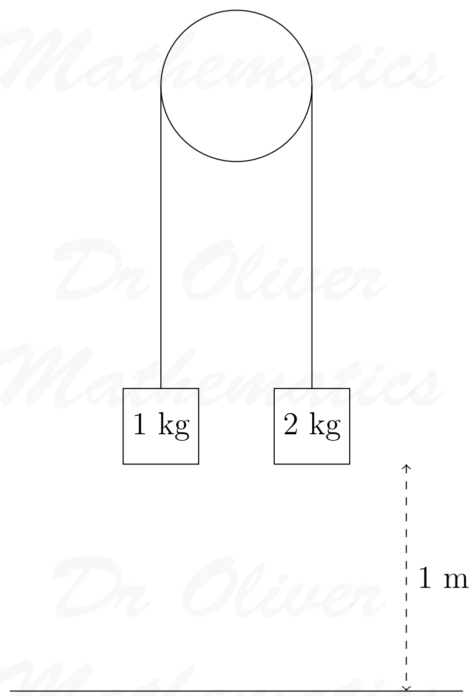
Dr Oliver Mathematics
Dynamics Worksheet

This worksheet consists of 2 questions. The total number of marks available is 27. If you require a numerical value for g use $g = 9.8\,\text{ms}^{-2}$

1. A 1 kg mass and a 2 kg mass are connected by a light inextensible string that runs over a smooth pulley, as shown in the diagram below.



Initially, the two masses are held at rest with each 1 m above a horizontal floor. The system is released from rest.

(a) Explain how you will you use of the following modelling assumptions in your working.

(i) that the string is light, (1)

> **Solution:**
> The mass of the string is considered to be negligible compared with the other masses in the question and so can be ignored.

(ii) that the string is inextensible, (1)

> **Solution:**
> The two masses will have the same acceleration.

(iii) that the pulley is smooth. (1)

Solution:
Since there is no friction the tension will be the same on either side of the pulley.

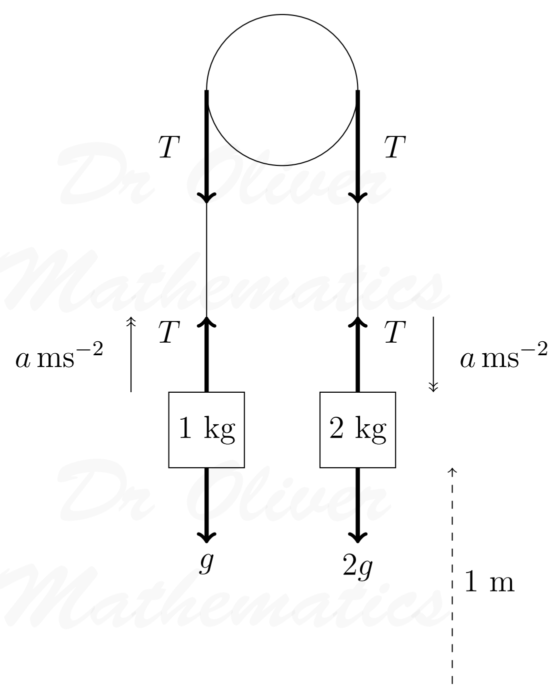
(b) Write down any two other modelling assumptions that you will make in your working. (2)

Solution:
For example, the two masses will be modelled as particles and air resistance will be ignored.

Assuming that the string is long enough for the 2 kg mass to reach the ground, find

(c) the acceleration of the system, (4)

Solution:
Begin by drawing on all of the relevant forces and accelerations:



Write down Newton's Second Law for each of the two masses:

$$T - g = a \quad \text{for the 1 kg mass,}$$
$$2g - T = 2a \quad \text{for the 2 kg mass.}$$

Add the two equations to eliminate T:

$$g = 3a$$

and so

$$\underline{\underline{a = \frac{1}{3}g.}}$$

(d) the time that it takes for the 2 kg mass to reach the floor. (3)

Solution:
Begin by listing what you know and what you want to find so that you can choose the appropriate kinematic equation:

$$s = 1,\ u = 0,\ a = \tfrac{1}{3}g,\ t = ?$$

So the appropriate equation to use is $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} 1 = 0 \times t + \tfrac{1}{2} \times \tfrac{1}{3}g \times t^2 &\Rightarrow 1 = \tfrac{1}{6}gt^2 \\ &\Rightarrow t^2 = \tfrac{30}{49} \\ &\Rightarrow t = \tfrac{\sqrt{30}}{7} \\ &\Rightarrow t = \underline{\underline{0.88 \text{ s (2 sf)}}}. \end{aligned}$$

Assuming that the 1 kg does not hit the pulley,

(e) find the maximum height above the floor that the 1 kg mass reaches. (4)

Solution:
When the 2 kg mass hits the ground the string will become slack and there is no longer any tension pulling the 1 kg mass upwards. So the only force acting on the 1 kg mass at this point is gravity. The initial upward speed of the 1 kg mass will be the same as the speed with which the 2 kg mass hits the floor.

Begin by listing what you know and what you want to find so that you can choose the appropriate kinematic equation:

$$s = 1,\ u = 0,\ a = \tfrac{1}{3}g,\ v = ?$$

So the appropriate equation to use is $v^2 = u^2 + 2as$:

$$v^2 = 0^2 + 2 \times \tfrac{1}{3}g \times 1 \Rightarrow v^2 = \tfrac{98}{15}$$
$$\Rightarrow v = \tfrac{7\sqrt{30}}{15}.$$

Now, we can list what we know and what we want to find out for the movement of the 1 kg mass:

$$s = ?,\ u = \tfrac{7\sqrt{30}}{15}, a = -g, v = 0$$

So the equation to use is $v^2 = u^2 + 2as$:

$$\begin{aligned} 0 &= \left(\tfrac{7\sqrt{30}}{15}\right)^2 + 2 \times (-g) \times s \\ \Rightarrow \quad 2gs &= \tfrac{98}{15} \\ \Rightarrow \quad s &= \tfrac{1}{3}. \end{aligned}$$

So, once the string goes slack, the 1 kg mass moves up another one-third of a metre. But the 1 kg mass was already 2 m above the floor at this point and so the maximum height that the 1 kg mass reaches is $\underline{\underline{2\tfrac{1}{3}\text{ m}}}$.

Another student proposes to re-run the initial experiment but this time using 4 kg and 8 kg masses instead.

(f) Are the following statements true or false?

(i) The initial acceleration of the system will be the same in the second experiment as the first. (1)

Solution:
This statement is true since the acceleration is the same provided the masses are in the same ratio. If you do not see this intuitively, then write down Newton's Second Law for each of the two masses:

$$T - 4g = 4a \qquad \text{for the 4 kg mass,}$$
$$8g - T = 8a \qquad \text{for the 8 kg mass.}$$

Add the two equations to eliminate T: $4g = 12a$ and so $\underline{\underline{a = \tfrac{1}{3}g}}$.

(ii) The 8 kg mass in the second experiment will hit the ground with the same speed as the 2 kg mass did in the first experiment. (1)

Solution:

This statement is true since s, u, and a are all the same as they were in the first experiment.

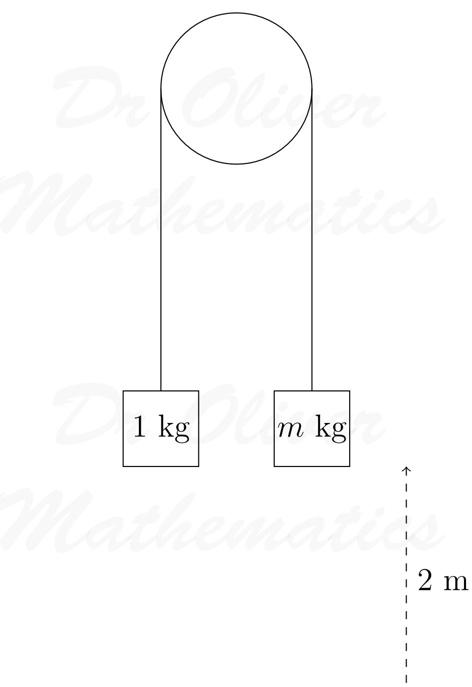
(iii) The 4 kg mass will reach the same height above the floor in the second experiment as the 1 kg mass did in the first experiment. (1)

Solution:
The only thing that changes in this calculation is the force that gravity exerts on the 4 kg mass:

$$\begin{aligned} & 0 = \left(\tfrac{7\sqrt{30}}{15}\right)^2 + 2 \times (-4g) \times s \\ \Rightarrow \quad & 8gs = 6\tfrac{8}{15} \\ \Rightarrow \quad & s = \tfrac{1}{12}. \end{aligned}$$

So the maximum height that the 4 kg mass reaches is $2\frac{1}{12}$ m and hence this statement is false.

2. A 1 kg mass and a m kg mass are connected by a light inextensible string that runs over a smooth pulley, as shown in the diagram below. (8)



Initially, the two masses are held at rest with each 2 m above a horizontal floor. The system is released from rest. Two seconds later, the heavier of the two masses hits the floor. (You may assume that the string is sufficiently long to prevent the lighter of the two masses from reaching the pulley.)

Find the possible values of m.

Solution:

Irrespective of which of the two masses is the heavier we can establish the acceleration of the system when it is released from rest:

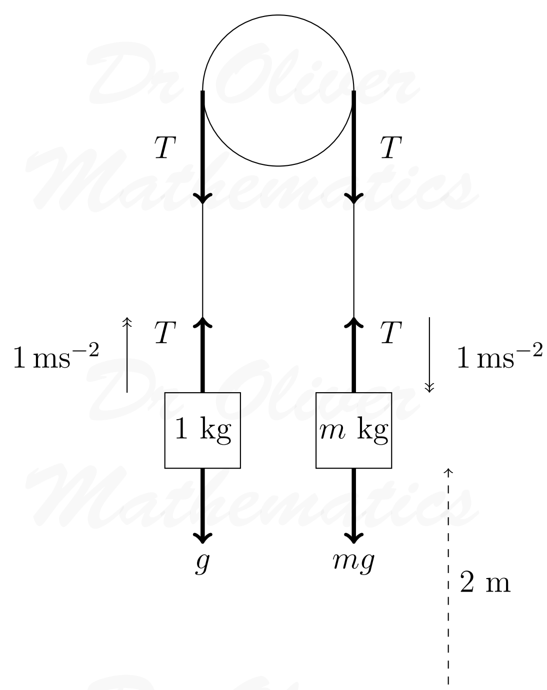
$$s = 2,\ u = 0,\ t = 2,\ a = ?$$

and so the appropriate equation to use is $s = ut + \frac{1}{2}at^2$:

$$\begin{aligned} 2 &= 0 + \tfrac{1}{2}a \times 2^2 \\ a &= 1. \end{aligned}$$

Is there any point in assuming $m = 1$? No, because the system is at rest.

Suppose that $m > 1$. Then the m kg mass will be the heavier of the two and will be the one that hits the floor. We can now draw on all of the relevant forces and accelerations:



Write down Newton's Second Law for each of the two masses:

$$\begin{aligned} T - g &= 1 \qquad \text{for the 1 kg mass,} \\ mg - T &= m \qquad \text{for the } m \text{ kg mass.} \end{aligned}$$

Add the two equations to eliminate T:

$$\begin{aligned} (m-1)g &= 1 + m \\ mg - g &= 1 + m \\ mg - m &= g + 1 \\ m(g-1) &= g + 1 \\ m &= \tfrac{g+1}{g-1} = \underline{\underline{\tfrac{27}{22}}}. \end{aligned}$$

Suppose now that $m < 1$. Write down Newton's Second Law for each of the two masses:

$$\begin{aligned} g - T &= 1 \qquad \text{for the 1 kg mass,} \\ T - mg &= m \qquad \text{for the } m \text{ kg mass,} \end{aligned}$$

and you should be able to show that

$$\begin{aligned} g - mg = m + 1 &\Rightarrow g - 1 = mg + m \\ &\Rightarrow g - 1 = m(g+1) \\ &\Rightarrow m = \tfrac{g-1}{g+1} \\ &\Rightarrow m = \underline{\underline{\tfrac{22}{27}}}. \end{aligned}$$

[Note: what do you notice about the values of the m kg?]