

Dr Oliver Mathematics

Non-constructive Proof

In this note, we will explore non-constructive ways to prove a given statement.

Example 1

Prove that a power of an irrational number to an irrational exponent may be rational.

Solution

Consider $\sqrt{2}^{\sqrt{2}}$.

If it is rational, we are done. Hence, the power of an irrational number to an irrational exponent is rational.

If it is irrational,

$$(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = (\sqrt{2})^2 = 2$$

and it is rational. ■