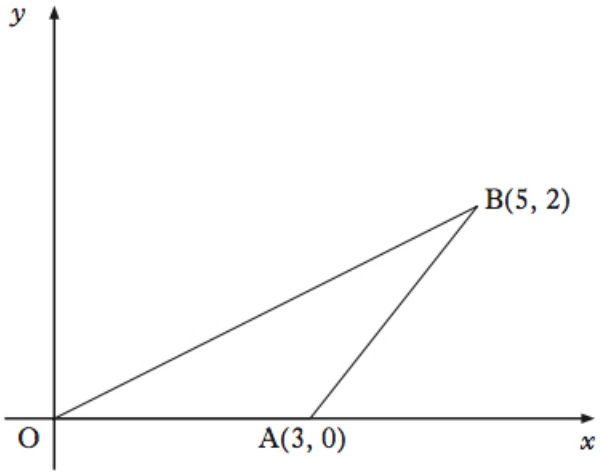
Dr Oliver Mathematics
Mathematics: Higher
2014 Paper 2: Calculator
1 hour 10 minutes

The total number of marks available is 60.
You must write down all the stages in your working.

1. $A(3,0)$, $B(5,2)$, and the origin are the vertices of a triangle as shown in the diagram.



   (a) Obtain the equation of the perpendicular bisector of AB. (4)

   The median from A has equation $y + 2x = 6$.

   (b) Find T, the point of intersection of this median and the perpendicular bisector of AB. (2)

   (c) Calculate the angle that AT makes with the positive direction of the x-axis. (2)

2. A curve has equation $y = x^4 - 2x^3 + 5$. (4)

   Find the equation of the tangent to this curve at the point where $x = 2$.

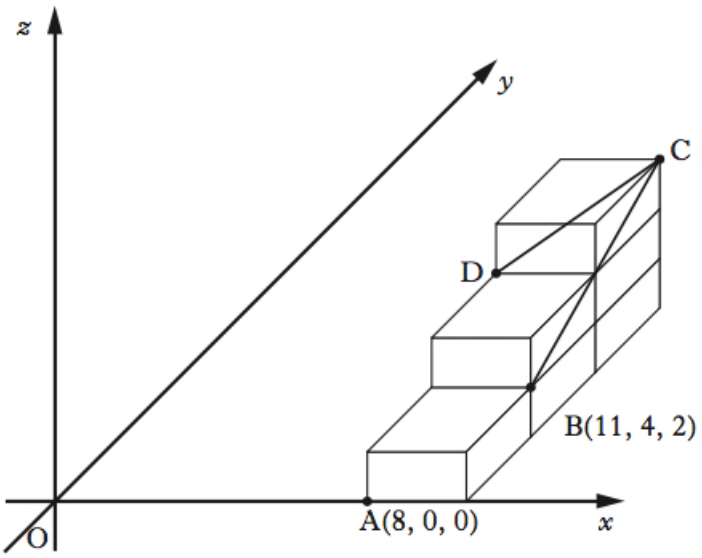
3. Functions f and g are defined on suitable domains by

$$\mathrm{f}(x) = x(x-1) + q \text{ and } \mathrm{g}(x) = x + 3.$$

   (a) Find an expression for $\mathrm{f}(\mathrm{g}(x))$. (2)

   (b) Hence, find the value of q such that the equation $\mathrm{f}(\mathrm{g}(x)) = 0$ has equal roots. (4)

4. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points $(8, 0, 0)$ and $(11, 4, 2)$ respectively.

(a) State the coordinates of C and D. (2)

(b) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$. (2)

(c) Find the size of the angle BCD. (5)

5. Given that (5)

$$\int_4^t (3x + 4)^{-\frac{1}{2}} \, \mathrm{d}x = 2,$$

find the value of t.

6. Solve the equation (5)

$$\sin x - 2\cos 2x = 1,$$

for $0 \leqslant x < 2\pi$.

7. Land enclosed between a path and a railway line is being developed for housing. This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents 300 m^2 of land.

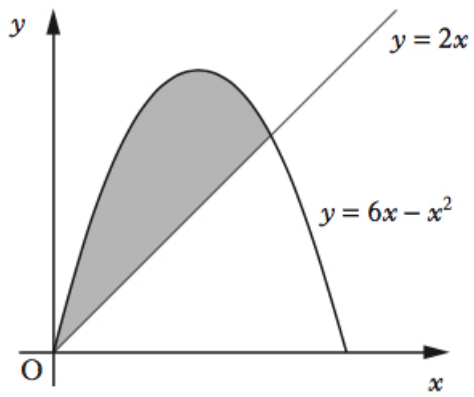


Diagram 1

(a) Calculate the area of land being developed. (5)

A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

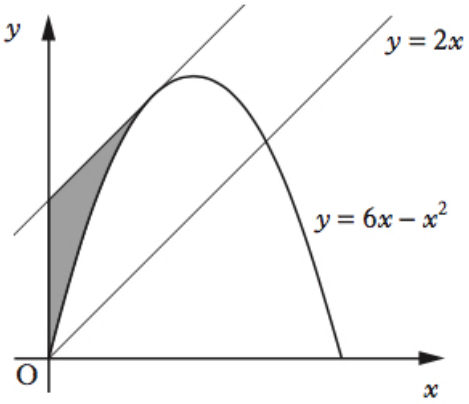


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

(b) Calculate the area of the car park. (5)

8. Given that the equation (5)

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$

represents a circle, determine the range of values of p.

9. Acceleration is defined as the rate of change of velocity.
An object is travelling in a straight line.

The velocity, v m/s, of this object, t seconds after the start of the motion, is given by

$$\mathrm{v}(t) = 8\cos(2t - \tfrac{1}{2}\pi).$$

(a) Find a formula for $\mathrm{a}(t)$, the acceleration of this object, t seconds after the start of the motion. (3)

(b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. (2)

Velocity is defined as the rate of change of displacement.

(c) Determine a formula for $\mathrm{s}(t)$, the displacement of the object, given that $\mathrm{s}(t) = 4$ when $t = 0$. (3)