Dr Oliver Mathematics
Indices: Part 1

1. (Non-calculator) Which is bigger,
$$\sqrt[3]{2} \text{ or } \sqrt[7]{5}?$$

Solution

How do we do this? Well, we raise to the

$$3 \times 7 = 21\text{st power :}$$

that will two integers to compare.

$$\begin{aligned}
(\sqrt[3]{2})^{21} &= [(2)^{\frac{1}{3}}]^{21} \\
&= (2)^7 \\
&= 128 \\
&> 125 \\
&> (5)^3 \\
&= [(5)^{\frac{1}{7}}]^{21} \\
&= (\sqrt[7]{5})^{21}
\end{aligned}$$

and this implies

$$\underline{\underline{\sqrt[3]{2} > \sqrt[7]{5}}}.$$

Note:

$$\begin{aligned}
\sqrt[3]{2} &= 1.259\,921\,05 \text{ (FCD)} \\
\sqrt[7]{5} &= 1.258\,498\,951 \text{ (FCD)}.
\end{aligned}$$