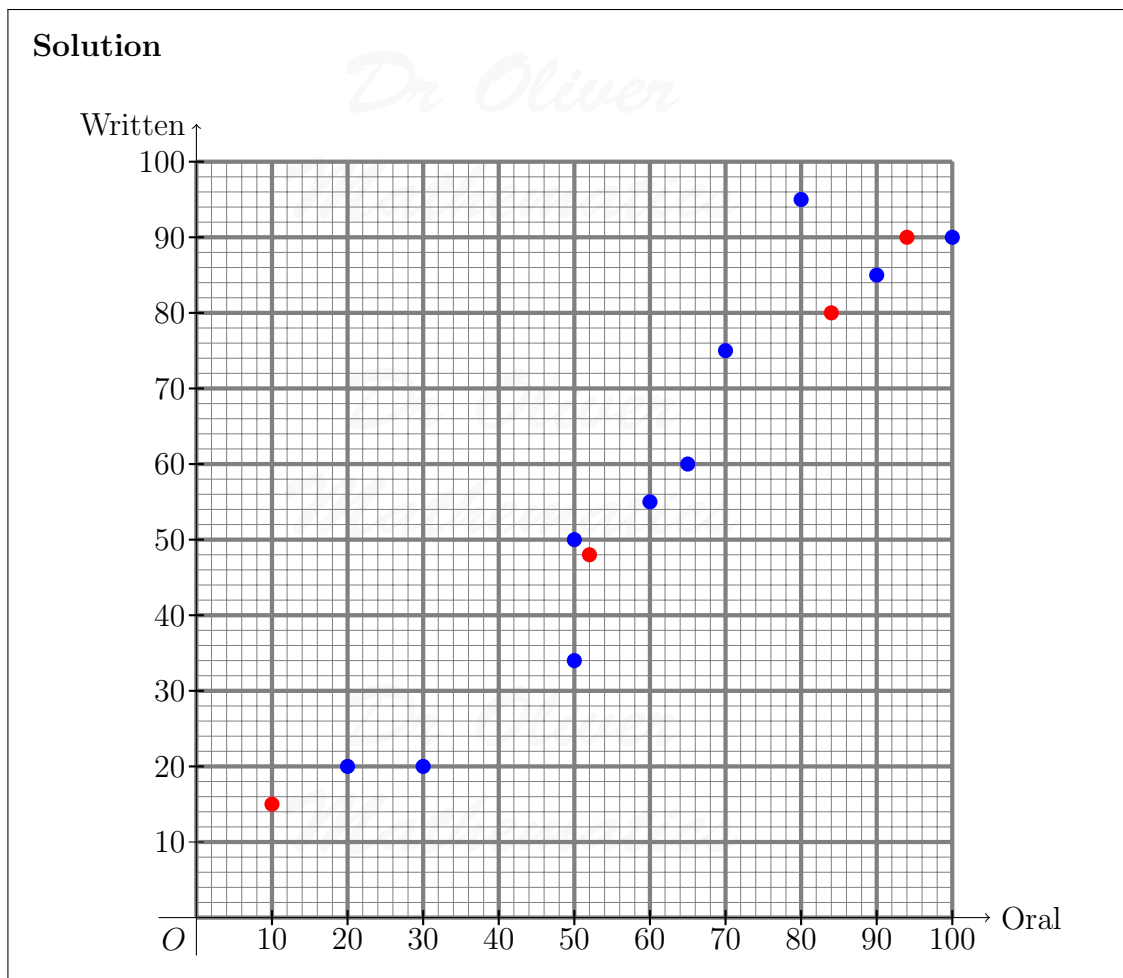(a) How many students scored 50 in their Oral test? (1)

Solution

2.

(b) Four **more** students take the same tests. (2)
The table shows their scores.

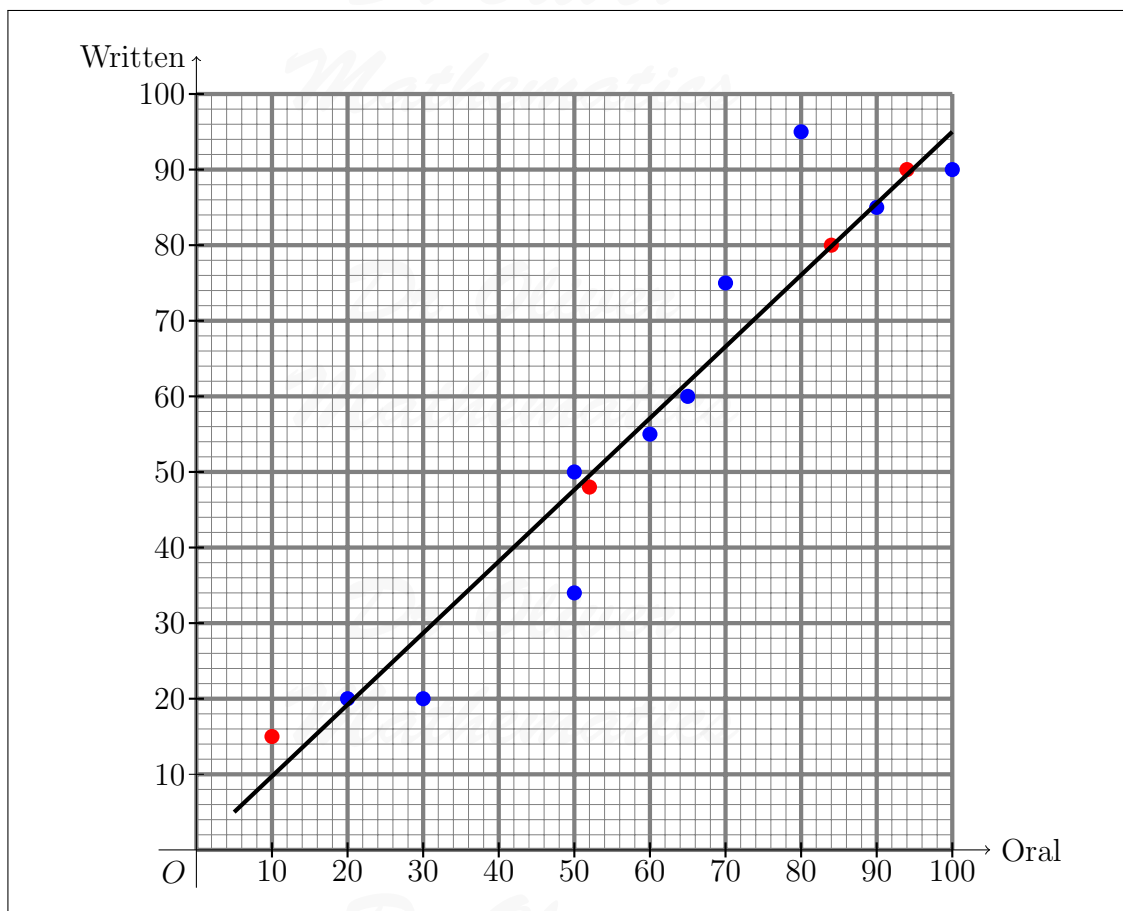| Oral	10	94	52	84
Written	15	90	46	80

Plot the scores on the scatter diagram.

Solution

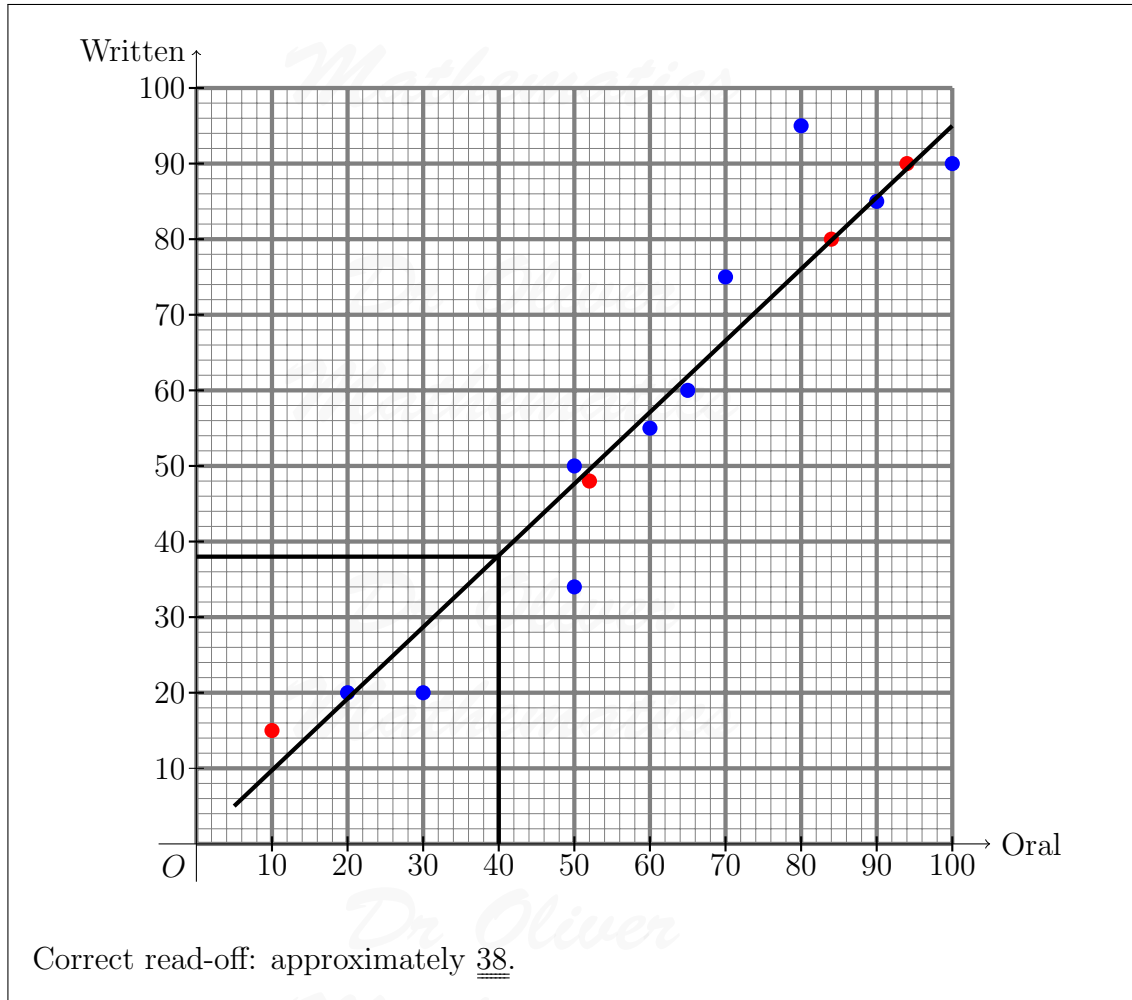(c) Draw a line of best fit on the scatter diagram. (1)

Solution

(d) Rob scored 40 in the Oral test. (1)
He was absent for the Written test.

Use your line of best fit to estimate a score for him in the Written test.

Solution

4

Written

100
90
80
70
60
50
40
30
20
10

O   10   20   30   40   50   60   70   80   90   100   → Oral

Correct read-off: approximately $\underline{\underline{38}}$.

3. Andrew is planning a survey about his local library.
   Here is one of his questions with a response section.

How many times do you go to the library?

1   2   3   5 or more

☐   ☐   ☐   ☐
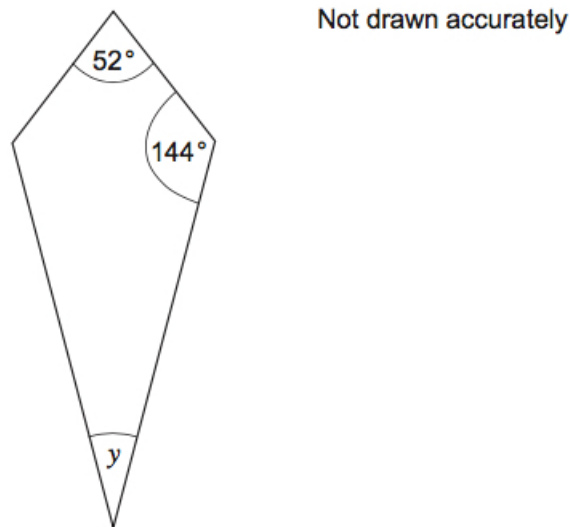
(a) Give **one** criticism of the **question**.   (1)

5

> **Solution**
>
> E.g., it is not specific enough (yesterday/last week/in your whole life).

(b) Give **two** criticisms of the **response** section. (2)

> **Solution**
>
> E.g., 0 is omitted, 4 is omitted.
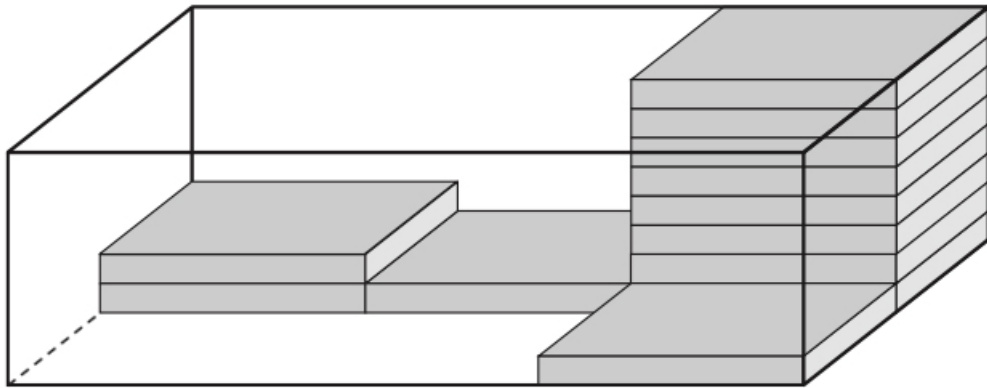
4. The diagram shows a kite. (2)

Not drawn accurately

Work out the size of angle y.

> **Solution**
>
> $$52 + 144 + y + 144 = 360 \Rightarrow y + 340 = 360$$
> $$\Rightarrow y = 20°.$$

5. DVD cases are packed in this box. (5)

Jenny buys a **full** box of cases for £2.43
She sells all the cases for 11 pence each.
She saves **two-thirds** of the profit.

How much money does she save?

Solution

There are
$$8 \times 6 = 48$$

DVD in each box. Now, the cost is
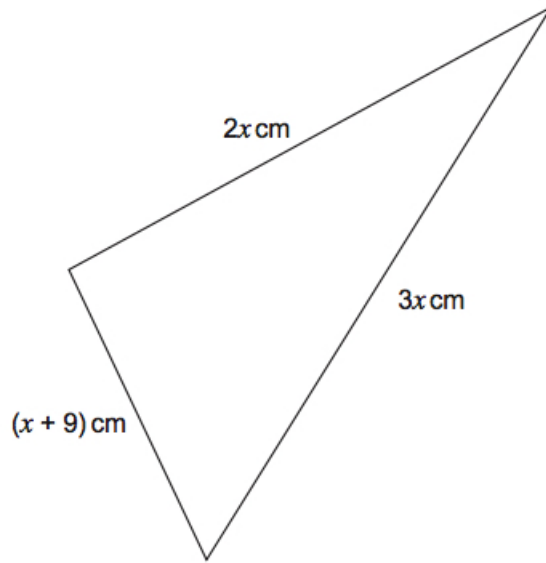$$48 \times 0.11 = £5.28$$

and the profit is
$$5.28 - 2.43 = £2.85.$$

Hence, two-thirds of the profit is

$$\tfrac{2}{3} \times 2.85 = \underline{\underline{£1.90}}.$$

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6. The perimeter of this triangle is 48 cm. (4)

Work out the value of x.

Solution

$$2x + (x + 9) + 3x = 48 \Rightarrow 6x + 9 = 48$$
$$\Rightarrow 6x = 39$$
$$\Rightarrow x = 6\tfrac{1}{2}.$$

7. Here are two ways of having a car for one year. (5)

Buy and sell	**Rent**
Buy it for £ 12 000	Normal Price: £ 195 per month
Sell it for £ 10 000 after one year	**Special Offer** 15% off

Which way is cheaper?
You **must** show your working.

> **Solution**
>
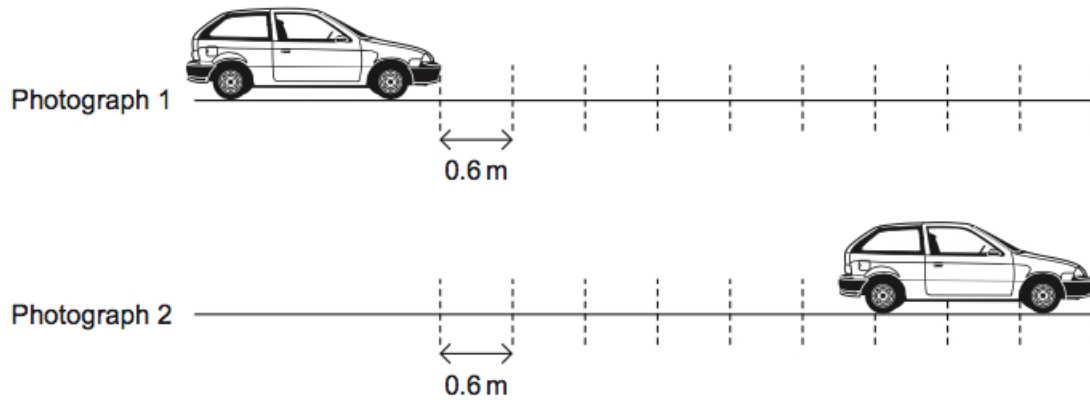> Buy and Sell:
> $$\text{Loss} = 12\,000 - 10\,000 = 2\,000.$$
>
> Rent:
> $$\text{Expenditure} = 195 \times 12 \times 0.85$$
> $$= 1\,989.$$
>
> The person must <u>Rent</u> and it is <u>£11</u> cheaper.

8. (a) A speed camera takes two photographs of a car. (3)

Photograph 1

0.6 m

Photograph 2

0.6 m

Photograph 2 was taken 0.5 seconds after Photograph 1.
Marks on the road are 0.6 metres apart.

Calculate the average speed of the car in m/s.

> **Solution**
>
> $$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$
> $$= \frac{0.6 \times 9 \text{ m}}{0.5 \text{ s}}$$
> $$= \frac{5.4 \text{ m}}{0.5 \text{ s}}$$
> $$= 10.8 \text{ m/s}.$$

(b) You are given that (3)

$$1 \text{ kilometre} = 1\,000 \text{ metres}$$

and

$$1 \text{ hour} = 3\,600 \text{ seconds}.$$

A lorry is travelling at 13.6 m/s.
The speed limit is 50 km/h.

Show that the lorry is travelling below the speed limit.

Solution

$$\begin{aligned} 13.6 \text{ m/s} &= \frac{13.6 \text{ m}}{1 \text{ s}} \\ &= \frac{0.0136 \text{ km}}{\frac{1}{3\,600} \text{ hr}} \\ &= 48.96 \text{ km/h}; \end{aligned}$$

hence, the lorry is travelling <u>below the speed limit</u>.

9. A tank contains 0.6 m³ of water. (3)
   The water is used to fill pots.
   Each pot can hold 1 250 cm³ of water.

   How many pots can be filled?

   Solution

   $$\begin{aligned} \frac{0.6 \text{ m}^3}{1\,250 \text{ cm}^3} &= \frac{0.6 \times 1\,000\,000 \text{ cm}^3}{1\,250 \text{ cm}^3} \\ &= \frac{600\,000 \text{ cm}^3}{1\,250 \text{ cm}^3} \\ &= \underline{\underline{480 \text{ pots}}}. \end{aligned}$$

10. 150 boys and 160 girls sit an examination.
    The table shows some of the probabilities that they came with or without a calculator.

	With calculator	Without calculator
Boy	0.92	0.08
Girl	0.95	

(a) What is the probability that a girl came **without** a calculator? (1)
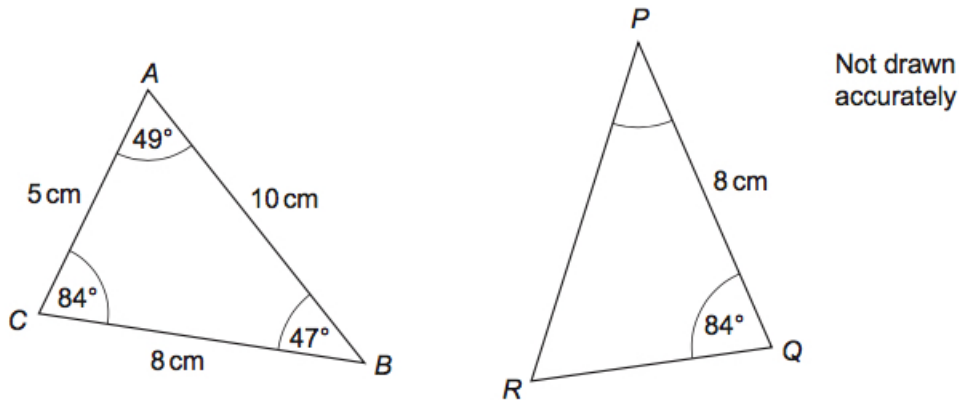Write your answer in the table.

> **Solution**
>
> | | With calculator | Without calculator |
> |-------|:---------------:|:------------------:|
> | Boy | 0.92 | 0.08 |
> | Girl | 0.95 | <u>0.05</u> |

(b) How many of the 150 boys came **with** a calculator? (2)

> **Solution**
> $$150 \times 0.92 = \underline{138}.$$

11. These two triangles are congruent.

Not drawn accurately

(a) What is the size of angle P? (1)
Circle your answer.

$$47° \qquad 49° \qquad 84° \qquad \text{none of these}$$

> **Solution**
> BC is the same as PQ: <u>47°</u>.

(b) What is the length of PR? (1)
Circle your answer.

$$5 \text{ cm} \qquad 8 \text{ cm} \qquad 10 \text{ cm} \qquad \text{none of these}$$
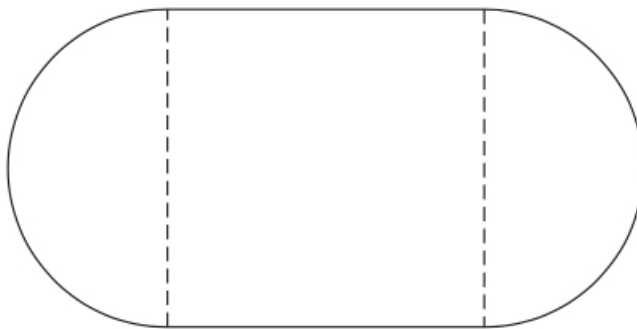
11

> **Solution**
>
> $\underline{\underline{10 \text{ cm}}}$.

12. The perimeter of this square is 48 cm. (4)
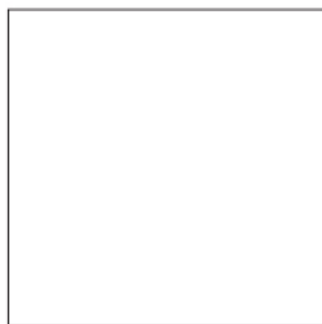
Semi-circles are joined to two sides of the square.

Work out the perimeter of this shape.

> **Solution**
>
> One side of the square is
> $$\frac{48}{4} = 12 \text{ cm}$$
> and half that is
> $$\frac{12}{2} = 6 \text{ cm}.$$

Hence,

$$\text{perimeter} = 12 + (\pi \times 6) + 12 + (\pi \times 6)$$
$$= \underline{\underline{(24 + 12\pi)}} \text{ or } \underline{\underline{61.7}} \text{ cm (3 sf).}$$

13. Amy raised £n for charity. (5)
    Chris raised £18 more than Amy.

    The mean amount raised by the two of them is £45.

    Work out how much money each one of them raised.

    Solution

    $$\frac{n + (n + 18)}{2} = 45 \Rightarrow 2n + 18 = 90$$
    $$\Rightarrow 2n = 72$$
    $$\Rightarrow n = 36$$
    $$\Rightarrow n + 18 = 54.$$

    Hence, Amy raised £$\underline{\underline{36}}$ and Chris raised £$\underline{\underline{54}}$.
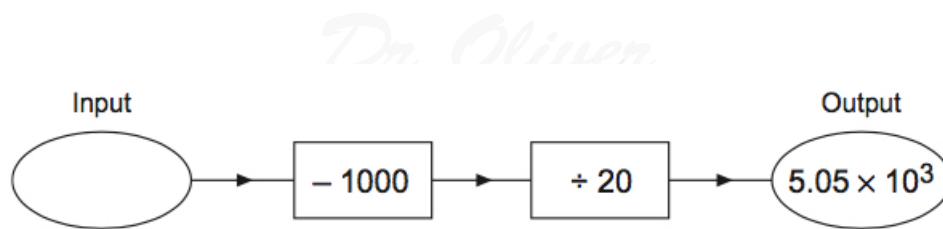
14. (a) Work out (2)
    $$(6.45 \times 10^6) \times (2.5 \times 10^{-4}).$$

    Write your answer in standard form.

    Solution

    $$(6.45 \times 10^6) \times (2.5 \times 10^{-4}) = (6.45 \times 2.5) \times (10^6 \times 10^{-4})$$
    $$= 16.125 \times 10^2$$
    $$= \underline{\underline{1.612\,5 \times 10^3}}.$$

    (b) Here is a number machine. (3)

13

Work out the input when the output is 5.05×10^3.

Write your answer in standard form.

Solution

$$\frac{\text{Input} - 1\,000}{20} = 5.05 \times 10^3 \Rightarrow \text{Input} - 1\,000 = 101\,000$$
$$\Rightarrow \text{Input} = 102\,000$$
$$\Rightarrow \text{Input} = 1.02 \times 10^5.$$

15. (a) Work out the value of (1)

$$x^3 - 2x + 7$$

when $x = -2.5$.

Solution

When $x = -2.5$, the answer is

$$(-2.5)^3 - 2(-2.5) + 7 = -15.625 + 5 + 7$$
$$= -3.625.$$

(b) Factorise fully (2)

$$4x^2 + 6xy.$$

Solution

$$4x^2 + 6xy = 2x(2x + 3y).$$

16. Here is part of a shopping bill for clothing. (5)

14

1 jacket at

1 shirt at £29

Total **cost** before discount =

10% discount

Total to pay **after** discount = £80.10

Work out the cost of the jacket **before** the discount.

Solution

$$\text{New price} = \text{old price} \times 0.9 \Rightarrow 80.1 = \text{old price} \times 0.9$$
$$\Rightarrow \text{old price} = \frac{80.1}{0.9}$$
$$\Rightarrow \text{old price} = 89.$$

Hence,

$$\text{jacket} + \text{shirt} = \text{cost} \Rightarrow \text{jacket} + 29 = 89$$
$$\Rightarrow \text{jacket} = \underline{£60}.$$

17. A is the point with coordinates $(x, 2y)$. (4)
B is the point with coordinates $(3x, 4y)$.
The midpoint of AB has coordinates $(-4, 15)$.
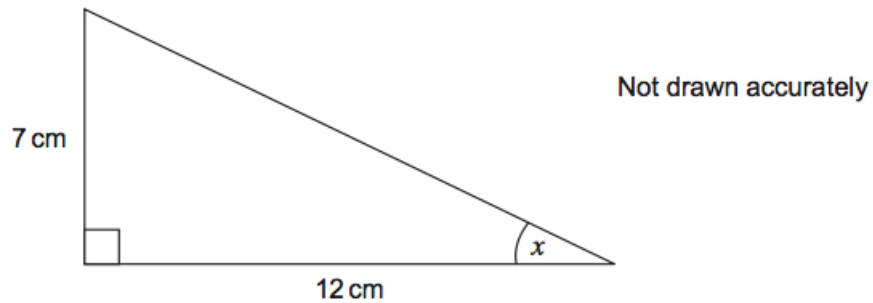
Work out the values of x and y.

Solution

$\underline{x}$:

$$\frac{x + 3x}{2} = -4 \Rightarrow 4x = -8$$
$$\Rightarrow \underline{\underline{x = -2}}.$$

15

y:

$$\frac{2y + 4y}{2} = 15 \Rightarrow 6y = 30$$
$$\Rightarrow \underline{\underline{y = 5}}.$$

18. (a) Work out the size of angle x. (3)
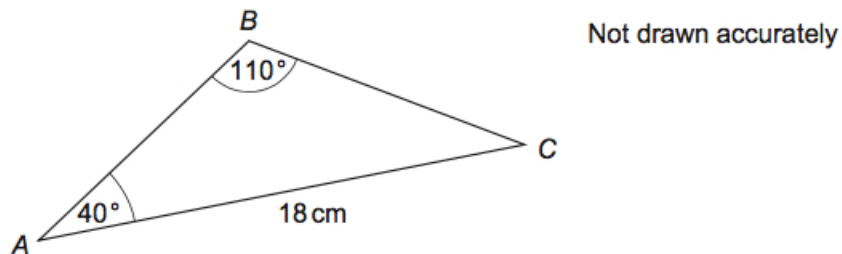
7 cm

Not drawn accurately

12 cm

x

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan x = \tfrac{7}{12}$$
$$\Rightarrow x = 30.256\,437\,16 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{x = 30.3^\circ \text{ (3 sf)}}}.$$
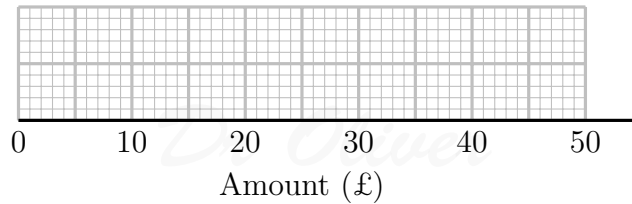
(b) Work out the length BC. (3)

B

110°

Not drawn accurately

C

40°   18 cm

A

Solution

$$\frac{BC}{\sin BAC} = \frac{AC}{\sin ABC} \Rightarrow \frac{BC}{\sin 40°} = \frac{18}{\sin 110°}$$
$$\Rightarrow BC = \frac{18 \sin 40°}{\sin 110°}$$
$$\Rightarrow BC = 12.312\,725\,16 \text{ (FCD)}$$
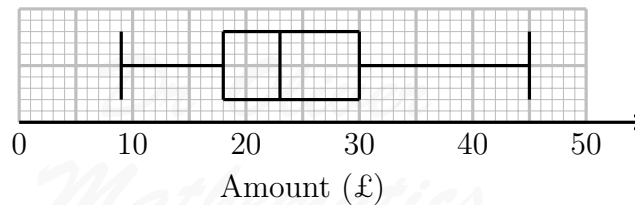$$\Rightarrow \underline{\underline{BC = 12.3 \text{ cm (3 sf)}}}.$$

19. (a) The table shows information about the travel expenses of employees at a company. All amounts are in £. (2)

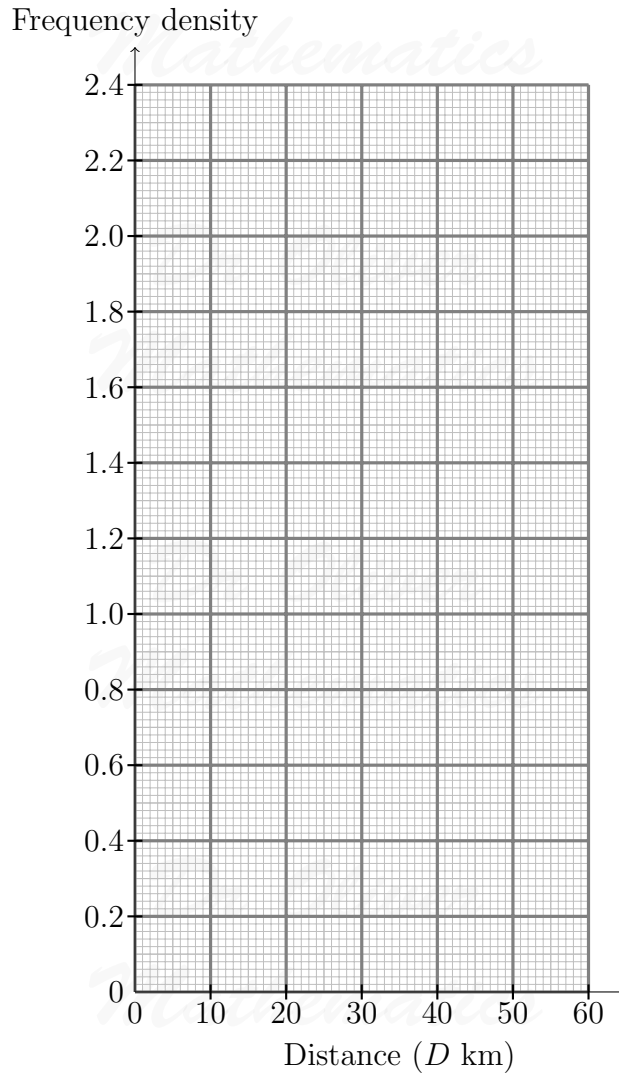| Minimum	Lower Quartile	Median	Upper Quartile	Maximum
9	18	23	30	45

Draw a box plot to show this information.

Solution

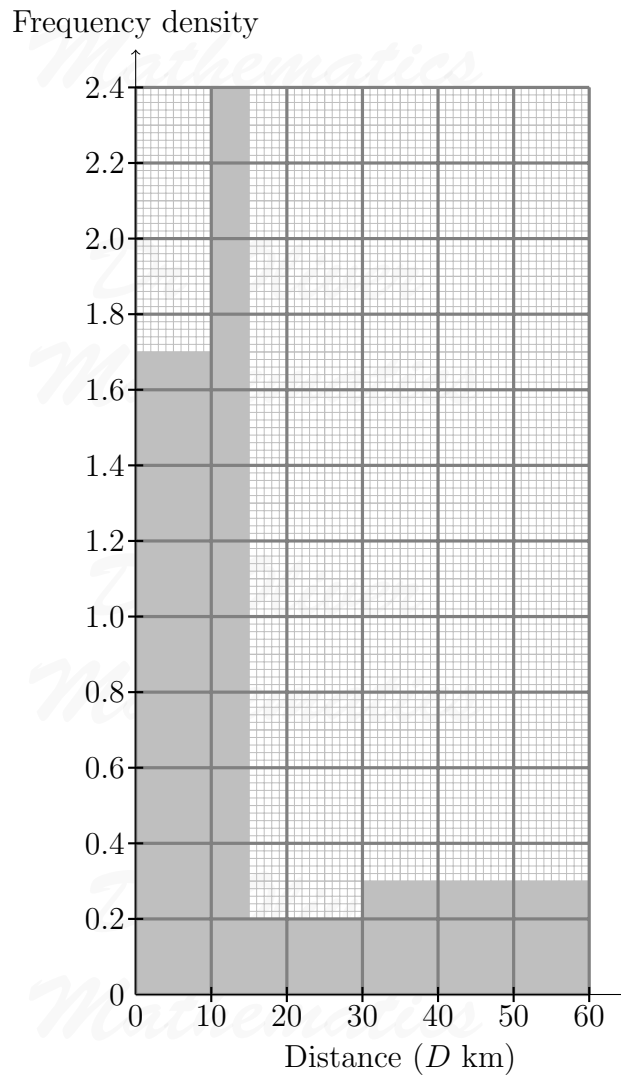(b) This table shows information about the distances the employees travel to work. (3)

Distance, D km	Frequency
$0 < D \leqslant 10$	17
$10 < D \leqslant 15$	12
$15 < D \leqslant 30$	3
$30 < D \leqslant 60$	9

Draw a histogram to show this information.

Frequency density

Distance (D km)

Solution

Distance, D km	Frequency	Width	Frequency Density
$0 < D \leqslant 10$	17	10	$\frac{17}{10} = 1.7$
$10 < D \leqslant 15$	12	5	$\frac{12}{5} = 2.4$
$15 < D \leqslant 30$	3	15	$\frac{3}{15} = 0.2$
$30 < D \leqslant 60$	9	30	$\frac{9}{30} = 0.3$

20. Solve the equation (3)

$$2x^2 + 8x + 5 = 0.$$

Give your answers to 2 decimal places.

Solution

$a = 2$, $b = 8$, and $c = 5$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 5}}{2 \times 2}$$
$$= \frac{-8 \pm \sqrt{24}}{4}$$
$$= -3.224\,744\,871, -0.775\,255\,128\,6 \text{ (FCD)}$$
$$= \underline{\underline{-3.22, -0.78 \text{ (2 dp)}}}.$$

21. The expression (3)
$$\frac{x^2 - 9}{x^2 + bx - 15}$$

simplifies to
$$\frac{x + 3}{x + 5}.$$

Work out the value of b.

Solution

$$\text{add to:} \quad 0 \atop \text{multiply to:} \quad -9 \Bigg\} - 3, +3$$

Hence,
$$x^2 - 9 = (x - 3)(x + 3).$$

In the denominator,
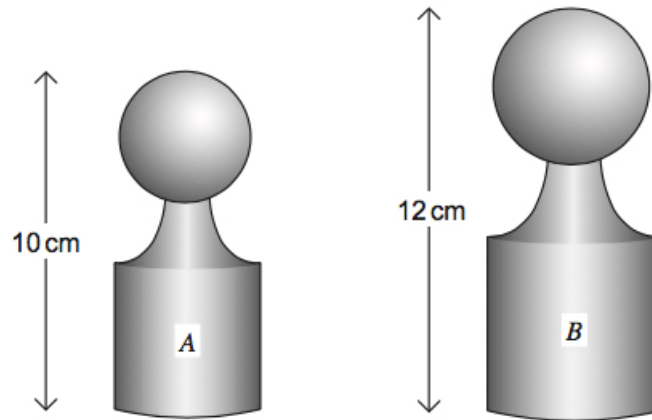$$\frac{-15}{+5} = -3$$

and so the denominator is
$$(x - 3)(x + 5).$$

$\times$	x	-3
x	x^2	$-3x$
$+5$	$+5x$	-15

20

Finally,
$$b = -3 + 5 = \underline{\underline{2}}.$$

22. A and B are two similar solids. (3)

The volume of A is 500 cm^3.

Work out the volume of B.

Solution

The length scale factor, LSF, is
$$\text{LSF} = \frac{12}{10} = 1.2$$

and the volume scale factor, VSF, is
$$\text{VSF} = 1.2^3 = 1.728.$$

Hence,
$$\text{volume of } B = 1.728 \times 500$$
$$= \underline{864 \text{ cm}^3}.$$

23. A bag contains 12 counters. (3)
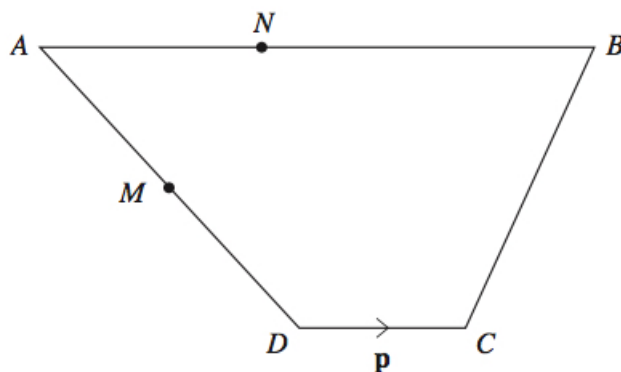   Five of the counters are white.

A counter is taken out of the bag at random and **not** replaced.
A second counter is taken out of the bag at random.

Calculate the probability that **only one** of the two counters is white.

Solution

$$P(\text{only one}) = P(\text{white, non-white}) + P(\text{non-white, white})$$
$$= \left(\tfrac{5}{12} \times \tfrac{7}{11}\right) + \left(\tfrac{7}{12} \times \tfrac{5}{11}\right)$$
$$= \tfrac{35}{132} + \tfrac{35}{132}$$
$$= \underline{\underline{\tfrac{35}{66}}}.$$

24. AB is parallel to DC.

Not drawn accurately

$\overrightarrow{AB} = 5\mathbf{p}$.
$\overrightarrow{DC} = \mathbf{p}$.
$\overrightarrow{DA} = 2\mathbf{q} - \mathbf{p}$.

(a) Show that (1)

$$\overrightarrow{CB} = 2\mathbf{q} + 3\mathbf{p}.$$

Solution

22

$$\overrightarrow{CB} = \overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{AB}$$
$$= -\mathbf{p} + (2\mathbf{q} - \mathbf{p}) + 5\mathbf{p}$$
$$= \underline{\underline{2\mathbf{q} + 3\mathbf{p}}},$$

as required.

(b) M is the midpoint of AD. (4)
$\overrightarrow{AN} : \overrightarrow{NB} = 2 : 3$.
Show that MN is parallel to CB.

Solution

$$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AN}$$
$$= \tfrac{1}{2}\overrightarrow{DA} + \tfrac{2}{5}\overrightarrow{AB}$$
$$= \tfrac{1}{2}(2\mathbf{q} - \mathbf{p}) + \tfrac{2}{5}(5\mathbf{p})$$
$$= \mathbf{q} - \tfrac{1}{2}\mathbf{p} + 2\mathbf{p}$$
$$= \mathbf{q} + \tfrac{3}{2}\mathbf{p}$$
$$= \tfrac{1}{2}(2\mathbf{q} + 3\mathbf{p})$$
$$= \tfrac{1}{2}\overrightarrow{CB};$$

hence, MN is <u>parallel</u> to CB

25. (a) On this grid, draw the graph of (1)
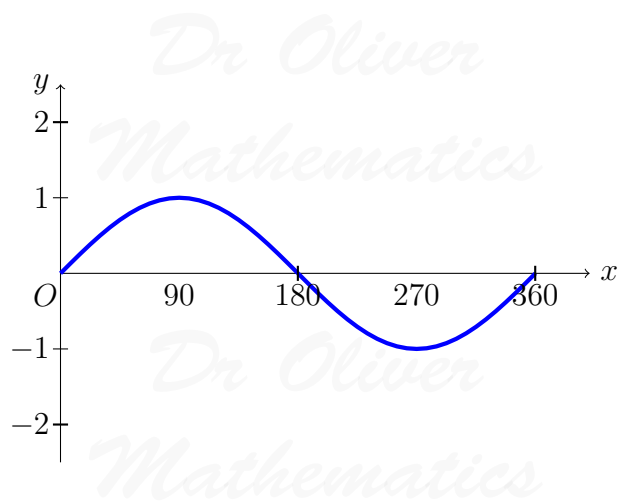
$$y = 1 + \sin x$$
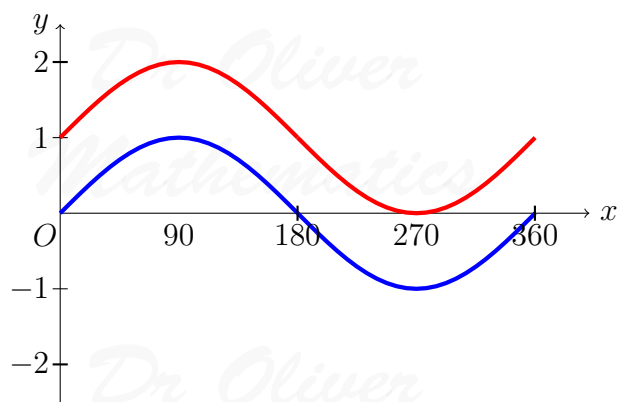
for values of x from $0°$ to $360°$.
The graph of

$$y = \sin x$$

has been drawn to help you.
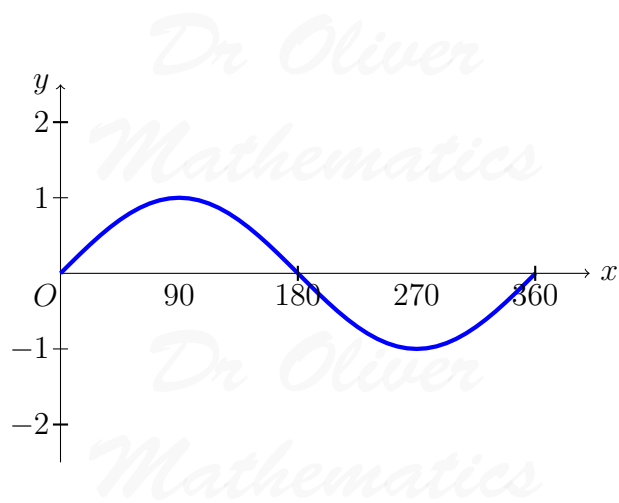
Solution

(b) On this grid, draw the graph of
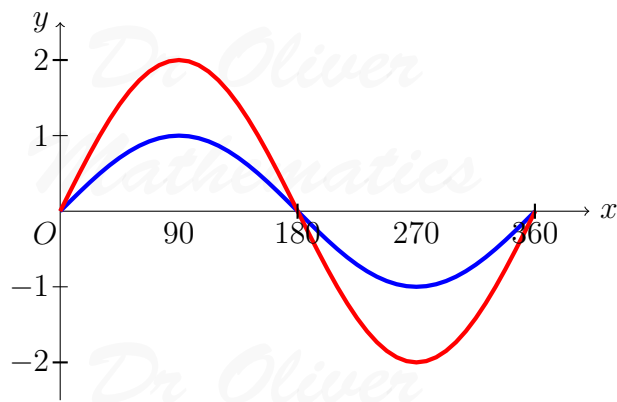$$y = 2 \sin x$$

(1)

for values of x from $0°$ to $360°$.

The graph of
$$y = \sin x$$

has been drawn to help you.

Solution

26. Solve the equation (6)
$$\frac{5}{x+2} + \frac{4}{x+1} = 2.$$

Solution

Multiply by $(x+2)(x+1)$:
$$\frac{5}{x+2} + \frac{4}{x+1} = 2 \Rightarrow 5(x+1) + 4(x+2) = 2(x+2)(x+1)$$

25

$\times$	x	$+2$
x	x^2	$+2x$
$+1$	$+x$	$+2$

$$\Rightarrow (5x+5) + (4x+8) = 2(x^2 + 3x + 2)$$
$$\Rightarrow 9x + 13 = 2x^2 + 6x + 4$$
$$\Rightarrow 2x^2 - 3x - 9 = 0$$

add to: -3
multiply to: $(+2) \times (+9) = -18$ $\Big\}-6, +3$

$$\Rightarrow 2x^2 - 6x + 3x - 9 = 0$$
$$\Rightarrow 2x(x-3) + 3(x-3) = 0$$
$$\Rightarrow (2x+3)(x-3) = 0$$
$$\Rightarrow 2x + 3 = 0 \text{ or } x - 3 = 0$$
$$\Rightarrow \underline{\underline{x = -1\tfrac{1}{2} \text{ or } x = 3.}}$$