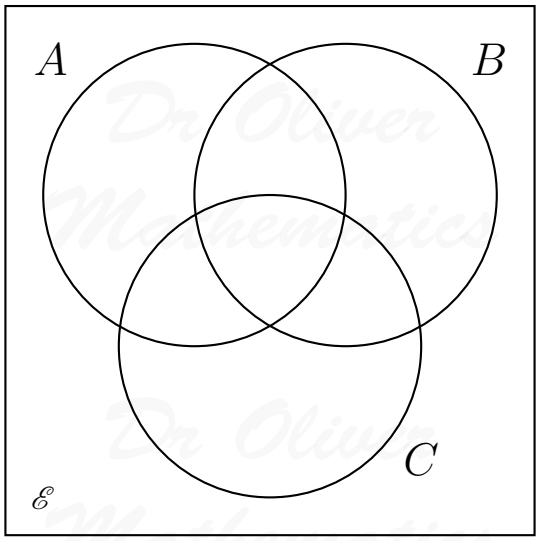
Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2008 November Paper 1: Calculator
2 hours

The total number of marks available is 80.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You must write down all the stages in your working.

1. Here is a Venn diagram.



   (a) Copy the Venn diagram above and shade the region that represents $A \cup (B \cap C)$. (1)

   (b) Copy the Venn diagram above and shade the region that represents $A \cap (B \cup C)$. (1)

   (c) Copy the Venn diagram above and shade the region that represents $(A \cup B \cup C)'$. (1)

2. Find the set of values of x for which (4)

$$(2x + 1)^2 > 8x + 9.$$

3. Prove that (4)

$$\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} \equiv 2 \operatorname{cosec} A.$$

4. A function f is such that (5)

$$f(x) = ax^3 + bx^2 + 3x + 4.$$

- When $\mathrm{f}(x)$ is divided by $(x-1)$, the remainder is 3.
- When $\mathrm{f}(x)$ is divided by $(2x+1)$, the remainder is 6.

Find the value of a and of b.

5. Given that $\mathbf{a} = 5\mathbf{i} - 12\mathbf{j}$ and that $\mathbf{b} = p\mathbf{i} + \mathbf{j}$, find

   (a) the unit vector in the direction of $\mathbf{a}$, (2)

   (b) the values of the constants p and q such that (3)

$$q\mathbf{a} + \mathbf{b} = 19\mathbf{i} - 23\mathbf{j}.$$

6. (a) Solve the equation (3)

$$2t = 9 + \frac{5}{t}.$$

   (b) Hence, or otherwise, solve the equation (3)

$$2x^{\frac{1}{2}} = 9 + 5x^{-\frac{1}{2}}.$$

7. (a) Express (3)

$$4x^2 - 12x + 3$$

   in the form

$$(ax+b)^2 + c,$$

   where a, b, and c are constants and $a > 0$.

   (b) Hence, or otherwise, find the coordinates of the stationary point of the curve. (2)

   (c) Given that (1)

$$\mathrm{f}(x) = 4x^2 - 12x + 3,$$

   write down the range of f.

8. A curve is such that (6)

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4\mathrm{e}^{-2x}.$$

   Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3$$

   when $x = 0$ and that the curve passes through the point $(2, \mathrm{e}^{-4})$, find the equation of the curve.

9. (a) Find, in ascending powers of x, the first 3 terms in the expansion of (3)

$$(2 - 3x)^5.$$

The first 3 terms in the expansion of

$$(a + bx)(2 - 3x)^5$$

in ascending powers of x are are

$$64 - 192x + cx^2.$$

(b) Find the value of a, of b, and of c. (5)

10. Functions f and g are defined, for $x \in \mathbb{R}$, by

$$\mathrm{f}(x) = 3 - x,$$
$$\mathrm{g}(x) = \frac{x}{x+2}, \text{ where } x \neq -2.$$

(a) (i) Find $\mathrm{f\,g}(x)$. (2)

(ii) Hence find the value of x for which (2)

$$\mathrm{f\,g}(x) = 10.$$

(b) A function h is defined, for $x \in \mathbb{R}$, by

$$\mathrm{h}(x) = 4 + \ln x, \text{ where } x > 1.$$

(i) Find the range of h. (1)

(ii) Find the value of $\mathrm{h}^{-1}(9)$. (2)

(iii) On the same axes, sketch the graphs of $y = \mathrm{h}(x)$ and $y = \mathrm{h}^{-1}(x)$. (3)

11. Solve the equation

(a) $\tan 2x - 3\cot 2x = 0$, for $0^\circ < x < 180^\circ$, (4)

(b) $\operatorname{cosec} y = 1 - 2\cot^2 y$, for $0^\circ < y < 360^\circ$, (5)

(c) $\sec(z + \frac{1}{2}\pi) = -2$, for $0 < z < \pi$ radians. (3)

EITHER

12. A curve has equation

$$y = \frac{x^2}{x+1}.$$

(a) Find the coordinates of the stationary points of the curve. (5)

The normal to the curve at the point where $x = 1$ meets the x-axis at M.

The tangent to the curve at the point where $x = -2$ meets the y-axis at N.

(b) Find the area of the triangle MNO, where O is the origin. (6)

OR

13. A curve has equation

$$y = \mathrm{e}^{x-2} - 2x + 6.$$

(a) Find the coordinates of the stationary point of the curve and determine the nature of the stationary point. (6)

The area of the region enclosed by the curve, the positive x-axis, the positive y-axis, and the line $x = 3$ is

$$k + \mathrm{e} - \mathrm{e}^{-2}.$$

(b) Find the value of k. (5)