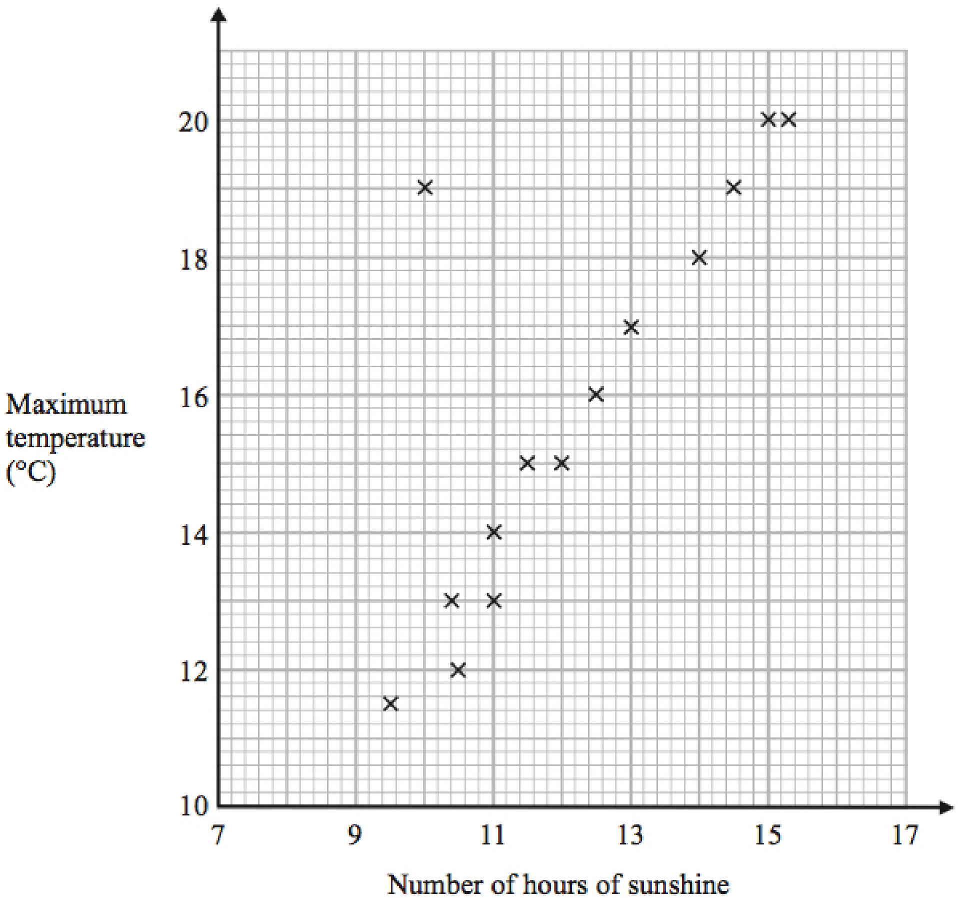
Dr Oliver Mathematics
GCSE Mathematics
2017 Paper 1H: Non-Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. The scatter graph shows the maximum temperature and the number of hours of sunshine in fourteen British towns on one day.



One of the points is an outlier.

(a) Write down the coordinates of this point. (1)

Solution

$\underline{\underline{(10, 19)}}$.

(b) For all the other points write down the type of correlation. (1)

Solution
(Strong) positive correlation.

On the same day, in another British town, the maximum temperature was 16.4°C.

(c) Estimate the number of hours of sunshine in this town. (2)

Solution
Draw a line of best fit drawn, go horizontally from 16.4 to the line of best fit, and down from that point, e.g., 12.6 hours of sunshine.

A weatherman says, "Temperatures are higher on days when there is more sunshine."

(d) Does the scatter graph support what the weatherman says? (1)
Give a reason for your answer.

Solution
Yes: as the majority of points for high temperature appear when there are more hours of sunshine.

2. Express 56 as the product of its prime factors. (2)

Solution

	56
2	28
2	14
2	7
7	1

So

$$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7.$$

3. Work out 54.6×4.3 (3)

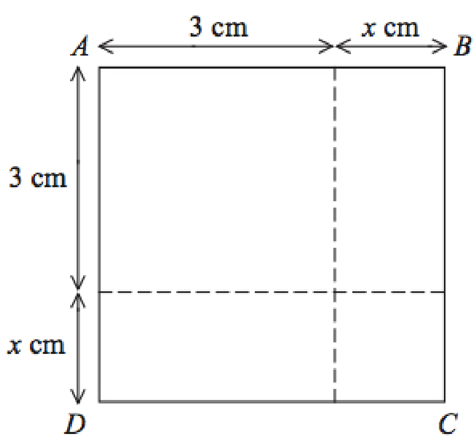
Solution

$$\begin{array}{ccccc} & & 5 & 4 & 6 \\ \times & & & 4 & 3 \\ \hline & 1 & 6 & 3 & 8 \\ 2 & 1 & 8 & 4 & \\ \hline 2 & 3 & 4 & 7 & 8 \\ \hline \end{array}$$

So

$$54.6 \times 4.3 = \underline{\underline{234.78}}.$$

4. The area of square $ABCD$ is 10 cm^2. (3)



Show that

$$x^2 + 6x = 1.$$

Solution

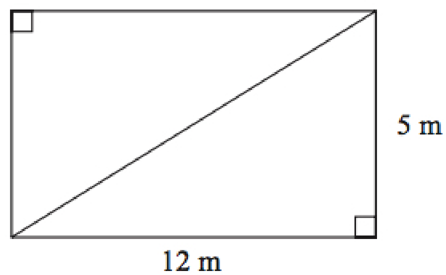
$\times$	3	$+x$
3	9	$+3x$
$+x$	$+3x$	$+x^2$

Now,

$$(x+3)^2 = 10 \Rightarrow x^2 + 6x + 9 = 10$$
$$\Rightarrow \underline{\underline{x^2 + 6x = 1}},$$

as required.

5. This rectangular frame is made from 5 straight pieces of metal. (5)



The weight of the metal is 1.5 kg per metre.

Work out the total weight of the metal in the frame.

Solution

The 'hypotenuse' is

$$\sqrt{5^2 + 12^2} = \sqrt{169} = 13.$$

So

$$5 + 12 + 5 + 12 + 13 = 47 \text{ m}$$

is in the frame and this works out be

$$47 \times 1.5 = 47 + 23.5 = \underline{\underline{70.5 \text{ kg}}}.$$

6. The equation of the line L_1 is $y = 3x - 2$. (2)
The equation of the line L_2 is $3y - 9x + 5 = 0$.

Show that these two lines are parallel.

Solution

$$\begin{aligned} 3y - 9x + 5 = 0 &\Rightarrow 3y = 9x - 5 \\ &\Rightarrow y = 3x - \tfrac{5}{3}; \end{aligned}$$

thus, the two lines are parallel.

7. There are 10 boys and 20 girls in a class. (3)
The class has a test.

The mean mark for all the class is 60.
The mean mark for the girls is 54.

Work out the mean mark for the boys.

Solution

Let x denote he mean mark for the boys. Then

$$\begin{aligned} 60 = \frac{(10 \times x) + (20 \times 54)}{30} &\Rightarrow 1\,800 = 10x + 1\,080 \\ &\Rightarrow 10x = 720 \\ &\Rightarrow x = 72. \end{aligned}$$

8. (a) Write 7.97×10^{-6} as an ordinary number. (1)

Solution

0.000 007 97.

(b) Work out the value of $(2.52 \times 10^5) \div (4 \times 10^{-3})$. (2)
Give your answer in standard form.

Solution

$$\begin{aligned} \frac{2.52 \times 10^5}{4 \times 10^{-3}} &= 0.63 \times 10^8 \\ &= 6.3 \times 10^7. \end{aligned}$$

9. Jules buys a washing machine. (2)

20% VAT is added to the price of the washing machine.
Jules then has to pay a total of £600.

What is the price of the washing machine with **no** VAT added?

Solution

Let x be the price of the washing machine with no VAT added. Then

$$600 = x \times 1.2 \Rightarrow x = \frac{600}{1.2}$$
$$\Rightarrow \underline{\underline{x = 500}}.$$

10. Show that $(x+1)(x+2)(x+3)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c, and d are positive integers. (3)

Solution

$\times$	x	$+1$
x	x^2	$+x$
$+2$	$+2x$	$+2$

and so

$$(x+1)(x+2) = x^2 + 3x + 2.$$

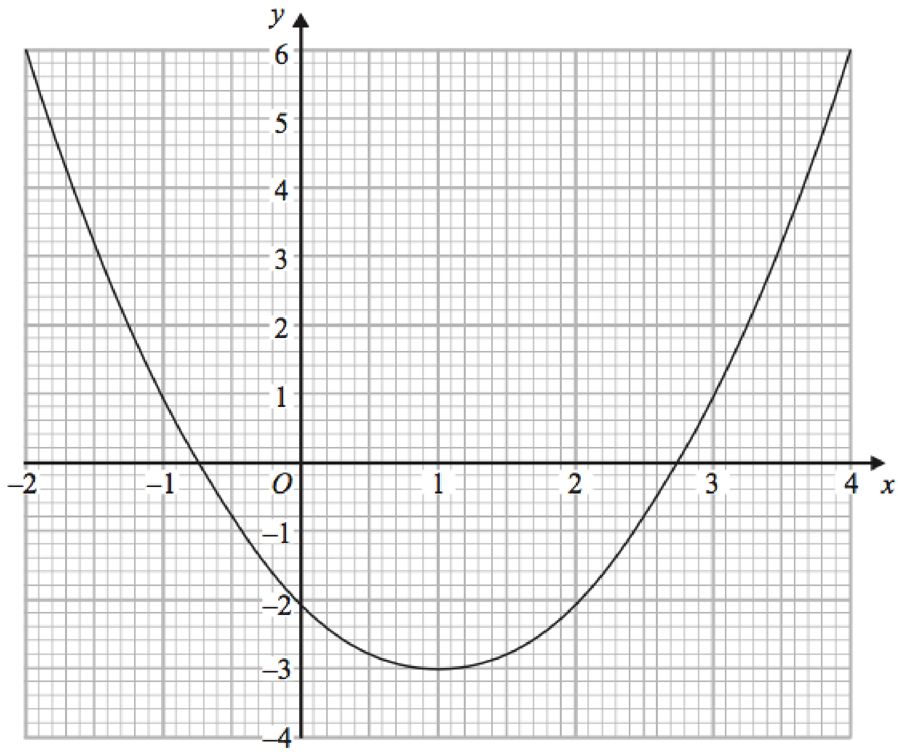
Now,

$\times$	x^2	$+3x$	$+2$
x	x^3	$+3x^2$	$+2x$
$+3$	$+3x^2$	$+9x$	$+6$

and so

$$(x+1)(x+2)(x+3) = \underline{\underline{x^3 + 6x^2 + 11x + 6}}.$$

11. The graph of $y = f(x)$ is drawn on the grid.



(a) Write down the coordinates of the turning point of the graph. (1)

Solution

$(1, -3)$.

(b) Write down estimates for the roots of $f(x) = 0$. (1)

Solution

About -0.75 and 2.75.

(c) Use the graph to find an estimate for $f(1.5)$. (1)

Solution

Draw a straight line through $x = 1.5$ and then read-off: about -2.8.

12. (a) Find the value of $81^{-\frac{1}{2}}$. (2)

Solution

$$\begin{aligned} 81^{-\frac{1}{2}} &= \frac{1}{81^{\frac{1}{2}}} \\ &= \frac{1}{\sqrt{81}} \\ &= \underline{\underline{\tfrac{1}{9}}}. \end{aligned}$$

(b) Find the value of $\left(\frac{64}{125}\right)^{\frac{2}{3}}$. (2)

Solution

$$\begin{aligned} \left(\frac{64}{125}\right)^{\frac{2}{3}} &= \left[\left(\frac{64}{125}\right)^{\frac{1}{3}}\right]^2 \\ &= \left[\tfrac{4}{5}\right]^2 \\ &= \underline{\underline{\tfrac{16}{25}}}. \end{aligned}$$

13. The table shows a set of values for x and y.

x	1	2	3	4
y	9	$2\frac{1}{4}$	1	$\frac{9}{16}$

y is inversely proportional to the square of x.

(a) Find an equation for y in terms of x. (2)

Solution

$$y \propto \frac{1}{x^2} \Rightarrow y = \frac{k}{x^2}$$

for some constant k. Now,

$$9 = \frac{k}{1^2} \Rightarrow k = 9$$

and so

$$\underline{\underline{y = \frac{9}{x^2}}}.$$

(b) Find the positive value of x when $y = 16$. (2)

Solution

$$16 = \frac{9}{x^2} \Rightarrow x^2 = \frac{9}{16}$$
$$\Rightarrow \underline{\underline{x = \tfrac{3}{4}}}.$$

14. White shapes and black shapes are used in a game. (4)
Some of the shapes are circles.
All the other shapes are squares.

The ratio of the number of white shapes to the number of black shapes is 3 : 7.
The ratio of the number of white circles to the number of white squares is 4 : 5.
The ratio of the number of black circles to the number of black squares is 2 : 5.

Work out what fraction of all the shapes are circles.

Solution

Let BC, BS, WC, and WS be the number of black circles, black squares, white circles, and white squares respectively. The ratio of the number of white circles to the number of white squares is 4 : 5 and we multiply the ratio of the number of white shapes to the number of black shapes by 3 to get

$$BC : BS : WC : WS = 6 : 15 : 4 : 5.$$

Finally,

$$\frac{6+4}{6+15+4+5} = \frac{10}{30} = \underline{\underline{\tfrac{1}{3}}}$$

are circles.

15. A cone has a volume of 98 cm^3.
The radius of the cone is 5.13 cm.

(a) Work out an estimate for the height of the cone. (3)

Solution

Approximate to 1 significant figure: let the cone has a volume of 100 cm^3 and the radius of the cone is 5 cm. Then

$$\begin{aligned} h &= \frac{3V}{\pi r^2} \\ &= \frac{3 \times 100}{3 \times 5^2} \\ &= \frac{300}{75} \\ &= \underline{\underline{4 \text{ cm}}}. \end{aligned}$$

John uses a calculator to work out the height of the cone to 2 decimal places.

(b) Will your estimate be more than John's answer or less than John's answer? (1)
Give reasons for your answer.

Solution

More than John's answer: we rounded up from 98 cm^3 to 100 cm^3, we rounded down from 5.13 cm to 5 cm, and we rounded down from π to 3.

16. n is an integer greater than 1. (4)
Prove algebraically that

$$n^2 - 2 - (n-2)^2$$

is always an even number.

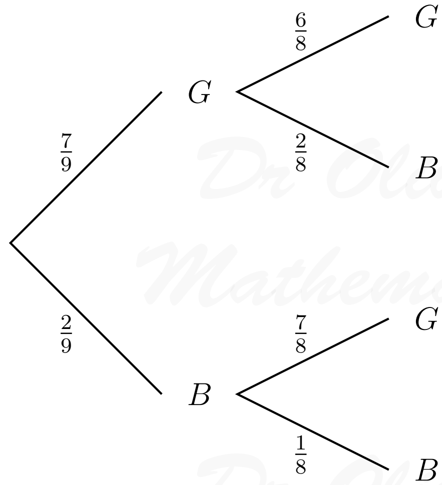
Solution

$$\begin{aligned} n^2 - 2 - (n-2)^2 &= n^2 - 2 - (n^2 - 4n + 4) \\ &= 4n - 6 \\ &= 2(2n-3) \end{aligned}$$

and $2(2n-3)$ is even since they are multpiles of 2.

17. There are 9 counters in a bag. (4)
7 of the counters are green.
2 of the counters are blue.
Ria takes at random two counters from the bag.
Work out the probability that Ria takes one counter of each colour.
You must show your working.

Solution



$\mathrm{P}(GG) = \frac{7}{9} \times \frac{6}{8} = \frac{42}{72}$

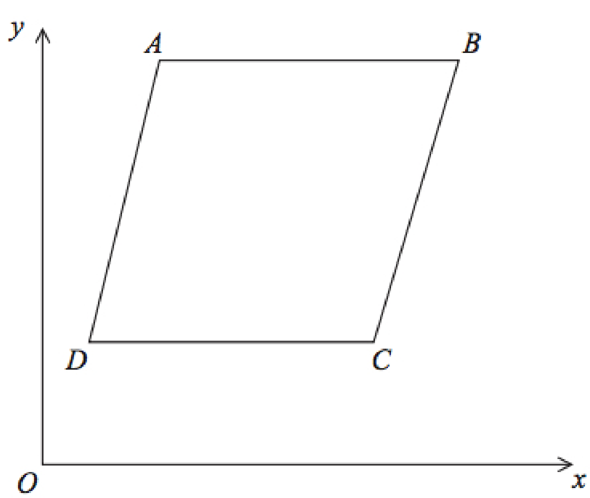
$\mathrm{P}(GB) = \frac{7}{9} \times \frac{2}{8} = \frac{14}{72}$

$\mathrm{P}(BG) = \frac{2}{9} \times \frac{6}{8} = \frac{14}{72}$

$\mathrm{P}(BB) = \frac{2}{9} \times \frac{1}{8} = \frac{2}{72}$

$$\mathrm{P}(GB) + \mathrm{P}(BG) = \frac{14}{72} + \frac{14}{72} = \frac{28}{72} = \underline{\underline{\frac{7}{18}}}.$$

18. $ABCD$ is a rhombus. (4)



The coordinates of A are $(5, 11)$.
The equation of the diagonal DB is $y = \frac{1}{2}x + 6$.
Find an equation of the diagonal AC.

Solution

The gradient of AC is

$$-\frac{1}{\frac{1}{2}} = -2.$$

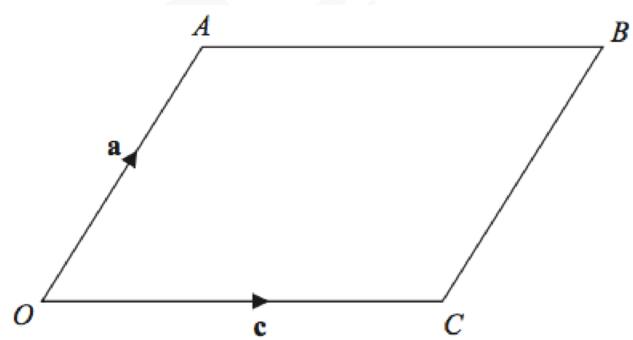
Now,

$$11 = -2 \times 5 + c \Rightarrow c = 21$$

and the the equation of the line is

$$\underline{\underline{y = -2x + 21}}.$$

19. $OABC$ is a parallelogram. (4)



$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.
X is the midpoint of the line AC.
OCD is a straight line so that $OC : CD = k : 1$.
Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$, find the value of k.

Solution

$$\begin{aligned}\overrightarrow{OX} &= \overrightarrow{OA} + \tfrac{1}{2}\overrightarrow{AC} \\ &= \mathbf{a} + \tfrac{1}{2}(\mathbf{c} - \mathbf{a}) \\ &= \tfrac{1}{2}(\mathbf{a} + \mathbf{c}).\end{aligned}$$

Now,

$$\begin{aligned}\overrightarrow{OD} &= \overrightarrow{OX} + \overrightarrow{XD} \\ &= \tfrac{1}{2}(\mathbf{a} + \mathbf{c}) + (3\mathbf{c} - \tfrac{1}{2}\mathbf{a}) \\ &= \tfrac{7}{2}\mathbf{c}\end{aligned}$$

and

$$\overrightarrow{CD} = \tfrac{7}{2}\mathbf{c} - \mathbf{c} = \tfrac{5}{2}\mathbf{c}.$$

Finally,

$$k = \frac{1}{\frac{5}{2}} = \underline{\underline{\tfrac{2}{5}}}.$$

20. Solve algebraically the simultaneous equations (5)

$$x^2 + y^2 = 25$$
$$y - 3x = 13.$$

Solution

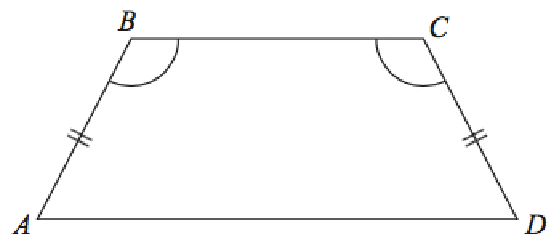
$$\begin{aligned} x^2 + (3x+13)^2 = 25 &\Rightarrow x^2 + (9x^2 + 78x + 169) = 25 \\ &\Rightarrow 10x^2 + 78x + 144 = 0 \\ &\Rightarrow 5x^2 + 39x + 72 = 0 \end{aligned}$$

$$\left.\begin{array}{ll} \text{add to:} & +39 \\ \text{multiply to:} & (+5)\times(+72) = +360 \end{array}\right\} +15,\ +24$$

$$\begin{aligned} &\Rightarrow 5x^2 + 15x + 24x + 72 = 0 \\ &\Rightarrow 5x(x+3) + 24(x+3) = 0 \\ &\Rightarrow (5x+24)(x+3) = 0 \\ &\Rightarrow x = -\tfrac{24}{5} \text{ or } x = -3 \\ &\Rightarrow y = -\tfrac{7}{5} \text{ or } y = 4; \end{aligned}$$

hence, $\underline{\underline{x = -\tfrac{24}{5}, y = -\tfrac{7}{5}}}$ or $\underline{\underline{x = -3, y = 4}}$.

21. $ABCD$ is a quadrilateral. (4)

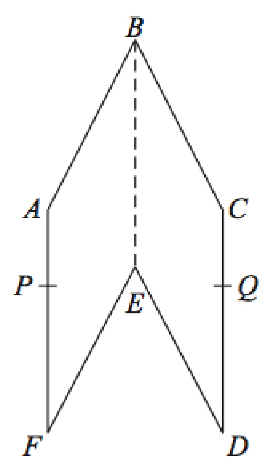


$AB = CD$.
Angle ABC = angle BCD.
Prove that $AC = BD$.

Solution

$AB = CD$.
Angle ABC = angle BCD.
BC is shared.
$\therefore \triangle ABC = \triangle BCD$ (SAS)
$\therefore \underline{\underline{AC = BD}}$.

22. The diagram shows a hexagon $ABCDEF$. (5)



$ABEF$ and $CBED$ are congruent parallelograms where $AB = BC = x$ cm.
P is the point on AF and Q is the point on CD such that $BP = BQ = 10$ cm.
Given that angle $ABC = 30°$, prove that

$$\cos PBQ = 1 - \frac{(2 - \sqrt{3})}{200} x^2.$$

Solution

$$\begin{aligned}\cos PBQ &= \frac{10^2 + 10^2 - PQ^2}{2 \times 10 \times 10} \\ &= \frac{200 - (x^2 + x^2 - 2 \times x \times x \times \cos 30^\circ)}{200} \\ &= \frac{200 - (2x^2 - x^2\sqrt{3})}{200} \\ &= \underline{\underline{1 - \frac{(2-\sqrt{3})}{200}x^2}},\end{aligned}$$

as required.