

Dr Oliver Mathematics
GCSE Mathematics
2010 June Paper 4H: Calculator
1 hour 45 minutes

The total number of marks available is 100.

You must write down all the stages in your working.

1. Here is a list of ingredients for making a trifle for 4 people.

(3)

Trifle for 4 people

120 g of raspberry jelly

8 sponge fingers

420 ml of custard

180 g of tinned fruit

Rob is going to make a trifle for 6 people.

Work out the amount of each ingredient he needs.

Solution

Raspberry jelly:

$$120 \times \frac{6}{4} = \underline{\underline{180 \text{ g}}}$$

Sponge fingers:

$$8 \times \frac{6}{4} = \underline{\underline{12}}$$

Custard:

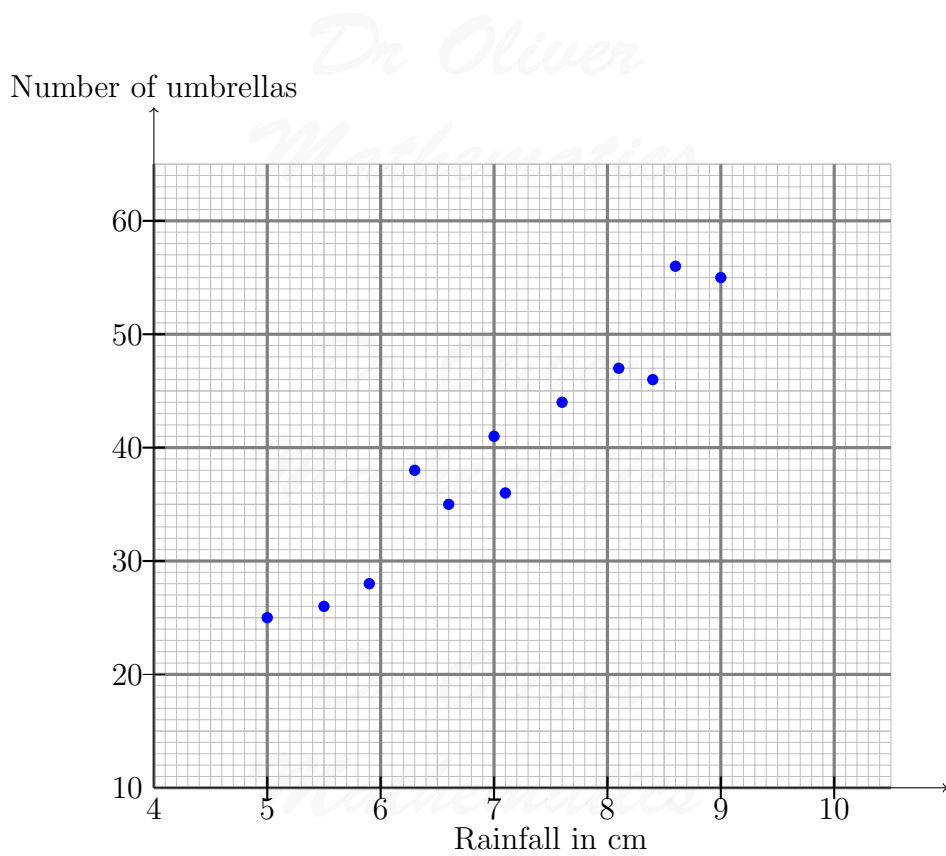
$$420 \times \frac{6}{4} = \underline{\underline{630 \text{ g}}}$$

Tinned fruit:

$$180 \times \frac{6}{4} = \underline{\underline{270 \text{ g}}}$$

2. Mr Wither sells umbrellas.

The scatter graph shows some information about the number of umbrellas he sold and the rainfall, in cm, each month last year.



In January of this year, the rainfall was 6.1 cm.
 During January, Mr Wither sold 32 umbrellas.

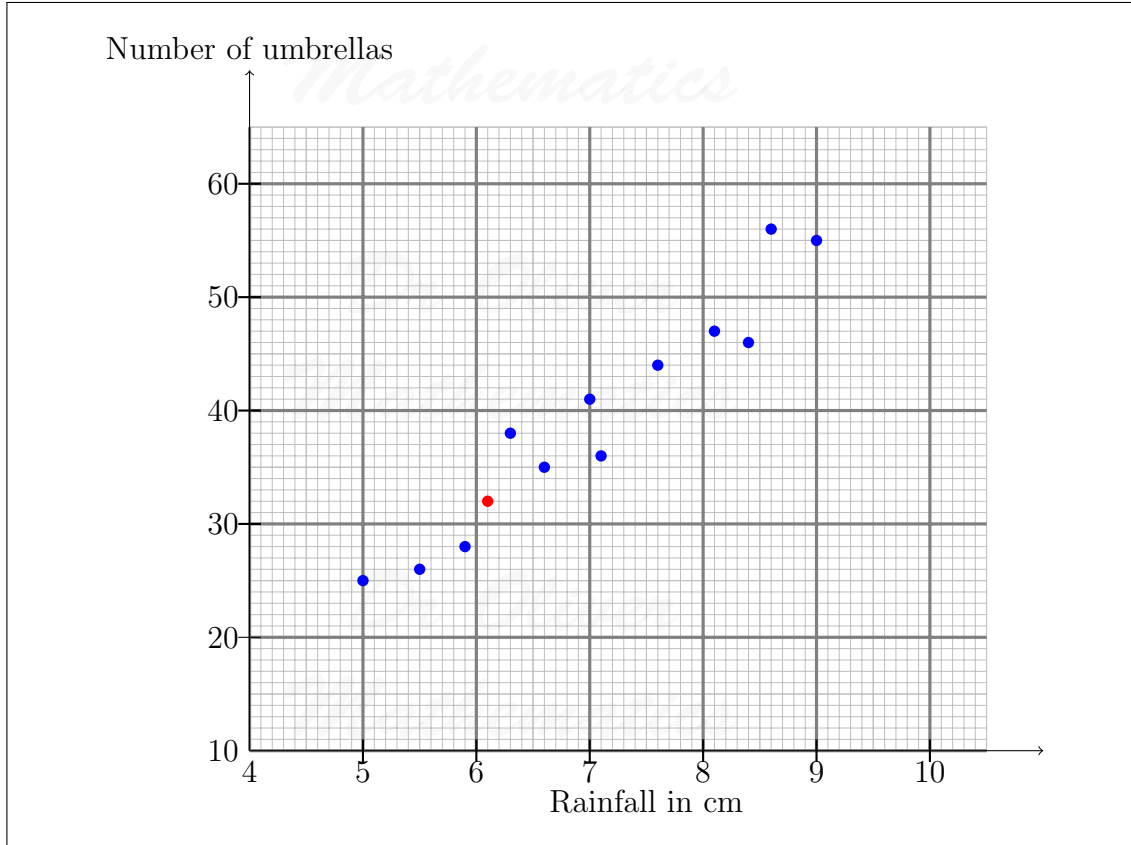
(a) Show this information on the scatter graph.

(1)

Solution

Dr Oliver Mathematics

Dr Oliver Mathematics



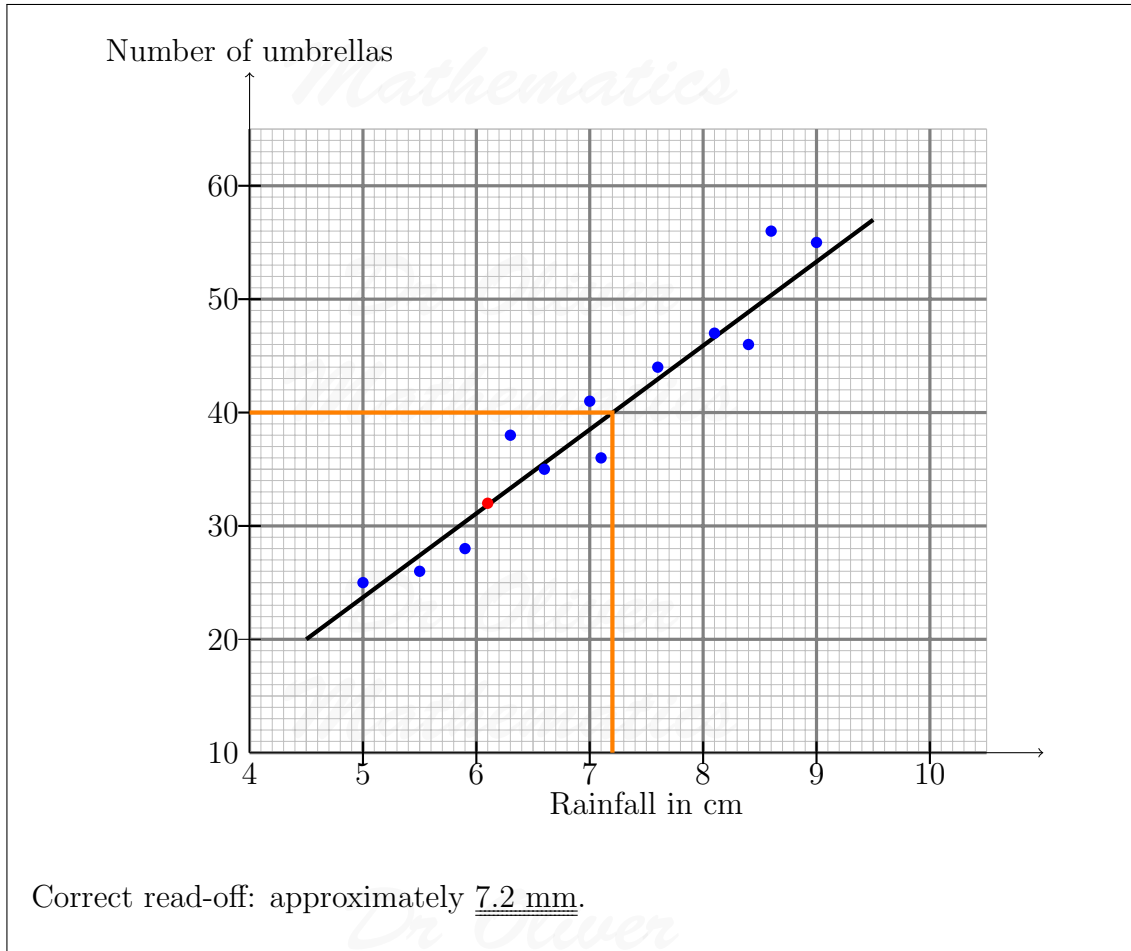
(b) What type of correlation does this scatter graph show? (1)

Solution
Positive correlation, i.e., the greater the rainfall, the more umbrella he sells.

In February of this year, Mr Wither sold 40 umbrellas.

(c) Estimate the rainfall for February. (2)

Solution



3. In August 2008, Eddie hired a car in Italy.

The cost of hiring the car was £620.

The exchange rate was £1 = €1.25.

(a) Work out the cost of hiring the car in euros (€).

(2)

Solution

$$620 \times 1.25 = \underline{\underline{€775.}}$$

Eddie bought some perfume in Italy.

The cost of the perfume in Italy was €50.

The cost of the same perfume in London was £42.

The exchange rate was still £1 = €1.25.

(b) Work out the difference between the cost of the perfume in Italy and the cost of the perfume in London.

(3)

Give your answer in pounds (£).

Solution

The cost in Italy is

$$\frac{50}{1.25} = 40$$

and the difference is

$$42 - 40 = \underline{\underline{\pounds 40}}.$$

4. (a) Complete the table of values for $y = 3x + 4$.

(2)

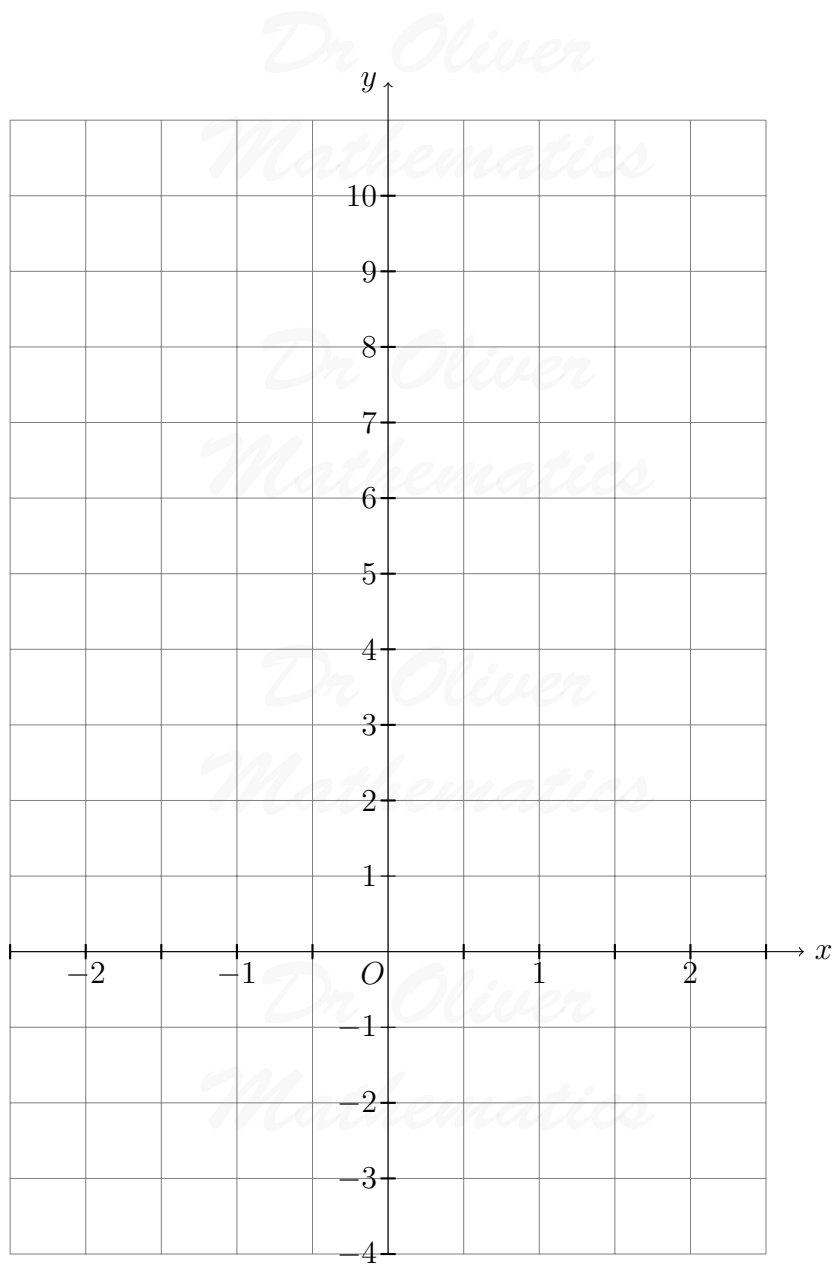
x	-2	-1	0	1	2
y		1			10

Solution

x	-2	-1	0	1	2
y	<u>-2</u>	1	<u>4</u>	<u>7</u>	10

- (b) On the grid, draw the graph of $y = 3x + 4$.

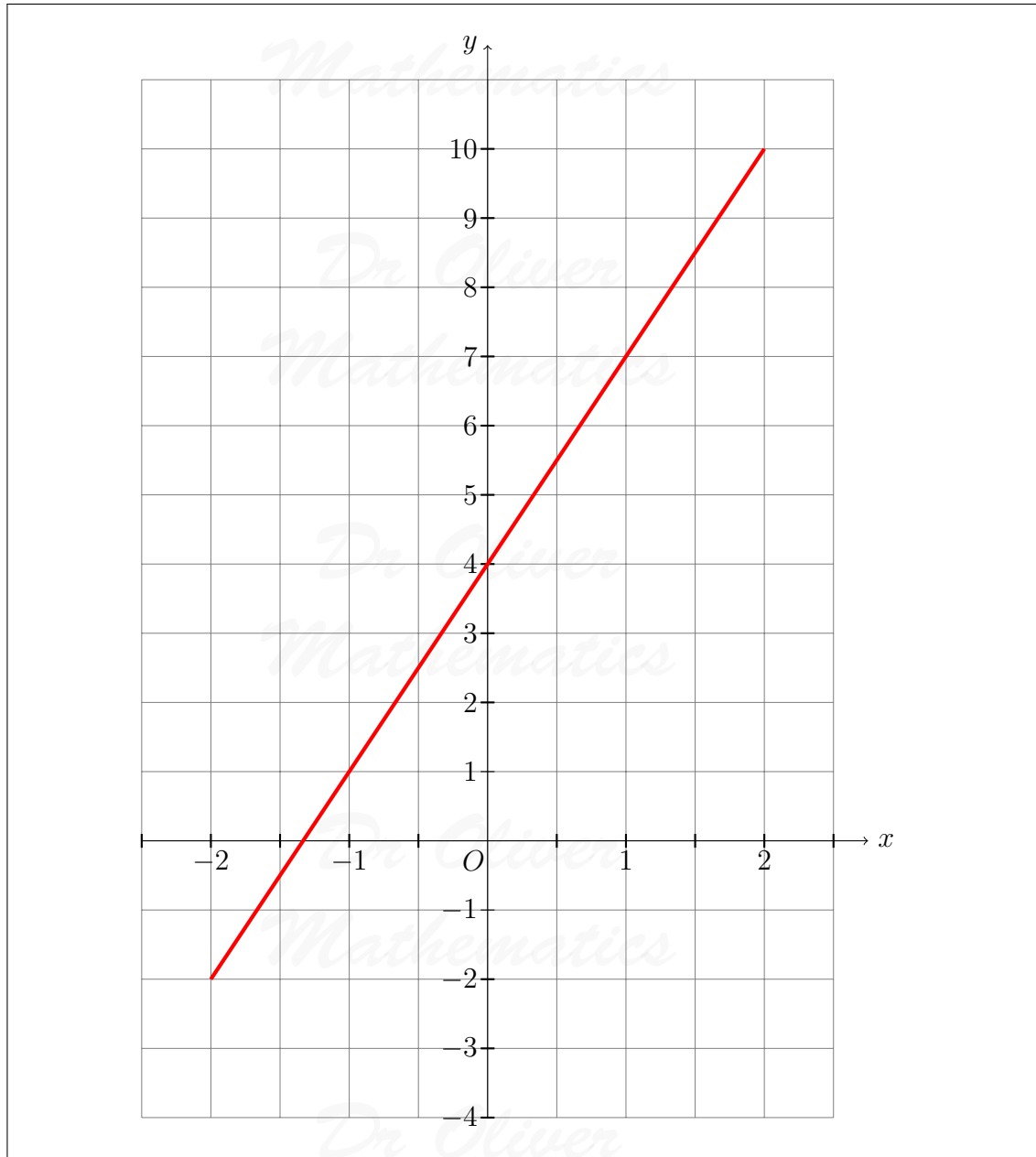
(2)



Solution

Dr. Oliver
Mathematics

Dr. Oliver
Mathematics



5. ANB is parallel to CMD .

(3)

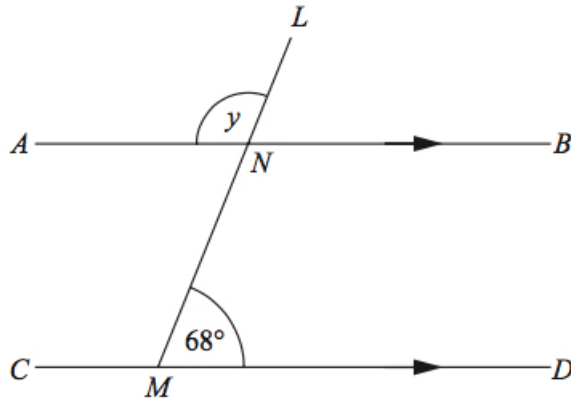


Diagram **NOT**
accurately drawn

$LN M$ is a straight line.
Angle $LMD = 68^\circ$.

- (a) Work out the size of the angle marked y .

Solution

112° .

- (b) Give reasons for your answer.

Solution

$\angle CMN = 68^\circ$ (corresponding angles).
 $\angle DMN = 180 - 68 = 112^\circ$ (angles on a straight line).

6. (a) Use your calculator to work out

(2)

$$\frac{2}{1.5 + 2.45}$$

Write down all the figures on your calculator display.
You must give your answer as a decimal.

Solution

$$\begin{aligned} \frac{2}{1.5 + 2.45} &= \frac{2}{3.95} \\ &= \underline{\underline{0.506\ 329\ 113\ 9}} \text{ (FCD)}. \end{aligned}$$

- (b) Write your answer to part (a) correct to 2 decimal places.

(1)

Solution

0.51 (2 dp).

7. A circle has a diameter of 12 cm.

(2)

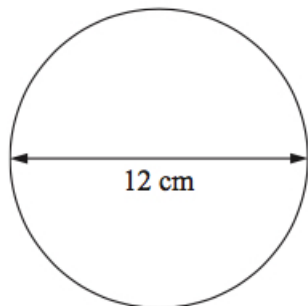


Diagram **NOT**
accurately drawn

Work out the circumference of the circle.
Give your answer correct to 3 significant figures.

Solution

The radius is 6 cm and the circumference of the circle is

$$\begin{aligned} 2 \times \pi \times 6 &= 37.699\ 111\ 84 \text{ (FCD)} \\ &= \underline{\underline{37.7 \text{ cm (3 sf)}}}. \end{aligned}$$

8. The equation

$$x^3 + 10x = 25$$

(4)

has a solution between 1 and 2.
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show **all** your working.

Solution

You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.

F(X)= and you type in $X^3 + 10X$; then you press $\boxed{=}$.

Start? and you enter 1; then you press $\boxed{=}$.

End? and you enter 2; then you press [=].

Step? and enter 0.05 – 1 decimal place divided by 2; then you press [=].

x	$f(x)$	Comment
1.85	24.831	too low
1.9	25.859	too high

Clearly,

$$1.85 < x < 1.9$$

and the answer is

$$\underline{\underline{x = 1.9 \text{ (1 dp)}}}.$$

9. Work out £84 as a percentage of £350.

(2)

Solution

$$\frac{84}{350} \times 100\% = \underline{\underline{24\%}}.$$

10. There are some ribbons in a box.

The ribbons are green or red or yellow or white.

The table shows each of the probabilities that a ribbon chosen at random will be green or red or white.

Colour	Green	Red	Yellow	White
Probability	0.15	0.30		0.35

- (a) Work out the probability that a ribbon chosen at random will be yellow.

(2)

Solution

$$1 - (0.15 + 0.30 + 0.35) = \underline{\underline{0.20}}.$$

There are 500 ribbons in the box.

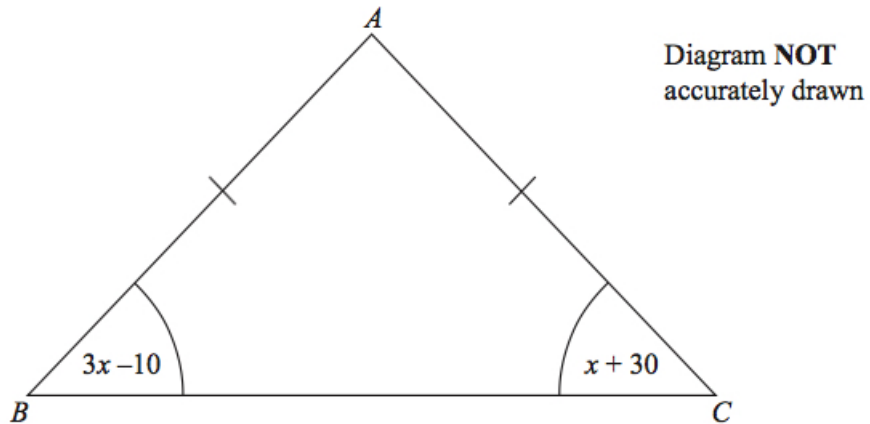
- (b) Work out the number of red ribbons.

(2)

Solution

$$500 \times 0.30 = \underline{150}.$$

11. ABC is an isosceles triangle.



$$AB = AC.$$

(a) Explain why $3x - 10 = x + 30$.

(1)

Solution

They must be as the base angles in an isosceles triangle are equal.

(b) Solve $3x - 10 = x + 30$.

(2)

Solution

$$\begin{aligned} 3x - 10 &= x + 30 \Rightarrow 2x = 40 \\ &\Rightarrow \underline{\underline{x = 20}}. \end{aligned}$$

12. ABC is a right-angled triangle.

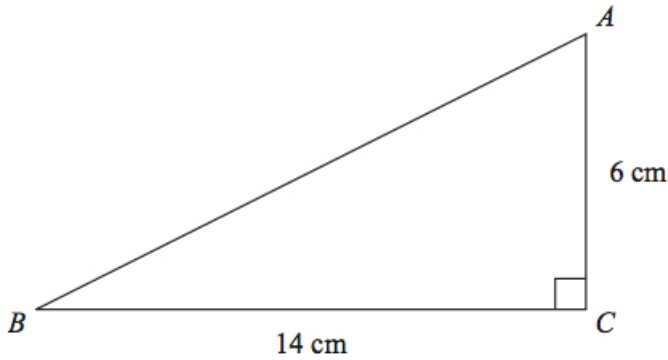


Diagram **NOT**
accurately drawn

$$AC = 6 \text{ cm.}$$

$$BC = 14 \text{ cm.}$$

- (a) Work out the area of triangle ABC . (2)

Solution

$$\begin{aligned} \text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 6 \times 14 \\ &= \underline{\underline{42 \text{ cm}^2}}. \end{aligned}$$

- (b) Calculate the length of AB .
Give your answer correct to 2 decimal places. (3)

Solution

$$\begin{aligned} AB &= \sqrt{AC^2 + BC^2} \\ &= \sqrt{6^2 + 14^2} \\ &= \sqrt{232} \\ &= 15.231\ 546\ 21 \text{ (FCD)} \\ &= \underline{\underline{15.23 \text{ cm (2 dp)}}}. \end{aligned}$$

13. The diagram shows a solid prism.

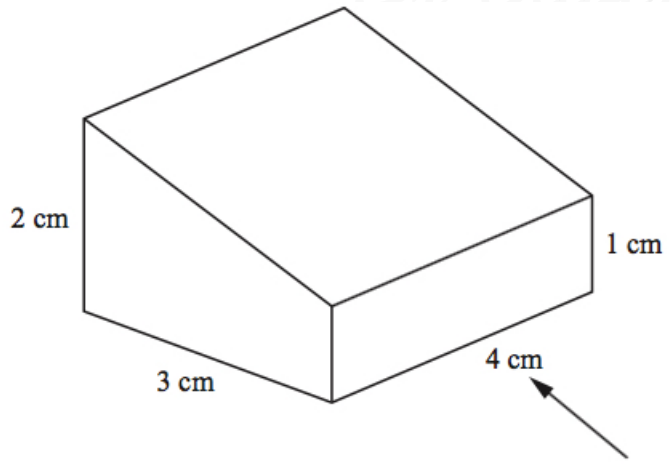
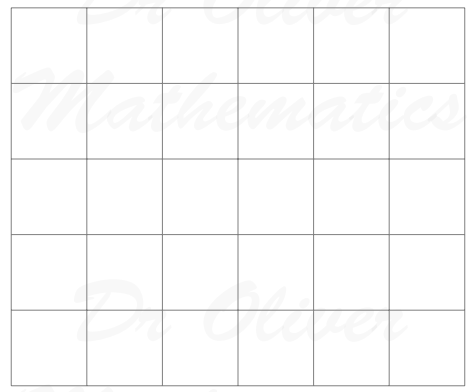


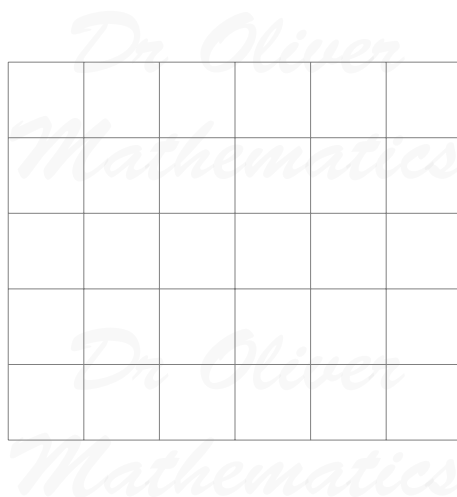
Diagram NOT accurately drawn

- (a) On the grid below, draw the front elevation of the prism from the direction of the arrow. (2)



Solution

- (b) On the grid below, draw the plan of the prism. (2)



Solution

14. The table gives information about the number of CDs sold in a shop during each of the last 30 weeks. (4)

Number of CDs (n)	Frequency
$0 < n \leq 40$	3
$40 < n \leq 80$	5
$80 < n \leq 120$	12
$120 < n \leq 160$	7
$160 < n \leq 200$	3

Calculate an estimate for the mean number of CDs sold each week.
Give your answer correct to 1 decimal place.

Solution

Number of CDs (n)	Frequency	Midpoint	Frequency \times Midpoint
$0 < n \leq 40$	3	20	60
$40 < n \leq 80$	5	60	300
$80 < n \leq 120$	12	100	1 200
$120 < n \leq 160$	7	140	980
$160 < n \leq 200$	3	180	540
Total	30		3 080

$$\begin{aligned} \text{Mean number of CDs sold each week} &\approx \frac{3\,080}{30} \\ &= 102\frac{2}{3} \\ &= \underline{\underline{102.7}} \text{ (1 dp)}. \end{aligned}$$

15. $-4 < n \leq 1$.

n is an integer.

(a) Write down all the possible values of n .

(2)

Solution

$-3, -2, -1, 0, 1$

(b) Solve $3x - 2 > x + 7$.

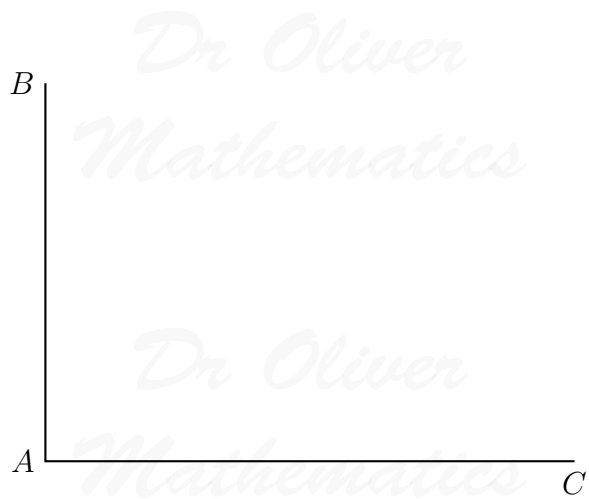
(2)

Solution

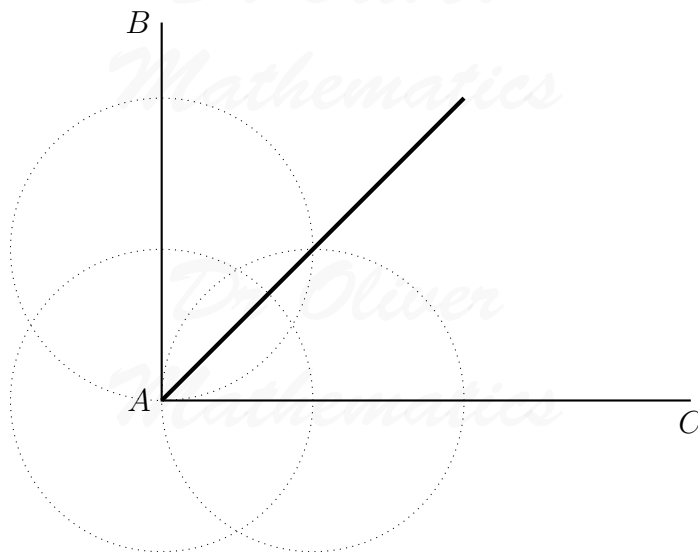
$$\begin{aligned} 3x - 2 > x + 7 &\Rightarrow 2x > 9 \\ &\Rightarrow \underline{\underline{x > 4\frac{1}{2}}}. \end{aligned}$$

16. Draw the locus of all points which are equidistant from the lines AB and AC .

(2)



Solution



17. Make A the subject of the formula

(2)

$$r = \sqrt{\frac{A}{3}}$$

Solution

$$r = \sqrt{\frac{A}{3}} \Rightarrow r^2 = \frac{A}{3}$$

$$\Rightarrow \underline{\underline{A = 3r^2.}}$$

18. (a) Write 15 500 in standard form. (1)

Solution

$$15\,500 = \underline{\underline{1.55 \times 10^4}}.$$

- (b) Write 2.48×10^{-3} as an ordinary number. (1)

Solution

$$2.48 \times 10^{-3} = \underline{\underline{0.00248}}.$$

- (c) Work out the value of (2)

$$24\,500 \div (1.25 \times 10^{-4}).$$

Give your answer in standard form.

Solution

$$24\,500 \div (1.25 \times 10^{-4}) = 196\,000\,000$$

$$= \underline{\underline{1.96 \times 10^8}}.$$

19. (a) Factorise $x^2 - 7x + 10$. (2)

Solution

$$x^2 - 7x + 10 = \underline{\underline{(x - 2)(x - 5)}}.$$

- (b) Solve $x^2 - 7x + 10$. (1)

Solution

$$x^2 - 7x + 10 = 0 \Rightarrow (x - 2)(x - 5) = 0$$

$$\Rightarrow \underline{x = 2} \text{ or } \underline{x = 5}.$$

20. ABC is a right-angled triangle.

(3)

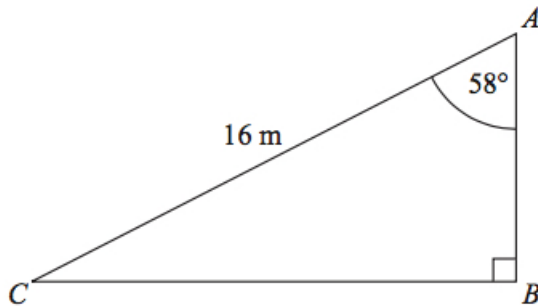


Diagram NOT
accurately drawn

$AC = 16$ m.

Angle $CAB = 58^\circ$.

Calculate the length of AB .

Give your answer correct to 3 significant figures.

Solution

$$\text{adj} = \text{hyp} \times \cos \Rightarrow AB = 16 \cos 58^\circ$$

$$\Rightarrow AB = 8.478\,708\,228 \text{ (FCD)}$$

$$\Rightarrow \underline{\underline{AB = 8.48 \text{ m (3 sf)}}}.$$

21. A field is in the shape of a rectangle.

The width of the field is 28 metres, measured to the nearest metre.

(a) Work out the upper bound of the width of the field.

(1)

Solution

Now,

$$27.5 \leq \text{width of the field} < 28.5$$

and so the upper bound 28.5 metres.

The length of the field is 145 metres, measured to the nearest 5 metres.

- (b) Work out the upper bound for the perimeter of the field. (3)

Solution

Now,

$$142.5 \leq \text{width of the field} < 147.5$$

and so the upper bound is

$$2(147.5 + 28.5) = \underline{\underline{352 \text{ metres}}}.$$

22. (a) Simplify $p^5 \times p^4$. (1)

Solution

$$p^5 \times p^4 = \underline{\underline{p^9}}.$$

- (b) Simplify $q^5 \div q^2$. (1)

Solution

$$q^5 \div q^2 = \underline{\underline{q^3}}.$$

- (c) Simplify $12tu^6 \div 6tu^5$. (2)

Solution

$$12tu^6 \div 6tu^5 = \underline{\underline{2u}}.$$

- (d) Simplify $(9w^2y^6)^{\frac{1}{2}}$. (2)

Solution

$$(9w^2y^6)^{\frac{1}{2}} = \underline{\underline{3wy^3}}.$$

- (e) For $x > 1$, write the following expressions in order of size. (2)
Start with the expression with the least value.

$$x^0 \quad x^2 \quad x \quad x^{-2} \quad x^{\frac{1}{2}}$$

Solution

$$\underline{x^{-2}} \quad \underline{x^0} \quad \underline{x^{\frac{1}{2}}} \quad \underline{x} \quad \underline{x^2}$$

23. **A** and **B** are two solid shapes which are mathematically similar. The shapes are made from the same material.

(4)

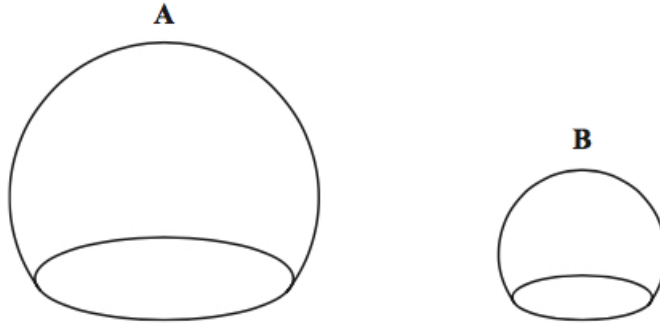


Diagram **NOT**
accurately drawn

The surface area of **A** is 50 cm^2 .

The surface area of **B** is 18 cm^2 .

The mass of **A** is 500 grams.

Calculate the mass of **B**.

Solution

$$\text{ASF}_{\mathbf{B} \rightarrow \mathbf{A}} = 18 : 50 = 9 : 25 = 3^2 : 5^2$$

and so the

$$\text{LSF}_{\mathbf{B} \rightarrow \mathbf{A}} = 3 : 5$$

and so the

$$\text{VSF}_{\mathbf{B} \rightarrow \mathbf{A}} = 3^3 : 5^3 = 27 : 125.$$

Finally, the mass of **B** is

$$500 \times \frac{27}{125} = \underline{\underline{108 \text{ grams}}}.$$

24. (a) Explain what is meant by a random sample.

(1)

Solution

In a random sample, every member of the population has an equal chance of being in the sample.

Chris collects stamps from different countries.
 He has 245 stamps from France.
 He wants to take a random sample of 10 of his stamps from France.

(b) Describe a method that Chris could use. (1)

Solution

E.g., he could number his stamps from 1 to 245. He would then use his calculator and use the random number option: simply exclude the '0.' and then repeat if he gets a number 246 or higher.

The table shows information about Chris' collection of 662 stamps.

Country	France	Germany	Spain	Italy	Total
Number of stamps	245	184	138	95	662

Chris takes a sample of 50 stamps stratified by country.

(c) Work out the number of stamps from Italy in this sample. (2)

Solution

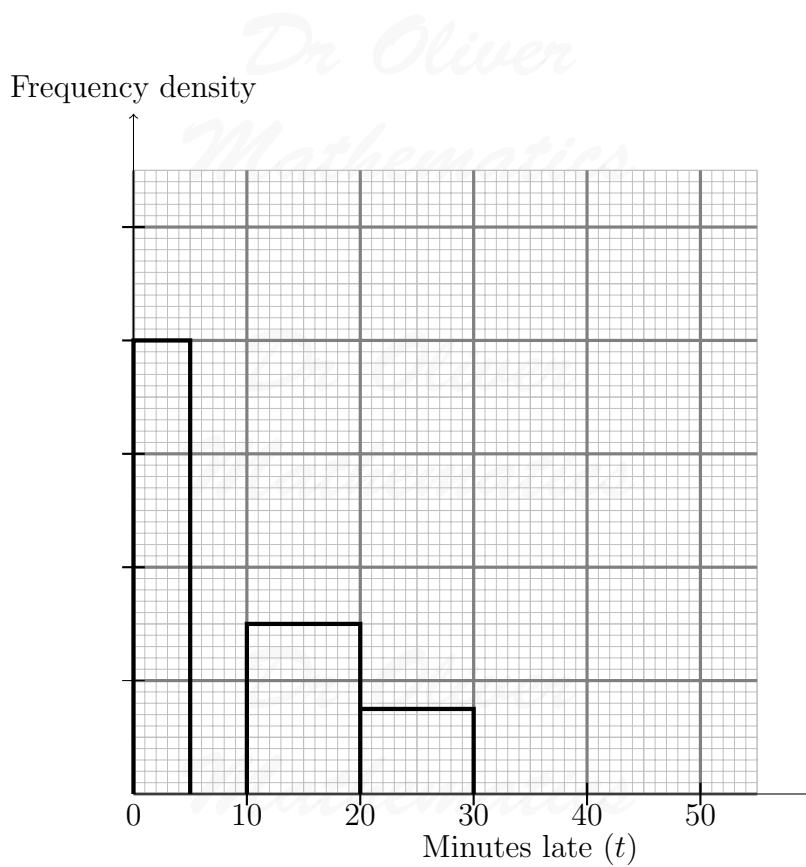
$$50 \times \frac{95}{662} = 7.17\dots;$$

hence, 7 stamps.

25. Some trains from Manchester to London were late.

The incomplete table and histogram gives some information about how late the trains were.

Minutes late (t)	Frequency
$0 < t \leq 5$	16
$5 < t \leq 10$	10
$10 < t \leq 20$	
$20 < t \leq 30$	
$30 < t \leq 50$	8



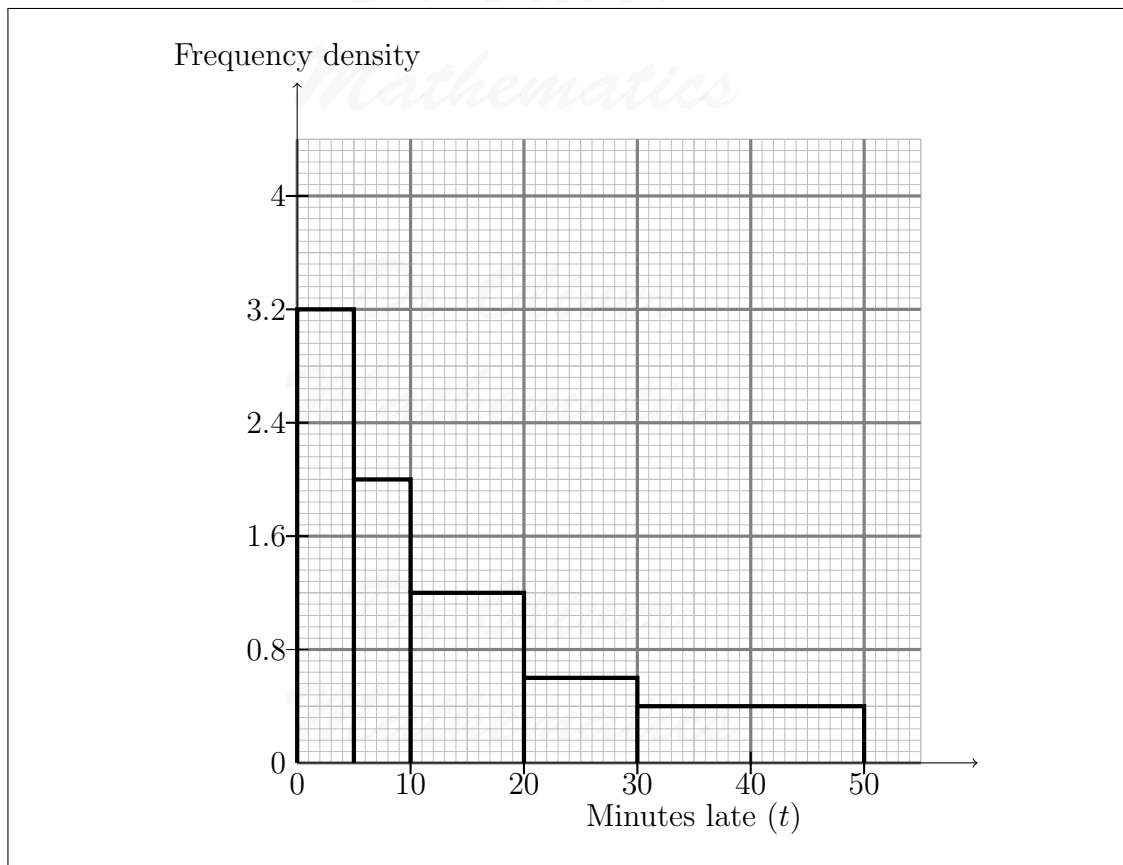
(a) Use the information in the histogram to complete the table. (2)

Solution

Minutes late (t)	Frequency	Width	Frequency density
$0 < t \leq 5$	16	5	$\frac{16}{5} = 3.2$
$5 < t \leq 10$	10	5	$\frac{10}{5} = 2$
$10 < t \leq 20$	<u>12</u>	10	$\frac{12}{10} = 1.2$
$20 < t \leq 30$	<u>6</u>	10	$\frac{6}{10} = 0.6$
$30 < t \leq 50$	8	20	$\frac{8}{20} = 0.4$

(b) Use the information in the table to complete the histogram. (2)

Solution



26. The diagram shows a sector of a circle with centre O .

(5)

The radius of the circle is 8 cm.

PRS is an arc of the circle.

PS is a chord of the circle.

Angle $POS = 40^\circ$.

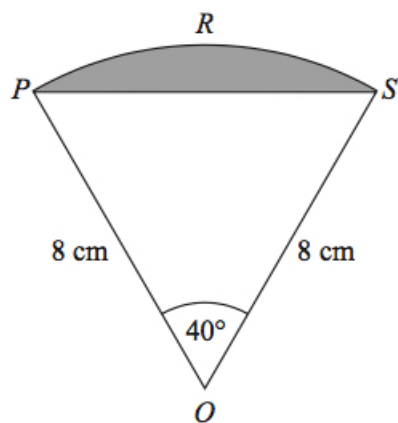


Diagram NOT accurately drawn

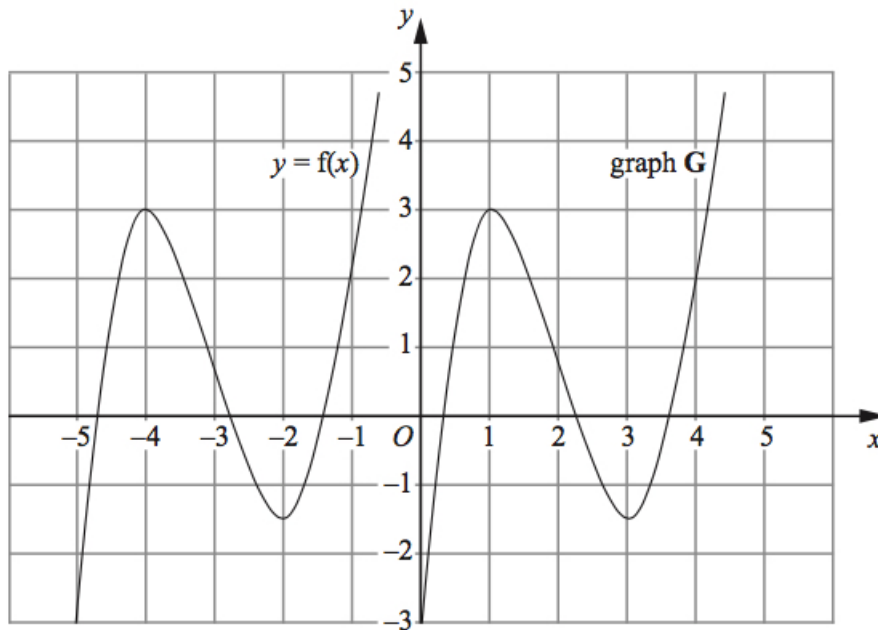
Calculate the area of the shaded segment.

Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Area} &= \frac{40}{360} \times \pi \times 8^2 - \frac{1}{2} \times 8^2 \times \sin 40^\circ \\ &= 1.771\,010\,916 \text{ (FCD)} \\ &= \underline{\underline{1.77 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

27. The graph of $y = f(x)$ is shown on the grid.



The graph **G** is a translation of the graph of $y = f(x)$.

(a) Write down, in terms of f , the equation of graph **G**.

(1)

Solution

$y = f(x - 5)$.

The graph of $y = f(x)$ has a maximum point at $(-4, 3)$.

(b) Write down the coordinates of the maximum point of the graph of $y = f(-x)$.

(2)

Solution

(4, 3).

*Dr Oliver
Mathematics*

*Dr Oliver
Mathematics*

*Dr Oliver
Mathematics*

*Dr Oliver
Mathematics*

*Dr Oliver
Mathematics*

*Dr Oliver
Mathematics*