Dr Oliver Mathematics
GCSE Mathematics
2005 November Paper 6H: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. Use your calculator to work out the value of

$$\frac{8.95 + \sqrt{7.84}}{2.03 \times 1.49}.$$

   (a) Write down all the figures on your calculator display. (2)

Solution

$$\frac{8.95 + \sqrt{7.84}}{2.03 \times 1.49} = \frac{8.95 + \sqrt{7.84}}{3.0247} = \underline{\underline{3.884\,682\,778 \text{ (FCD)}}}.$$

   (b) Write down your answer to part (a) correct to 3 significant figures. (1)

Solution

$\underline{\underline{3.88 \text{ (3 sf)}}}$.

2. The equation (4)

$$x^3 + 10x = 21$$

has a solution between 1 and 2.
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **all** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 + 10X$; then you press [=].
Start? and you enter 1; then you press [=].

End? and you enter 2; then you press $\boxed{=}$.
Step? and enter 0.05 – 1 decimal place divided by 2; then you press $\boxed{=}$.

x	$\mathrm{f}(x)$	Comment
1.65	20.992	too low
1.7	21.913	too high

Clearly,

$$1.65 < x < 1.7$$

and the answer is

$$\underline{\underline{x = 1.7 \text{ (1 dp)}}}.$$

3. Ann, Bill, and Colin are travelling in a car from Glasgow to Poole.
Ann, Bill, and Colin share the driving so that the distances they drive are in the ratio 3 : 4 : 4.
Ann drives a distance of 210 km.

(a) Calculate the total distance they travelled from Glasgow to Poole. (3)

Solution

$$3 + 4 + 4 = 11$$

and so the total distance they travelled from Glasgow to Poole is

$$\frac{11}{3} \times 210 = \underline{\underline{770 \text{ km}}}.$$

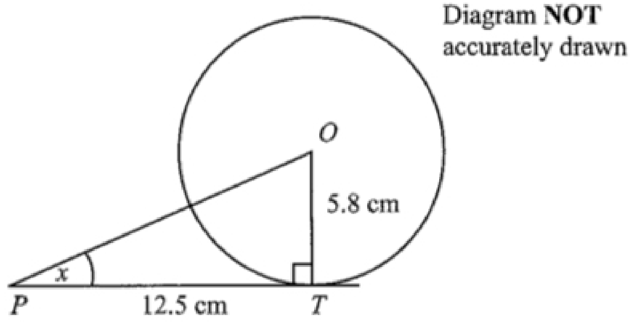
Ann drives the 210 km in 2 hours 40 minutes.

(b) Work out Ann's average speed. (4)

Solution

$$\frac{210}{2\frac{2}{3}} = \underline{\underline{78\tfrac{3}{4} \text{ km/h}}}.$$

4. In the diagram, T is a point on a circle, centre O.



PT is the tangent to the circle at T.

(a) Angle OTP is a right angle. (1)
Give a reason why.

Solution
It is right angle because the radius is perpendicular to tangent.

The radius of the circle is 5.8 cm.
$PT = 12.5$ cm.

(b) Calculate the size of angle x. (3)
Give your answer correct to 1 decimal place.

Solution

$$\tan = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan x = \frac{5.8}{12.5}$$
$$\Rightarrow x = 24.891\,300\,12 \text{ (FCD)}$$
$$\Rightarrow x = 24.9 \text{ (1 dp)}.$$

C is the point on the circle where the straight line OP crosses the circle.

(c) Calculate the length of PC. (4)
Give your answer correct to 3 significant figures.

Solution

$$OP = \sqrt{OT^2 + TP^2}$$
$$= \sqrt{5.8^2 + 12.5^2}$$
$$= 13.780\,058\,06 \text{ (FCD)}$$

and so

$$CP = 13.780\ldots - 5.8 = \underline{\underline{7.98 \text{ cm (3 sf)}}}.$$

5. (a) $4x + 3y < 12$. (2)

x and y are both integers.

Write down two possible pairs of values that satisfy this inequality.

Solution

E.g., $\underline{\underline{x = 1, y = 1}}$ and $\underline{\underline{x = -10, y = -20}}$.

(b) $4x + 3y < 12$. (3)

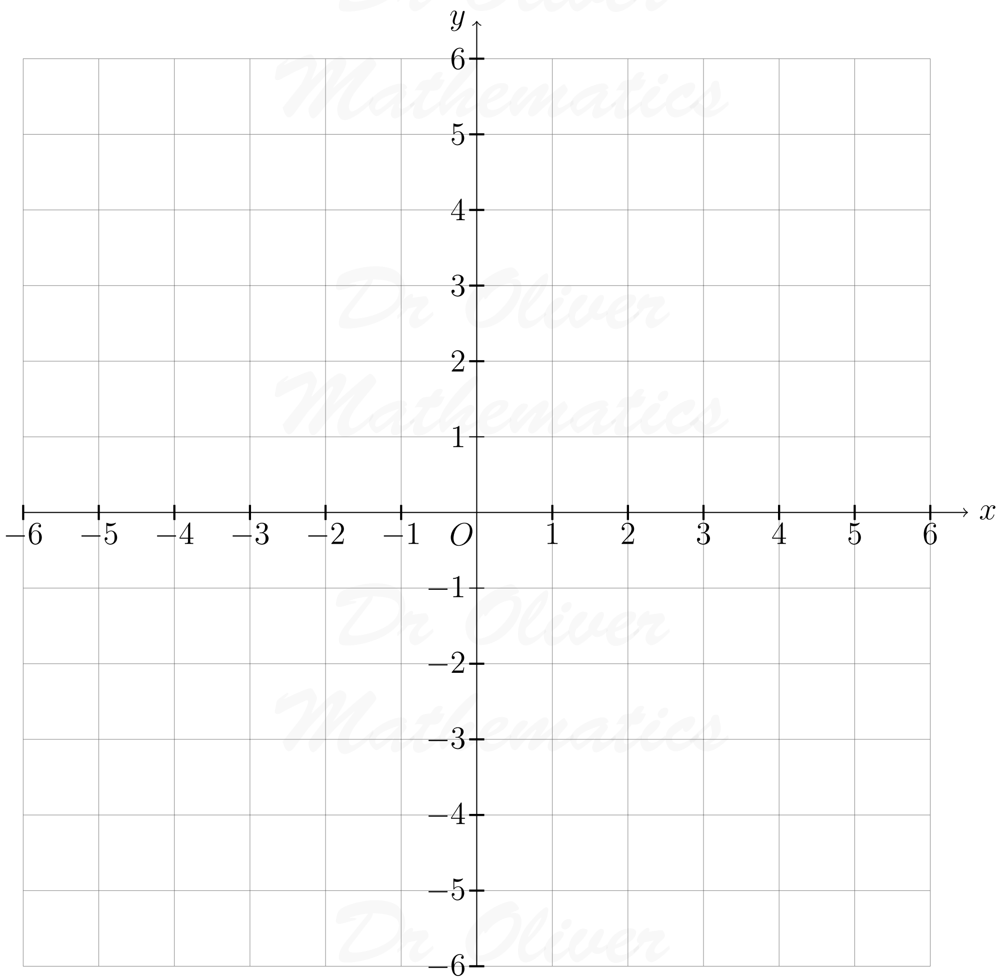
$y < 3x$.

$y > 0$.

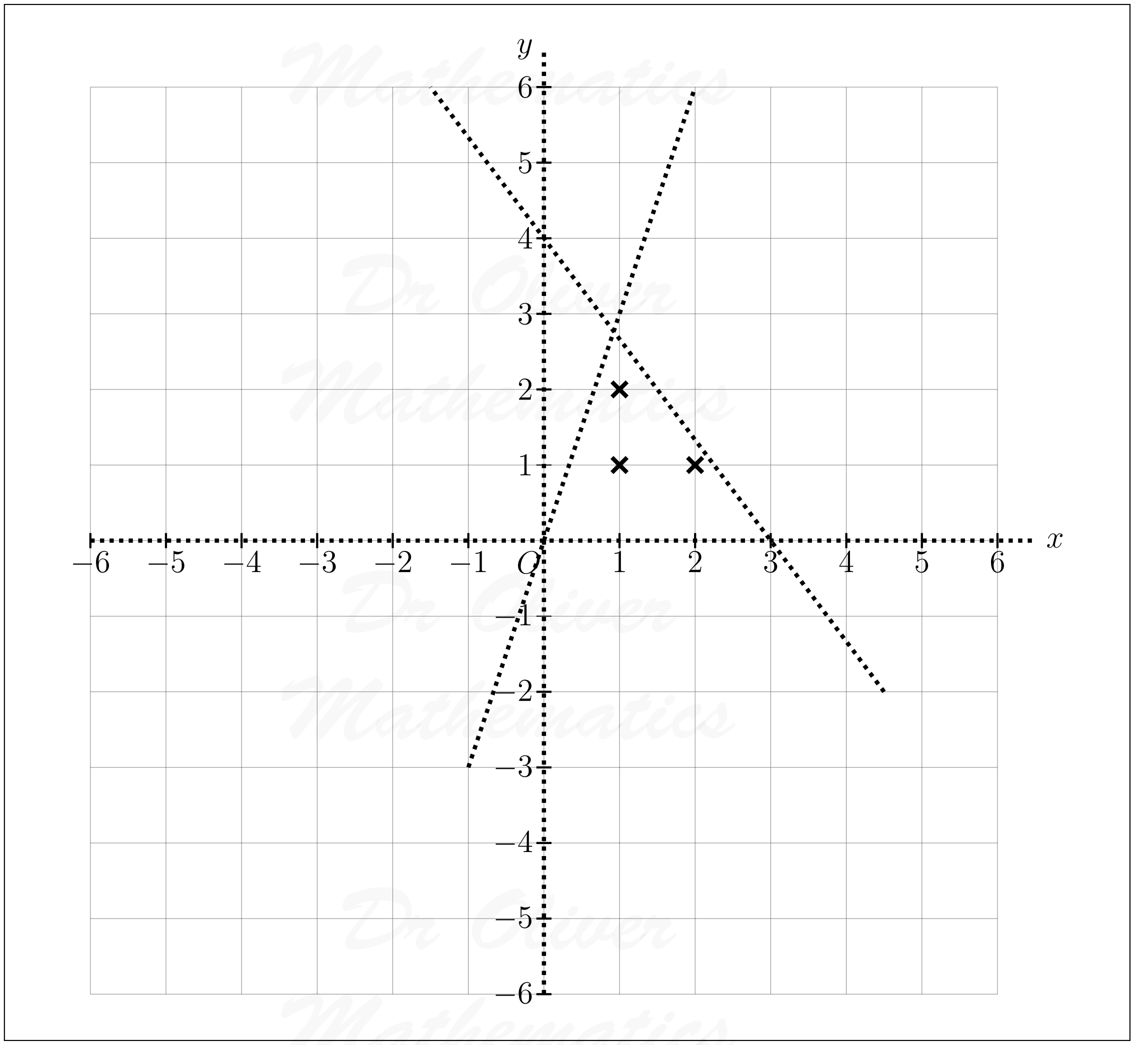
$x > 0$.

x and y are both integers.

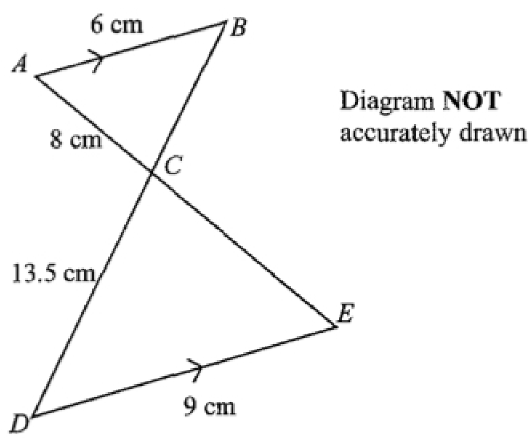
On the grid, mark with a cross ($\times$), each of the three points which satisfy all these four inequalities.



Solution



6. AB is parallel to DE. (3)



ACE and BCD are straight lines.
$AB = 6$ cm.
$AC = 8$ cm.
$CD = 13.5$ cm.
$DE = 9$ cm.

(a) Work out the length of CE.

Solution

$$\frac{CE}{AC} = \frac{DE}{AB} \Rightarrow CE = \frac{8 \times 9}{6}$$
$$\Rightarrow \underline{\underline{CE = 12 \text{ cm}}}.$$

(b) Work out the length of BC.

Solution

$$\frac{BC}{AB} = \frac{CD}{DE} \Rightarrow BC = \frac{13.5 \times 6}{9}$$
$$\Rightarrow \underline{\underline{BC = 9 \text{ cm}}}.$$

7. Solve the simultaneous equations (4)

$$3x + 7y = 26$$
$$4x + 5y = 13.$$

Solution

$$3x + 7y = 26 \quad (1)$$
$$4x + 5y = 13 \quad (2)$$

E.g.,

$$5 \times (1): \quad 15x + 35y = 130 \quad (3)$$
$$7 \times (2): \quad 28x + 35y = 91 \quad (4)$$

(4) − (3):

$$\begin{aligned}13x = -39 &\Rightarrow \underline{\underline{x = -3}} \\ &\Rightarrow -9 + 7y = 26 \\ &\Rightarrow 7y = 35 \\ &\Rightarrow \underline{\underline{y = 5}}.\end{aligned}$$

8. Lisa said that -2 is the only value of x that satisfies the equation (2)

$$x^2 + 4x + 4 = 0.$$

Was Lisa correct?
Show working to justify your answer.

Solution

$$\begin{aligned}x^2 + 4x + 4 = 0 &\Rightarrow (x+2)^2 = 0 \\ &\Rightarrow x + 2 = 0 \\ &\Rightarrow x = -2\end{aligned}$$

so, yes, Lisa said that -2 is the only value of x.

9. Bytes is a shop that sells computers and digital cameras.
In 2003, Bytes sold 620 computers.
In 2004, Bytes sold 708 computers.

(a) Work out the percentage increase in the number of computers sold. (4)
Give your answer to an appropriate degree of accuracy.

Solution

$$\frac{708 - 620}{620} \times 100 = 14.193\,548\,39 \text{ (FCD)};$$

the percentage increase in the number of computers sold is 14.2% (1 dp).

In a sale, normal prices are reduced by 14%.
The sale price of a digital camera is £129.86.

(b) Work out the normal price of the digital camera. (3)

Solution

$$100 - 14 = 0.86$$

and

$$\frac{129.86}{0.86} = \underline{\underline{\pounds 151}}.$$

The table shows the number of digital cameras Bytes sold each month in the first six months of 2005.

Month	Jan	Feb	Mar	Apr	May	Jun
Number sold	30	19	20	15	27	39

The first 3-month moving average for this data is 23.

(c) Work out the **second** 3-month moving average for this data. (2)

Solution

$$\frac{19 + 20 + 15}{3} = \frac{54}{3} = \underline{\underline{18}}.$$

10. Fred did a survey on the areas of pictures in a newspaper.
The table gives information about the areas.

Area (A cm^2)	Frequency
$0 < A \leqslant 10$	38
$10 < A \leqslant 25$	36
$25 < A \leqslant 40$	30
$40 < A \leqslant 60$	46

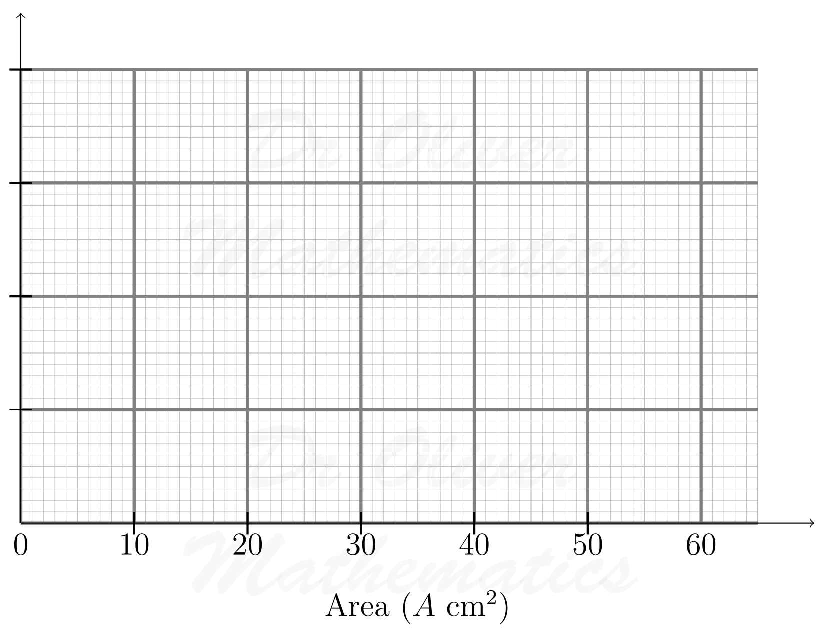
(a) Work out an estimate for the mean area of a picture. (4)

Solution

Area (A cm^2)	Frequency	Midpoint	Freq×Midpoint
$0 < A \leqslant 10$	38	5	$38 \times 5 = 190$
$10 < A \leqslant 25$	36	17.5	$36 \times 17.5 = 630$
$25 < A \leqslant 40$	30	32.5	$30 \times 32.5 = 975$
$40 < A \leqslant 60$	46	50	$46 \times 20 = 2\,300$
Total	150		$2\,715$

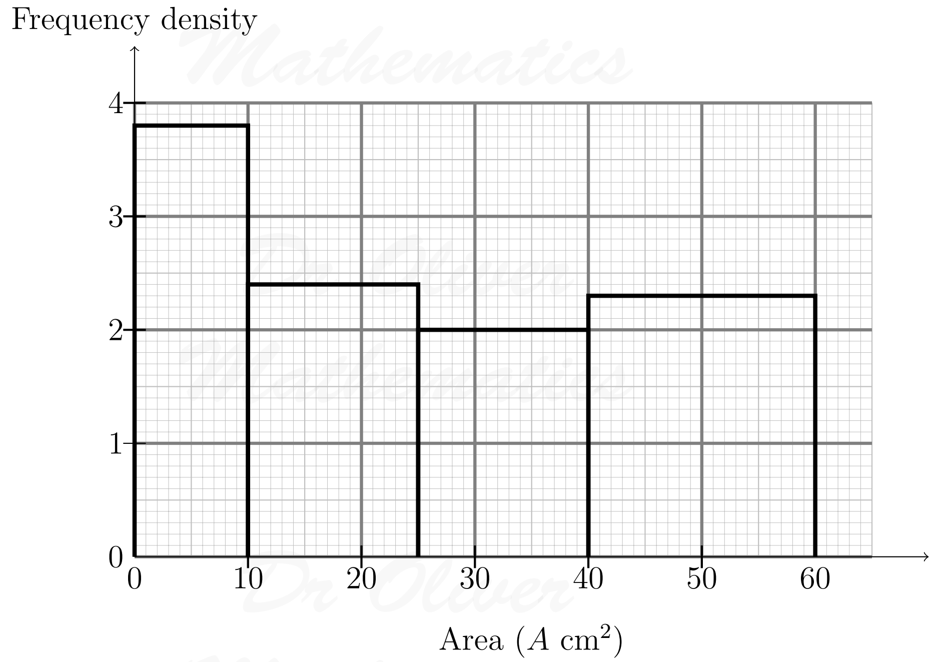
$$\begin{aligned} \text{Mean} &= \frac{\Sigma f x}{\Sigma f} \\ &\approx \frac{4\,095}{150} \\ &= \underline{\underline{27.3 \text{ cm}^2}}. \end{aligned}$$

(b) Draw a histogram for the information given in the table. (3)

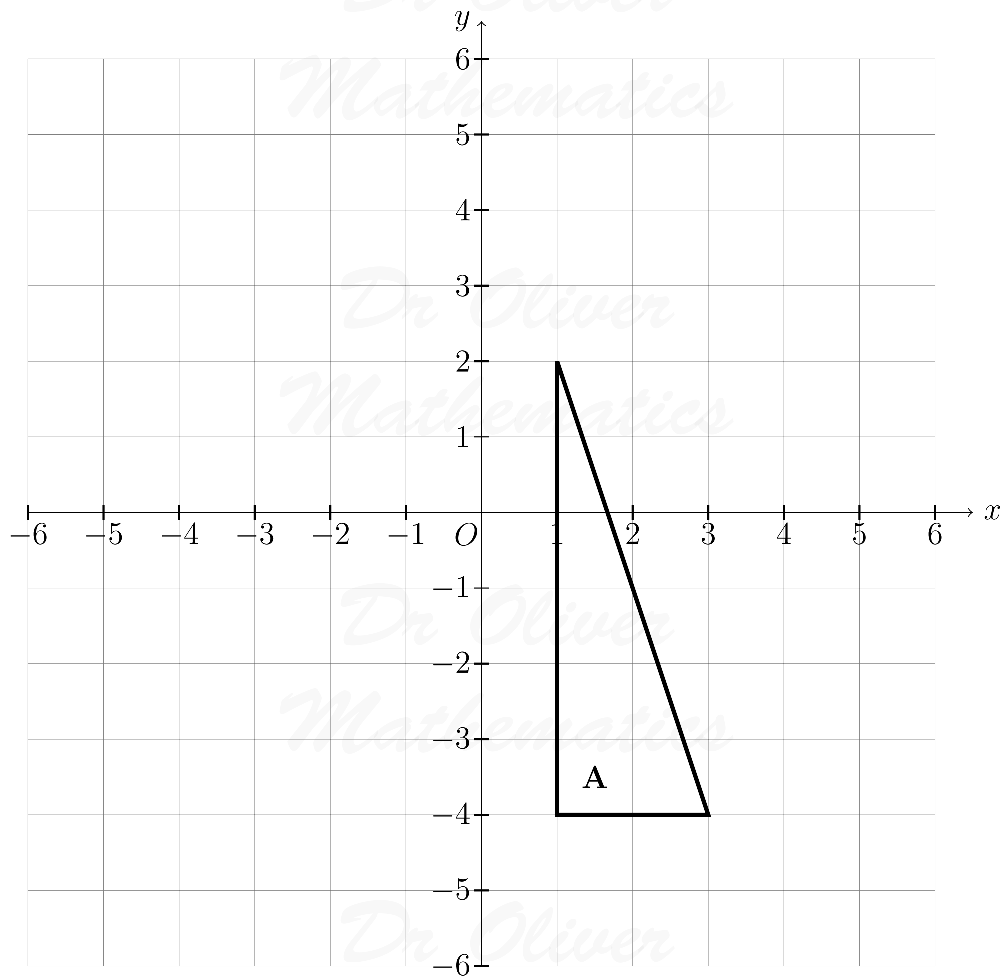


Solution

Area (A cm^2)	Frequency	Width	Frequency Density
$0 < A \leqslant 10$	38	10	$\frac{38}{10} = 3.8$
$10 < A \leqslant 25$	36	15	$\frac{36}{15} = 2.4$
$25 < A \leqslant 40$	30	15	$\frac{30}{15} = 2$
$40 < A \leqslant 60$	46	20	$\frac{46}{20} = 2.3$

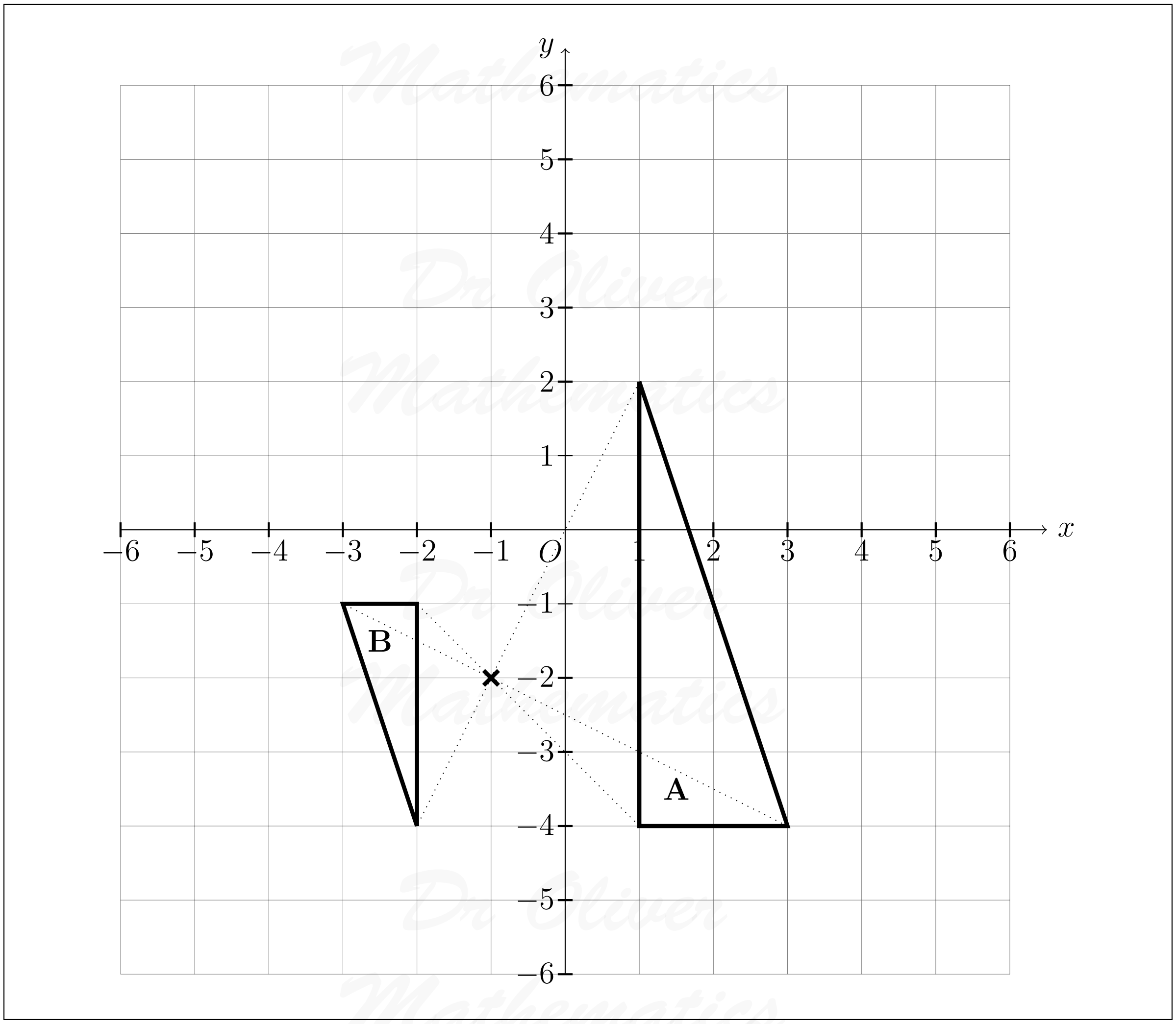


11. Enlarge triangle **A** by scale factor $-\frac{1}{2}$, centre $(-1, -2)$. (3)



Label your triangle **B**.

Solution



12. Make x the subject of (4)

$$5(x-3) = y(4-3x).$$

Solution

$$\begin{aligned} 5(x-3) = y(4-3x) &\Rightarrow 5x - 15 = 4y - 3xy \\ &\Rightarrow 5x + 3xy = 4y + 15 \\ &\Rightarrow x(5+3y) = 4y + 15 \\ &\Rightarrow x = \frac{4y+15}{5+3y}. \end{aligned}$$

13. The distance, D, travelled by a particle is directly proportional to the square of the time, t, taken.
When $t = 40$, $D = 30$.

(a) Find a formula for D in terms of t. (3)

Solution

$$D \propto t^2 \Rightarrow D = kt^2$$

for some constant k. Now,

$$30 = k \times 40^2 \Rightarrow k = \tfrac{3}{160}$$

and so

$$\underline{\underline{D = \tfrac{3}{160}t^2}}.$$

(b) Calculate the value of D when $t = 64$. (1)

Solution

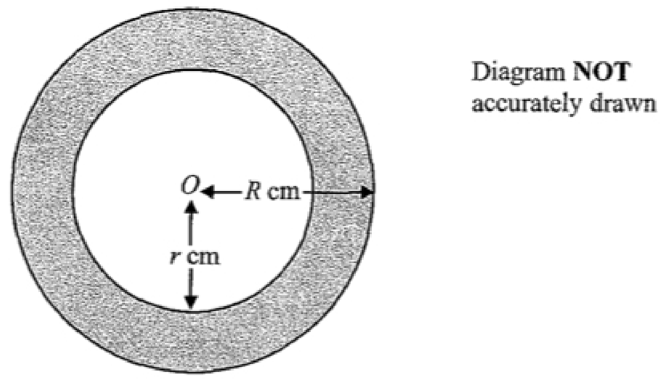
$$D = \tfrac{3}{160} \times 64^2 = \underline{\underline{76.8}}.$$

(c) Calculate the value of t when $D = 12$. (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} 12 = \tfrac{3}{160}t^2 &\Rightarrow t^2 = 640 \\ &\Rightarrow t = 25.298\,221\,28 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{t = 25.3 \text{ (3 sf)}}}. \end{aligned}$$

14. The diagram shows two circles.



O is the centre of both circles.
The radius of the outer circle is R cm.
The radius of the inner circle is r cm.
$R = 15.8$ correct to 1 decimal place.
$r = 14.2$ correct to 1 decimal place.

(a) John says that the minimum possible diameter of the inner circle is 28.35 cm. Explain why John is wrong. (2)

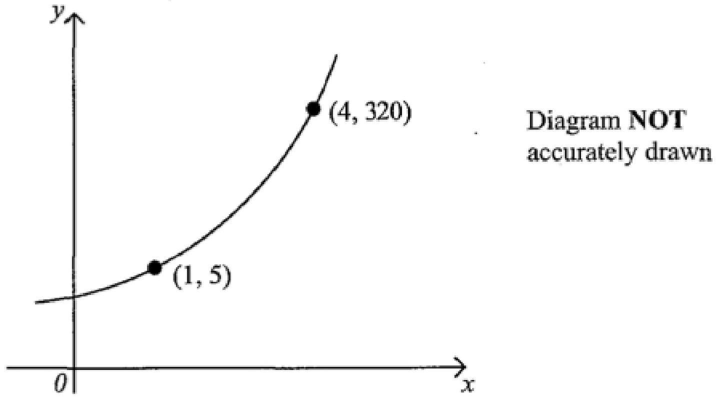
Solution
Well,
$$15.75 \leqslant R < 15.85$$
and
$$14.15 \leqslant r < 14.25.$$
Hence, the minimum of of $2r$ is
$$2 \times 14.15 = 28.3$$
so John is wrong.

The upper bound for the area, in cm^2, of the shaded region is $k\pi$.

(b) Find the **exact** value of k. (4)

Solution
$$\begin{aligned} \text{Upper bound} &= \pi \times 15.85^2 - \pi \times 14.15^2 \\ &= \pi(15.85^2 - 14.15^2) \\ &= 51\pi \text{ cm}^2, \end{aligned}$$
so $k = 51$.

15. The sketch graph shows a curve with equation $y = pq^x$. (3)



The curve passes through the points $(1, 5)$ and $(4, 320)$.
Calculate the value of p and the value of q.

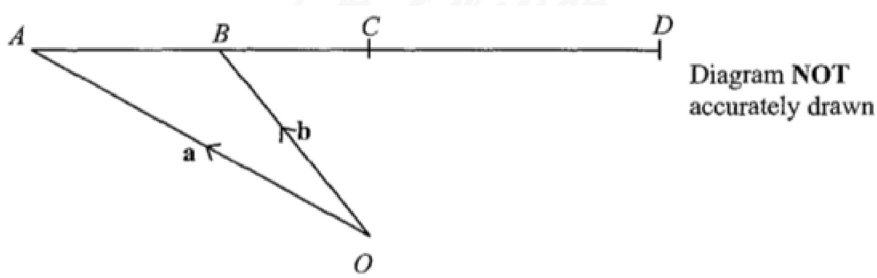
Solution

$$(1,5): \quad 5 = pq \quad (1)$$
$$(4,320): \quad 320 = pq^4 \quad (2)$$

$(2) \div (1)$:

$$q^3 = 64 \Rightarrow \underline{\underline{q = 4}}$$
$$\Rightarrow \underline{\underline{p = 1.25}}.$$

16. $ABCD$ is a straight line. (3)



O is a point so that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.
B is the midpoint of AC.

C is the midpoint of AD.
Express, in terms of **a** and **b**, the vectors

(a) $\overrightarrow{AC}$,

Solution

$$\begin{aligned}\overrightarrow{AC} &= 2\overrightarrow{AB} \\ &= \underline{\underline{2(\mathbf{b}-\mathbf{a})}}.\end{aligned}$$

(b) $\overrightarrow{OD}$.

Solution

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OA} + \overrightarrow{AD} \\ &= \overrightarrow{OA} + 2\overrightarrow{AC} \\ &= \mathbf{a} + 4(\mathbf{b}-\mathbf{a}) \\ &= \underline{\underline{4\mathbf{b} - 3\mathbf{a}}}.\end{aligned}$$

17. Simplify fully (3)

$$\frac{25-x^2}{25+5x}.$$

Solution

$$\begin{aligned}\frac{25-x^2}{25+5x} &= \frac{5^2-x^2}{5(5+x)} \\ &= \frac{(5-x)(5+x)}{5(5+x)} \\ &= \underline{\underline{\frac{5-x}{5}}}.\end{aligned}$$

18. (a) Solve the equation (3)

$$19x^2 - 124x - 224 = 0.$$

Solution

E.g.,

$$\left.\begin{array}{ll}\text{add to:} & -124 \\ \text{multiply to:} & (+19)\times(-224) = -4\,256\end{array}\right\} -152,\ +28$$

$$\begin{aligned}19x^2 - 124x - 224 = 0 &\Rightarrow 19x^2 - 152x + 28x - 224 = 0 \\ &\Rightarrow 19x(x-8) + 28(x-8) = 0 \\ &\Rightarrow (19x+28)(x-8) = 0 \\ &\Rightarrow \underline{\underline{x = -\tfrac{28}{19} \text{ or } x = 8}}.\end{aligned}$$

A bag contains red counters and blue counters and white counters.
There are n red counters.
There are 2 more blue counters than red counters.
The number of white counters is equal to the total number of red counters and blue counters.

(b) Show that the number of counters in the bag is $4(n+1)$. (1)

Solution

There are

$$n + (n+2) + 2(n+2) = 4n + 4 = \underline{\underline{4(n+1)}}$$

counters in the bag.

Bob and Ann play a game.
Bob will take a counter at random from the bag.
He will record the colour and put the counter back in the bag.
Ann will then take a counter at random from the bag.
She will record its colour.
The probability that Bob's counter is red and Ann's counter is **not** red is $\frac{14}{81}$.

(c) Prove that (5)

$$19n^2 - 124n - 224 = 0.$$

Solution

$$\frac{n}{4(n+1)} \times \frac{3n+4}{4(n+1)} = \frac{14}{81}$$
$$\Rightarrow \quad 81n(3n+4) = 14 \times 16(n+1)^2$$
$$\Rightarrow \quad 243n^2 + 324n = 224(n^2 + 2n + 1)$$
$$\Rightarrow \quad 243n^2 + 324n = 224n^2 + 448n + 224$$
$$\Rightarrow \quad \underline{\underline{19n^2 - 124n - 224 = 0}},$$

as required.

(d) Using your answer to part (a), or otherwise, show that the number of counters in the bag is 36. (1)

Solution

Using $x = 8$, the number of counters in the bag is

$$8 + 10 + 18 = \underline{\underline{36}}.$$

Bob and Ann play the game with all 36 counters in the bag.

(e) Calculate the probability that Bob and Ann will take counters with **different** colours. (3)

Solution

$$\begin{aligned} \text{P(same colour)} &= \text{P}(RR) + \text{P}(BB) + \text{P}(WW) \\ &= \left(\tfrac{8}{36}\right)^2 + \left(\tfrac{10}{36}\right)^2 + \left(\tfrac{18}{36}\right)^2 \\ &= \tfrac{61}{162} \end{aligned}$$

and

$$\text{P(different colour)} = 1 - \tfrac{61}{162} = \underline{\underline{\tfrac{101}{162}}}.$$

19. The diagram shows some of the markings on a baseball field.

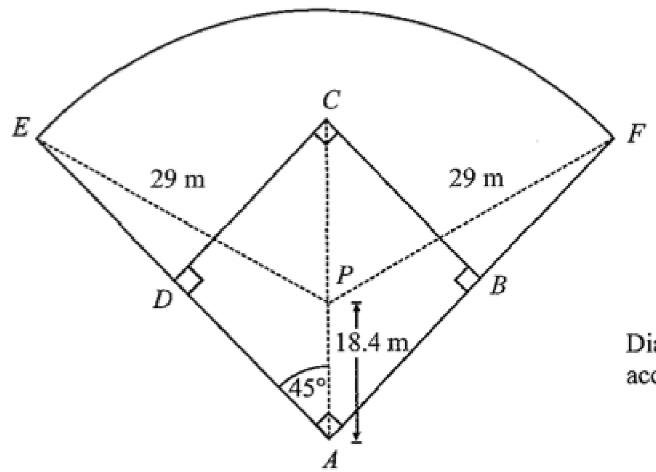


Diagram **NOT** accurately drawn

$ABCD$ is a square.
AC is a diagonal of $ABCD$.
P is a point on AC.
ADE and ABF are straight lines.
$AP = 18.4$ m.
Angle $PAE = 45°$.
EF is an arc of the circle, centre P and radius 29 m.

(a) By considering triangle PAE, calculate the size of angle AEP. (3)
Give your answer correct to 3 significant figures.

Solution

$$\frac{\sin \angle AEP}{18.4} = \frac{\sin 45°}{29} \Rightarrow \sin \angle AEP = \frac{18.4 \sin 45°}{29}$$
$$\Rightarrow \sin \angle AEP = 0.448\,647\,061\,2 \text{ (FCD)}$$
$$\Rightarrow \angle AEP = 26.656\,913\,88 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{\angle AEP = 26.7° \text{ (3 sf)}}}.$$

(b) Calculate the length of the arc EF. (4)
Give your answer correct to 3 significant figures.

Solution

$$\angle CPE = 45 + 26.656\ldots = 71.656\ldots$$

and

$$\angle EPF = 143.313\ldots$$

Finally,

$$\begin{aligned}
\text{arc } EF &= \frac{143.313\ldots}{360} \times 2 \times \pi \times 29 \\
&= 72.537\,646\,59 \text{ (FCD)} \\
&= \underline{\underline{72.5 \text{ m (3 sf)}}}.
\end{aligned}$$