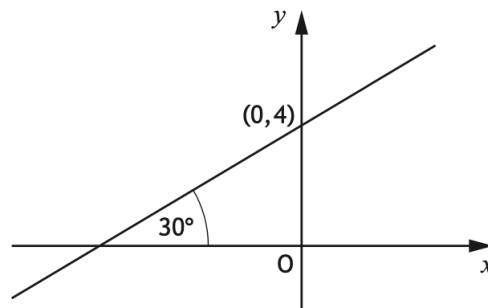


**Dr Oliver Mathematics**  
**Mathematics: Higher**  
**2024 Paper 1: Non-Calculator**  
**1 hour 15 minutes**

The total number of marks available is 55.

You must write down all the stages in your working.

1. A line passes through the point  $(0, 4)$  and makes an angle of  $30^\circ$  with the positive direction of the  $x$ -axis, as shown in the diagram. (3)



Determine the equation of the line.

2. A sequence is defined by the recurrence relation with

$$u_{n+1} = \frac{1}{5}u_n + 12 \text{ with } u_1 = 20.$$

- (a) Calculate the value of  $u_2$ . (1)  
(b) (i) Explain why this sequence approaches a limit as  $n \rightarrow \infty$ . (1)  
(ii) Calculate this limit. (2)
3. Given that (2)

$$y = (5x^2 + 3)^7,$$

find  $\frac{dy}{dx}$ .

4.  $P$  and  $Q$  have coordinates  $(-6, 1, 2)$  and  $(-1, 11, -8)$  respectively. (2)

Find the coordinates of the point  $R$  which divides  $PQ$  in the ratio  $2 : 3$ .

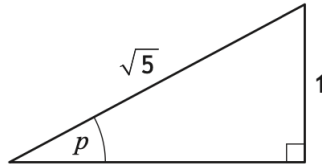
5. A function,  $h$ , is defined by where (3)

$$h(x) = 2x^3 - 7, \text{ where } x \in \mathbb{R}.$$

Find the inverse function,  $h^{-1}(x)$ .

6. The right-angled triangle in the diagram is such that and

$$\sin p = \frac{1}{\sqrt{5}} \text{ and } 0 < p < \frac{1}{4}\pi.$$



(a) Determine the value of:

(i)  $\sin 2p,$  (3)

(ii)  $\cos 2p.$  (1)

(b) Hence determine the value of  $\sin 4p.$  (1)

7. The line (4)

$$y = 2x$$

is a tangent to the circle with equation

$$x^2 + y^2 - 14x - 8y + 45 = 0.$$

Determine the coordinates of the point of contact.

8. The equation (4)

$$x^2 + (m - 4)x + (2m - 3) = 0$$

has no real roots.

Determine the range of values for  $m.$

**Justify your answer.**

9. Express (3)

$$\log_a 5 + \log_a 80 - 2 \log_a 10$$

in the form

$$\log_a k,$$

where  $k$  is a positive integer.

10. (a) Show that  $(x - 1)$  is a factor of (2)

$$2x^4 + 3x^3 - 4x^2 - 3x + 2.$$

- (b) Hence, or otherwise, factorise (4)

$$2x^4 + 3x^3 - 4x^2 - 3x + 2$$

fully.

11. (a) Express (4)

$$\cos x^\circ + \sqrt{3} \sin x^\circ$$

in the form

$$k \cos(x - \alpha)^\circ,$$

where  $k > 0$  and  $0 < \alpha < 360$ .

- (b) Hence, or otherwise, sketch the graph with equation (3)

$$y = \cos x^\circ + \sqrt{3} \sin x^\circ, 0 \leq x \leq 360.$$

12. The function  $f$  is given by (4)

$$f(x) = 12\sqrt[3]{x}, x > 0.$$

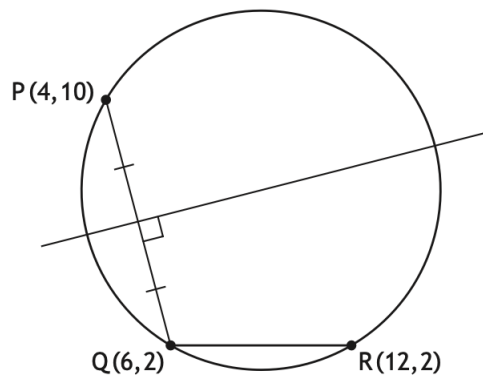
When  $x = a$ , the rate of change of  $f$  with respect to  $x$  is 1.

Determine the value of  $a$ .

13.  $P$  and  $Q$  are the points  $(4, 10)$  and  $(6, 2)$  respectively.

- (a) Find the equation of the perpendicular bisector of  $PQ$ . (4)

The point  $R$  has coordinates  $(12, 2)$ .



- A circle passes through the points  $P$ ,  $Q$ , and  $R$ .
- The chord  $QR$  is horizontal.

- (b) Find the equation of the circle. (4)