Dr Oliver Mathematics
Further Mathematics
Populations and Samples
Past Examination Questions

This booklet consists of 19 questions across a variety of examination topics.
The total number of marks available is 109.

1. A magazine has a large number of subscribers who each pay a membership fee that is due on January 1st each year. Not all subscribers pay their fee by the due date. Based on correspondence from the subscribers, the editor of the magazine believes that 40% of subscribers wish to change the name of the magazine. Before making this change the editor decides to carry out a sample survey to obtain the opinions of the subscribers. He uses only those members who have paid their fee on time.

   (a) Define the population associated with the magazine. (1)

   (b) Suggest a suitable sampling frame for the survey. (1)

   (c) Identify the sampling units. (1)

   (d) Give one advantage and one disadvantage that would have resulted from the editor using a census rather than a sample survey. (2)

2. Explain what you understand by

   (a) a sampling frame, (1)

   (b) a statistic. (2)

3. (a) Explain what you understand by (i) a population and (ii) a sampling frame. (2)

   The population and the sampling frame may not be the same.

   (b) Explain why this might be the case. (1)

   (c) Give an example, justifying your choices, to illustrate when you might use

      (i) a census, (2)

      (ii) a sample. (2)

4. Explain what you understand by

   (a) a sampling unit, (1)

   (b) a sampling frame, (1)

   (c) a sampling distribution. (2)

5. A bag contains a large number of coins. Half of them are 1p coins, one third are 2p coins, and the remainder are 5p coins.

(a) Find the mean and variance of the value of the coins. (4)

A random sample of 2 coins is chosen from the bag.

(b) List all the possible samples that can be drawn. (3)

(c) Find the sampling distribution of the mean value of these samples. (6)

6. Before introducing a new rule the secretary of a golf club decided to find out how members might react to this rule.

   (a) Explain why the secretary decided to take a random sample of club members rather than ask all the members. (1)

   (b) Suggest a suitable sampling frame. (1)

   (c) Identify the sampling units. (1)

7. (a) Define a statistic. (2)

   A random sample X_1, X_2, ..., X_n is taken from a population with unknown mean μ.

   (b) For each of the following state whether or not it is a statistic.

   (i) $\dfrac{X_1 + X_4}{2}$, (1)

   (ii) $\dfrac{\Sigma X^2}{n} - \mu^2$. (1)

8. A bag contains a large number of coins: 75% are 10p coins and 25% are 5p coins.
   A random sample of 3 coins is drawn from the bag.
   Find the sampling distribution for the median of the values of the 3 selected coins.

9. (a) Explain what you understand by a census. (1)

   Each cooker produced at GT Engineering is stamped with a unique serial number. GT Engineering produces cookers in batches of 2000. Before selling them, they test a random sample of 5 to see what electric current overload they will take before breaking down.

   (b) Give one reason, other than to save time and cost, why a sample is taken rather than a census. (1)

   (c) Suggest a suitable sampling frame from which to obtain this sample. (1)

   (d) Identify the sampling units. (1)

10. A random sample X_1, X_2, ..., X_n is taken from a population with unknown mean μ and unknown variance σ^2. A statistic Y is based on this sample.

   (a) Explain what you understand by the statistic Y. (2)

   (b) Explain what you understand by the sampling distribution of Y. (1)

(c) State, giving a reason which of the following is not a statistic based on this sample. (2)

(i) $\displaystyle\sum_{i=1}^{n} \frac{(X_i - \overline{X})^2}{n}$

(ii) $\displaystyle\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2$

(iii) $\displaystyle\sum_{i=1}^{n} X_i^2$

11. A bag contains a large number of coins. It contains only 1p and 2 p coins in the ratio 1 : 3.

(a) Find the mean μ and the variance σ^2 of the values of this population of coins. (3)

A random sample of size 3 is taken from the bag.

(b) List all the possible samples. (2)

(c) Find the sampling distribution of the mean value of the samples. (6)

12. Explain what you understand by

(a) a population, (1)

(b) a statistic. (1)

A researcher took a sample of 100 voters from a certain town and asked them who they would vote for in an election. The proportion who said they would vote for Dr Smith was 35%.

(c) State the population and the statistic in this case. (2)

(d) Explain what you understand by the sampling distribution of this statistic. (1)

13. A factory produces components. Each component has a unique identity number and it is assumed that 2% of the components are faulty. On a particular day, a quality control manager wishes to take a random sample of 50 components.

(a) Identify a sampling frame. (1)

The statistic F represents the number of faulty components in the random sample of size 50.

(b) Specify the sampling distribution of F. (2)

14. A bag contains a large number of balls: 65% are numbered 1 and 35% are numbered 2. A random sample of 3 balls is taken from the bag. Find the sampling distribution for the range of the numbers on the 3 selected balls. (6)

15. A bag contains a large number of 1p, 2p, and 5p coins: 50% are 1p coins, 20% are 2p coins, and 30% are 5p coins A random sample of 3 coins is chosen from the bag.

(a) List all the possible samples of size 3 with median 5p. (2)

(b) Find the probability that the median value of the sample is 5p. (4)

(c) Find the sampling distribution of the median of samples of size 3. (5)

16. A bag contains a large number of counters. A third of the counters have a number 5 on them and the remainder have a number 1.

A random sample of 3 counters is selected.

(a) List all possible samples. (2)

(b) Find the sampling distribution for the range. (3)

17. A bag contains a large number of counters. Each counter has a single digit number on it and the mean of all the numbers in the bag is the unknown parameter μ. The number 2 is on 40% of the counters and the number 5 is on 25% of the counters. All the remaining counters have numbers greater than 5 on them. A random sample of 10 counters is taken from the bag.

State whether or not each of the following is a statistic.

(a) S = the sum of the numbers on the counters in the sample, (1)

(b) D = the difference between the highest number in the sample and μ, (1)

(c) F = the number of counters in the sample with a number 5 on them. (1)

18. A bag contains a large number of 10p, 20p, and 50p coins in the ratio 1 : 2 : 2. (7)

A random sample of 3 coins is taken from the bag.

Find the sampling distribution of the median of these samples.

19. A bag contains a large number of counters with one of the numbers 4, 6 or 8 written on each of them in the ratio 5 : 3 : 2 respectively.

A random sample of 2 counters is taken from the bag.

(a) List all the possible samples of size 2 that can be taken. (2)

The random variable M represents the mean value of the 2 counters. Given that

$$\mathrm{P}(M = 4) = \tfrac{1}{4} \text{ and } \mathrm{P}(M = 8) = \tfrac{1}{25},$$

(b) find the sampling distribution for M. (5)

A sample of n sets of 2 counters is taken. The random variable Y represents the number of these n sets that have a mean of 8.

(c) Calculate the minimum value of n such that $\mathrm{P}(Y \geqslant 1) > 0.9$. (3)