

**Dr Oliver Mathematics**  
**Mathematics: Higher**  
**2010 Paper 1: Non-Calculator**  
**1 hour 30 minutes**

The total number of marks available is 70.  
You must write down all the stages in your working.

**Section A**

1. A line  $L$  is perpendicular to the line with equation (2)

$$2x - 3y - 6 = 0.$$

What is the gradient of the line  $L$ ?

- A.  $-\frac{3}{2}$   
B.  $-\frac{1}{2}$   
C.  $\frac{2}{3}$   
D. 2
2. A sequence is defined by the recurrence relation (2)

$$u_{n+1} = 2u_n + 3 \text{ and } u_0 = 1.$$

What is the value of  $u_2$ ?

- A. 7  
B. 10  
C. 13  
D. 16
3. Given that (2)

$$\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix},$$

find  $3\mathbf{u} - 2\mathbf{v}$  in component form.

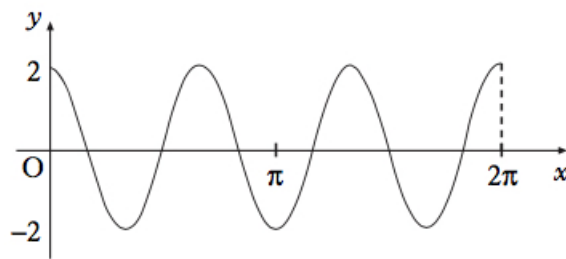
A.  $\begin{pmatrix} 4 \\ -1 \\ -5 \end{pmatrix}$

B.  $\begin{pmatrix} 4 \\ -4 \\ 11 \end{pmatrix}$

C.  $\begin{pmatrix} 8 \\ -1 \\ 5 \end{pmatrix}$

D.  $\begin{pmatrix} 8 \\ -4 \\ -5 \end{pmatrix}$

4. The diagram shows the graph with equation of the form  $y = a \cos bx$  for  $0 \leq x \leq 2\pi$ . (2)



What is the equation of this graph?

- A.  $y = 2 \cos 3x$   
 B.  $y = 2 \cos 2x$   
 C.  $y = 3 \cos 2x$   
 D.  $y = 4 \cos 3x$
5. When  $x^2 + 8x + 3$  (2)

is written in the form

$$(x + p)^2 + q,$$

what is the value of  $q$ ?

- A. -19  
 B. -13  
 C. -5  
 D. 19
6. The roots of the equation  $kx^2 - 3x + 2 = 0$  (2)

are equal. What is the value of  $k$ ?

- A.  $-\frac{9}{8}$
- B.  $-\frac{8}{9}$
- C.  $\frac{8}{9}$
- D.  $\frac{9}{8}$

7. A sequence is generated by the recurrence relation (2)

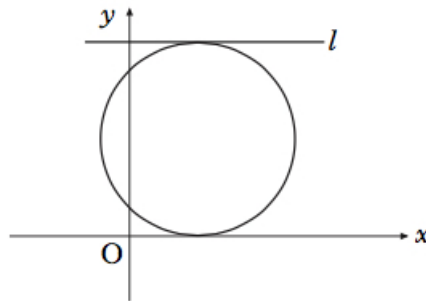
$$u_{n+1} = \frac{1}{4}u_n + 7 \text{ and } u_0 = -2.$$

What is the limit of this sequence as  $n \rightarrow \infty$ ?

- A.  $\frac{1}{28}$
- B.  $\frac{28}{5}$
- C.  $\frac{28}{3}$
- D. 28

8. The equation of the circle shown in the diagram is (2)

$$x^2 + y^2 - 6x - 10y + 9 = 0.$$



The  $x$ -axis and the line  $l$  are parallel tangents to the circle.

- A.  $y = 5$
- B.  $y = 10$
- C.  $y = 18$
- D.  $y = 20$

9. Find (2)

$$\int (2x^{-4} + \cos 5x) dx.$$

- A.  $-\frac{2}{5}x^{-5} - 5 \sin 5x + c$

- B.  $-\frac{2}{5}x^{-5} + \frac{1}{5} \sin 5x + c$   
 C.  $-\frac{2}{3}x^{-3} + \frac{1}{5} \sin 5x + c$   
 D.  $-\frac{2}{3}x^{-3} - 5 \sin 5x + c$

10. The vectors

$$x\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} \text{ and } -3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

(2)

are perpendicular.

What is the value of  $x$ ?

- A. 0  
 B. 1  
 C.  $\frac{4}{3}$   
 D.  $\frac{10}{3}$

11. Functions  $f$  and  $g$  are defined on suitable domains by

$$f(x) = \cos x \text{ and } g(x) = x + \frac{\pi}{6}.$$

(2)

What is the value of  $f\left(g\left(\frac{\pi}{6}\right)\right)$ ?

- A.  $\frac{1}{2} + \frac{\pi}{6}$   
 B.  $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$   
 C.  $\frac{\sqrt{3}}{2}$   
 D.  $\frac{1}{2}$

12. If

$$f(x) = \frac{1}{\sqrt[5]{x}}, \quad x \neq 0,$$

(2)

what is  $f'(x)$ ?

- A.  $-\frac{1}{5}x^{-\frac{6}{5}}$   
 B.  $-\frac{1}{5}x^{-\frac{4}{5}}$   
 C.  $-\frac{5}{2}x^{-\frac{7}{2}}$   
 D.  $-\frac{5}{2}x^{-\frac{3}{5}}$

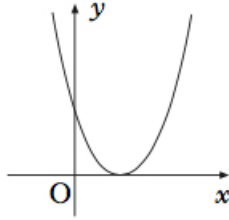
13. Which of the following diagrams shows a parabola with equation

(2)

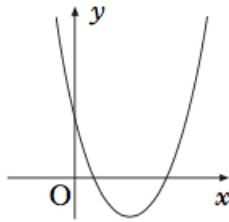
$$y = ax^2 + bx + c,$$

where  $a > 0$  and  $b^2 - 4ac > 0$ ?

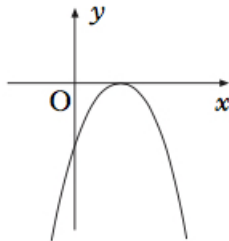
A



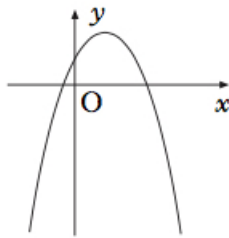
B



C



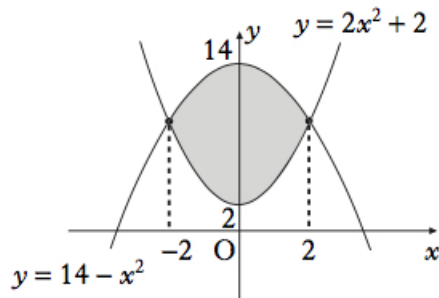
D



14. The diagram shows graphs with equations

(2)

$$y = 14 - x^2 \text{ and } y = 2x^2 + 2.$$



Which of the following represents the shaded area?

- A.  $\int_2^{14} (12 - 3x^2) dx$
- B.  $\int_2^{14} (3x^2 - 12) dx$
- C.  $\int_{-2}^2 (12 - 3x^2) dx$
- D.  $\int_{-2}^2 (3x^2 - 12) dx$

15. The derivative of a function  $f$  is given by (2)

$$f'(x) = x^2 - 9.$$

Here are two statements about  $f$ :

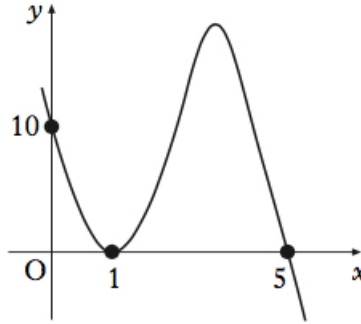
- (1)  $f$  is increasing at  $x = 1$ ;
- (2)  $f$  is stationary at  $x = -3$ .

Which of the following is true?

- A. Neither statement is correct.
- B. Only statement (1) is correct.
- C. Only statement (2) is correct.
- D. Both statements are correct.

16. The diagram shows the graph with equation (2)

$$y = k(x - 1)^2(x + t).$$



What are the values of  $k$  and  $t$ ?

- A.  $k = -2$  and  $t = -5$
- B.  $k = -2$  and  $t = 5$
- C.  $k = 2$  and  $t = -5$
- D.  $k = 2$  and  $t = 5$

17. If

$$s(t) = t^2 - 5t + 8,$$

(2)

what is the rate of change of  $s$  with respect to  $t$  when  $t = 3$ ?

- A.  $-5$
- B.  $1$
- C.  $2$
- D.  $9$

18. What is the solution of

$$x^2 + 4x > 0,$$

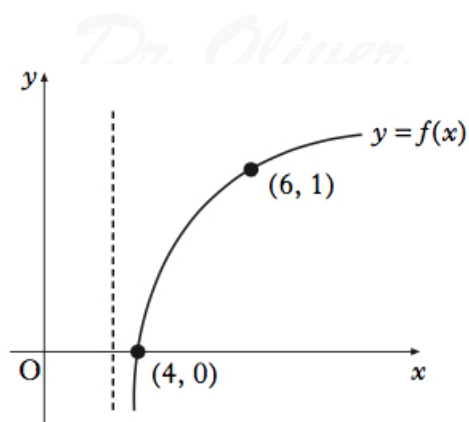
(2)

where  $x$  is a real number?

- A.  $-4 < x < 0$
- B.  $x < -4$  or  $x > 0$
- C.  $0 < x < 4$
- D.  $x < 0$  or  $x > 4$

19. The diagram shows the graph of  $y = f(x)$  where  $f$  is a logarithmic function.

(2)



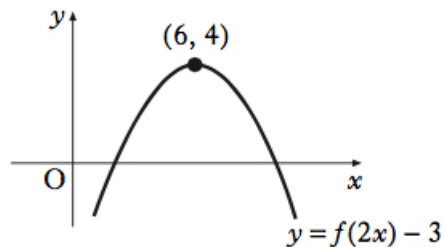
What is  $f(x)$ ?

- A.  $f(x) = \log_6(x - 3)$
- B.  $f(x) = \log_3(x + 3)$
- C.  $f(x) = \log_3(x - 3)$
- D.  $f(x) = \log_6(x + 3)$

20. The diagram shows the graph of

$$y = f(2x) - 3.$$

(2)



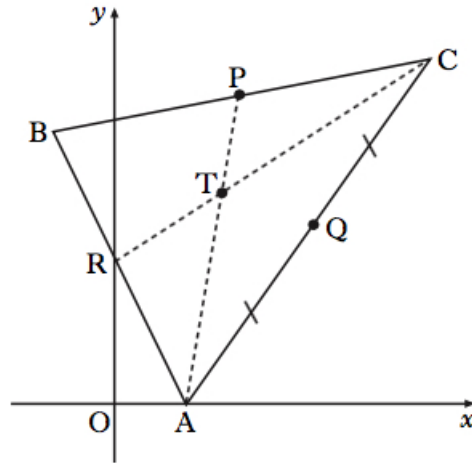
What are the coordinates of the turning point on the graph of  $y = f(x)$ ?

- A. (12, 7)
- B. (12, 1)
- C. (3, 7)
- D. (3, 1)

## Section B

21. Triangle  $ABC$  has vertices  $A(4, 0)$ ,  $B(-4, 16)$ , and  $C(18, 20)$ , as shown in the diagram below.





Medians  $AP$  and  $CR$  intersect at the point  $T(6, 12)$ .

- (a) Find the equation of median  $BQ$ . (3)
  - (b) Verify that  $T$  lies on  $BQ$ . (1)
  - (c) Find the ratio in which  $T$  divides  $BQ$ . (2)
22. (a) (i) Show that  $(x - 1)$  is a factor of  $f(x) = 2x^3 + x^2 - 8x + 5$ . (5)
- (ii) Hence factorise  $f(x)$  fully.
  - (b) Solve (1)

$$2x^3 + x^2 - 8x + 5 = 0.$$

The line with equation  $y = 2x - 3$  is a tangent to the curve with equation  $y = 2x^3 + x^2 - 6x + 2$  at the point  $G$ .

- (c) Find the coordinates of  $G$ . (5)

This tangent meets the curve again at the point  $H$ .

- (d) Write down the coordinates of  $H$ . (1)

23. Diagram 1 shows a right-angled triangle, where the line  $OA$  has equation  $3x - 2y = 0$ .

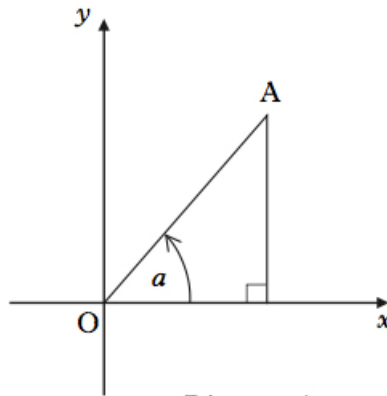


Diagram 1

- (a) (i) Show that  $\tan a = \frac{3}{2}$ . (4)  
 (ii) Find the value of  $\sin a$ .

A second right angled triangle is added as shown in Diagram 2.

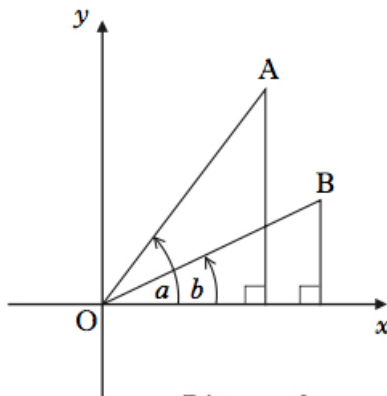


Diagram 2

The line  $OB$  has equation  $3x - 4y = 0$ .

- (b) Find the values of  $\sin b$  and  $\cos b$ . (4)  
 (c) (i) Find the value of  $\sin(a - b)$ . (4)  
 (ii) State the value of  $\sin(b - a)$ .