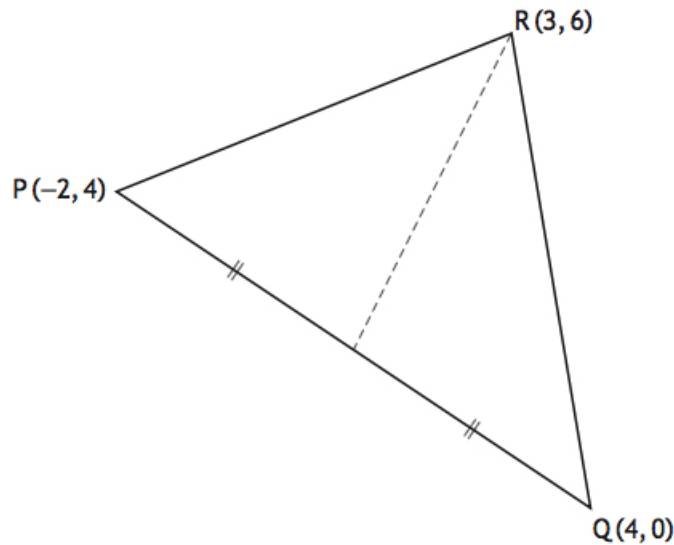# Dr Oliver Mathematics
Mathematics: Higher
2018 Paper 1: Non-Calculator
1 hour 10 minutes

The total number of marks available is 60.
You must write down all the stages in your working.

1. PQR is a triangle with vertices $P(-2, 4)$, $Q(4, 0)$, and $R(3, 6)$. (3)

   Find the equation of the median through R.

2. A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by (3)

$$g(x) = \tfrac{1}{5}x - 4.$$

   Find the inverse function, $g^{-1}(x)$.
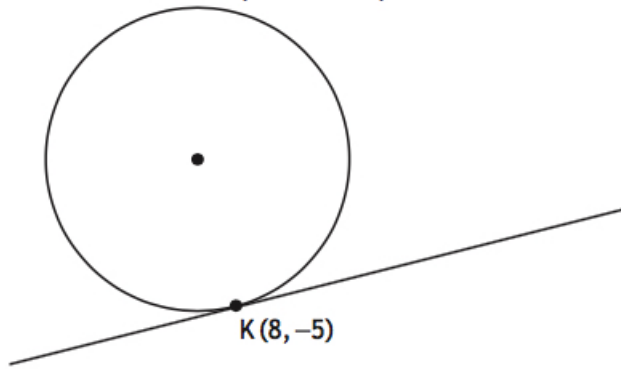
3. Given (3)
$$h(x) = 3\cos 2x,$$
   find the value of $h'(\tfrac{1}{6}\pi)$.

4. The point $K(8, -5)$ lies on the circle with equation (4)

$$x^2 + y^2 - 12x - 6y - 23 = 0.$$

$$x^2 + y^2 - 12x - 6y - 23 = 0$$

K(8, −5)

Find the equation of the tangent to the circle at K.

5. $A(-3, 4, -7)$, $B(5, t, 5)$, and $C(7, 9, 8)$ are collinear.

   (a) State the ratio in which B divides AC.   (1)

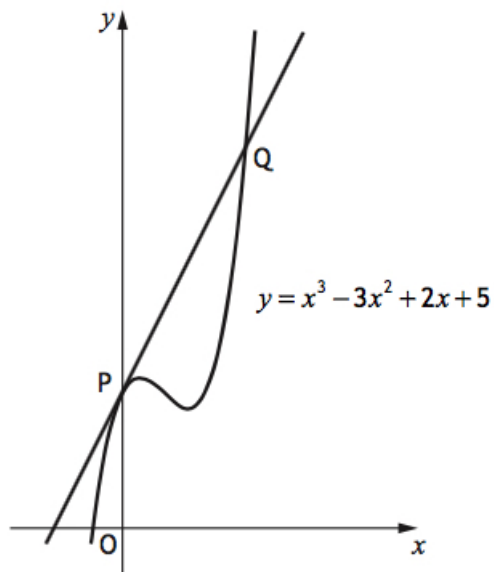   (b) State the value of t.   (1)

6. Find the value of   (3)

$$\log_5 250 - \tfrac{1}{3} \log_5 8.$$

7. The curve with equation

$$y = x^3 - 3x^2 + 2x + 5$$

is shown on the diagram.

$$y = x^3 - 3x^2 + 2x + 5$$

2

(a) Write down the coordinates of P, the point where the curve crosses the y-axis . (1)

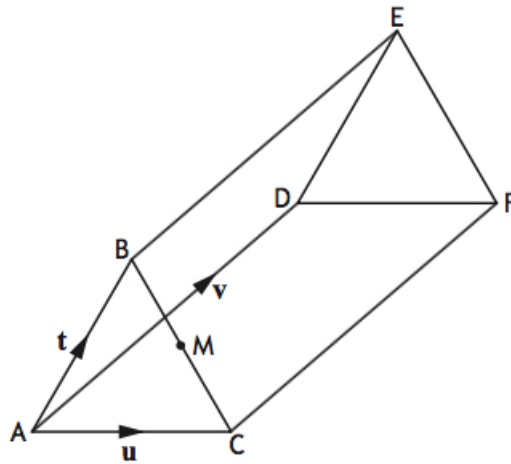(b) Determine the equation of the tangent to the curve at P. (3)

(c) Find the coordinates of Q, the point where this tangent meets the curve again. (4)

8. A line has equation (2)

$$y - \sqrt{3}x + 5 = 0.$$

Determine the angle this line makes with the positive direction of the x-axis.

9. The diagram shows a triangular prism $ABCDEF$.

$\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$, and $\overrightarrow{AD} = \mathbf{v}$.

(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. (1)

M is the midpoint of BC.

(b) Express MD in terms of $\mathbf{t}$, $\mathbf{u}$, and $\mathbf{v}$. (2)
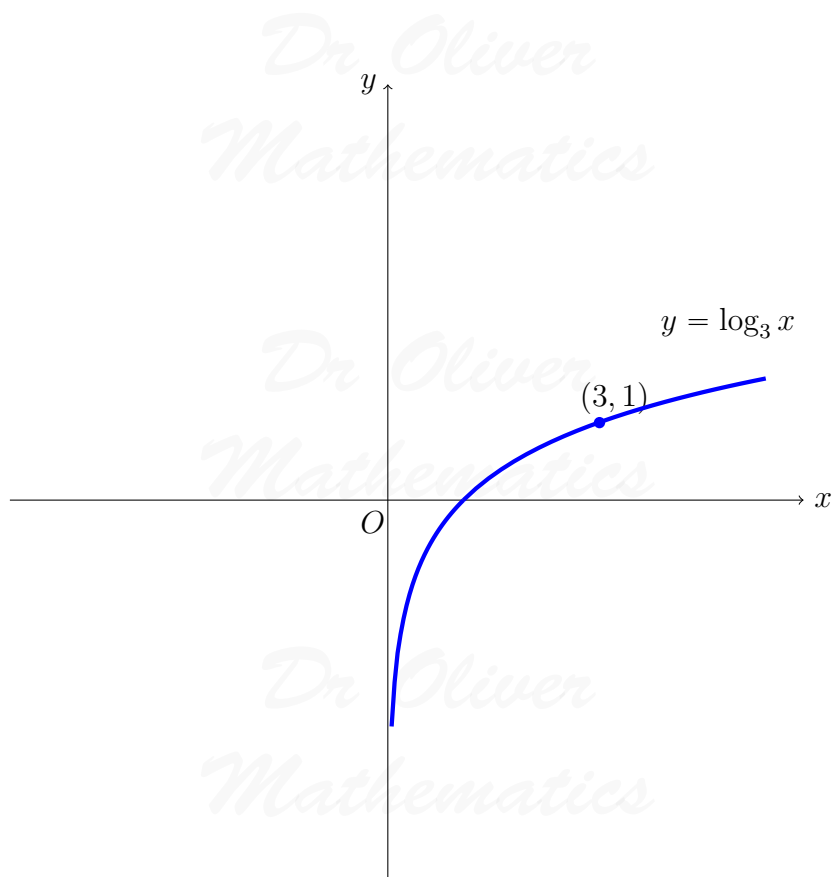
10. Given that (4)

- $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 3x + 4$ and
- $y = 14$ when $x = 2$,

express y in terms of x.

11. The diagram shows the curve with equation

$$y = \log_3 x.$$

(a) Sketch the curve with equation (2)

$$y = 1 - \log_3 x.$$

(b) Determine the exact value of the x-coordinate of the point of intersection of the two curves. (3)

12. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that

$$\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \text{ and } \mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}.$$
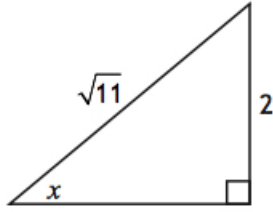
(a) Express $2\mathbf{a} + \mathbf{b}$ in component form. (1)

(b) Hence find the values of p for which (3)

$$|2\mathbf{a} + \mathbf{b}| = 7.$$

13. The right-angled triangle in the diagram is such that

$$\sin x = \frac{2}{\sqrt{11}} \text{ and } 0 < x < \tfrac{1}{4}\pi.$$

(a) Find the exact value of:
    (i) $\sin 2x$,   (3)
    (ii) $\cos 2x$ (1)

(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$.   (3)

14. Evaluate   (5)

$$\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x + 9)^2}} \, \mathrm{d}x.$$

15. A cubic function, f, is defined on the set of real numbers.   (4)

- $(x + 4)$ is a factor of $f(x)$,
- $x = 2$ is a repeated root of $f(x)$,
- $f'(-2) = 0$, and
- $f'(x) > 0$ where the graph with equation $y = f(x)$ crosses the y-axis.

Sketch a possible graph of $y = f(x)$.