

Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2010 Paper
1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. Differentiate the following, simplifying your answers as appropriate.

(a) $f(x) = e^{2x} \tan x, -\frac{1}{2}\pi < x < \frac{1}{2}\pi.$ (3)

(b) $g(x) = \frac{\cos 2x}{x^3}.$ (4)

2. Find the term in a^6 in the binomial expansion of (4)

$$\left(\frac{1}{a} + 3a\right)^{10}.$$

3. (a) Express (3)

$$\frac{3x}{(x+1)^2}$$

in partial fractions.

- (b) Hence obtain (2)

$$\int \frac{3x}{(x+1)^2} dx.$$

4. An industrial process is modelled by the differential equation (7)

$$\frac{dy}{dt} = \frac{9te^{3t}}{y}$$

where $y > 0$ and $t \geq 0$.

Given that $y = 2$ when $t = 0$, find y explicitly in terms of t .

5. (a) Find the value(s) of m for which the matrix (3)

$$\begin{pmatrix} m & 1 & 1 \\ 0 & m & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

is singular.

The matrix

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & -3 \end{pmatrix}.$$

(b) Use elementary row operations to obtain \mathbf{B}^{-1} . (4)

(c) Hence, or otherwise, solve the system of equations (2)

$$x + y - z = 3$$

$$y + z = -2$$

$$x - 3z = 7.$$