

Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2012 November Paper 1 Variant 1: Calculator
2 hours

The total number of marks available is 80.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You must write down all the stages in your working.

1. It is given that

$$\mathbf{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \text{ and } \mathbf{c} = \begin{pmatrix} 21 \\ 2 \end{pmatrix}.$$

- (a) Find

$$|\mathbf{a} + \mathbf{b} + \mathbf{c}|.$$

(2)

- (b) Find λ and μ such that

$$\lambda \mathbf{a} + \mu \mathbf{b} = \mathbf{c}.$$

(3)

2. (a) Find the inverse of the matrix

$$\begin{pmatrix} 2 & -1 \\ -1 & 1.5 \end{pmatrix}.$$

(2)

- (b) Hence find the matrix $\mathbf{A}$ such that

$$\begin{pmatrix} 2 & -1 \\ -1 & 1.5 \end{pmatrix} \mathbf{A} = \begin{pmatrix} 1 & 6 \\ -0.5 & 4 \end{pmatrix}.$$

(3)

3. (a) Show that

$$\cot \theta + \frac{\sin \theta}{1 + \cos \theta} \equiv \operatorname{cosec} \theta.$$

(5)

- (b) Explain why the equation

$$\cot \theta + \frac{\sin \theta}{1 + \cos \theta} = \frac{1}{2}$$

(1)

has no solution.

4. Given that

$$\log_a pq = 9 \text{ and } \log_a p^2q = 15,$$

find the value of

(a) $\log_a p$ and of $\log_a q$, (4)

(b) $\log_p a + \log_q a$. (2)

5. The line (7)

$$x - 2y = 6$$

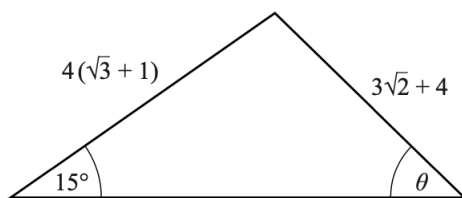
intersects the curve

$$x^2 + xy + 10y + 4y^2 = 156$$

at the points A and B .

Find the length of AB .

6. Here is a picture. (5)

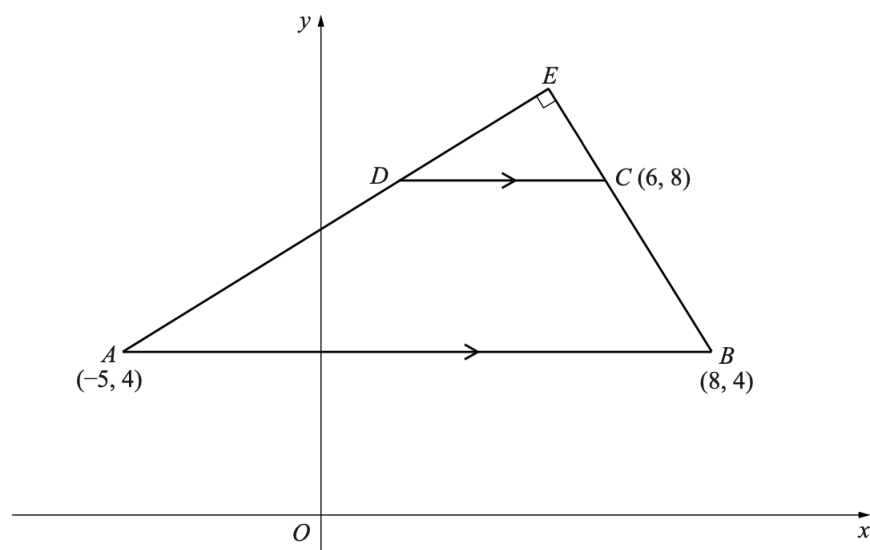


Using

$$\sin 15^\circ = \frac{\sqrt{2}}{4}(\sqrt{3} - 1),$$

and without using a calculator, find the value of $\sin \theta$ in the form $a + b\sqrt{2}$, where a and b are integers.

7. Solutions to this question by accurate drawing will not be accepted.



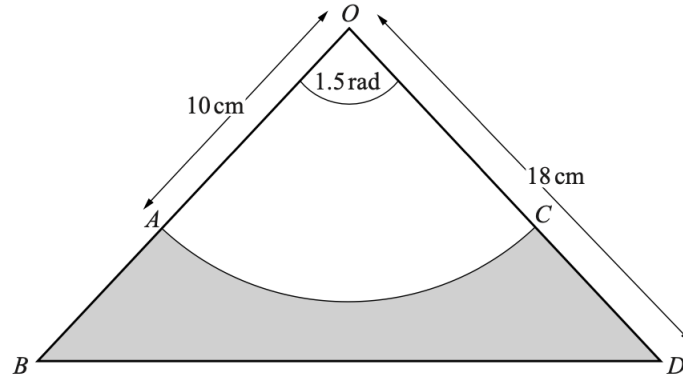
The vertices of the trapezium $ABCD$ are the points $A(-5, 4)$, $B(8, 4)$, $C(6, 8)$, and D .

- The line AB is parallel to the line DC .
- The lines AD and BC are extended to meet at E .
- Angle $AEB = 90^\circ$.

(a) Find the coordinates of D and of E . (6)

(b) Find the area of the trapezium $ABCD$. (2)

8. The diagram shows an isosceles triangle OBD in which $OB = OD = 18$ cm and angle $BOD = 1.5$ radians.



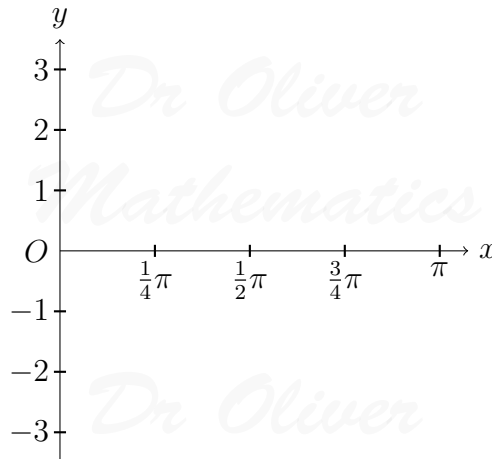
An arc of the circle, centre O and radius 10 cm, meets OB at A and OD at C .

(a) Find the area of the shaded region. (3)

(b) Find the perimeter of the shaded region. (4)

9. (a) (i) Using the axes below, sketch for $0 \leq x \leq \pi$, the graphs of (4)

$$y = \sin 2x \text{ and } y = 1 + \cos 2x.$$



- (ii) Write down the solutions of the equation (2)

$$\sin 2x - \cos 2x = 1, \text{ for } 0 \leq x \leq \pi.$$

- (b) (i) Write down the amplitude and period of (2)

$$5 \cos 4x - 3.$$

- (ii) Write down the period of (1)

$$4 \tan 3x.$$

10. A function f is such that

$$f(x) = 4x^3 + 4x^2 + ax + b.$$

It is given that $(2x - 1)$ is a factor of both $f(x)$ and $f'(x)$.

- (a) Show that $b = 2$ and find the value of a . (5)

Using the values of a and b from part (a),

- (b) find the remainder when $f(x)$ is divided by $(x + 3)$, (2)

- (c) express $f(x)$ in the form (2)

$$f(x) = (2x - 1)(px^2 + qx + r),$$

where p , q , and r are integers to be found,

- (d) find the values of x for which $f(x) = 0$. (2)

EITHER

11. A curve is such that

$$y = \frac{5x^2}{1 + x^2}.$$

- (a) Show that (4)

$$\frac{dy}{dx} = \frac{kx}{(1 + x^2)^2},$$

where k is an integer to be found.

- (b) Find the coordinates of the stationary point on the curve and determine the nature of this stationary point. (3)

- (c) By using your result from part (a), find (4)

$$\int \frac{x}{(1 + x^2)^2} dx$$

and hence evaluate

$$\int_{-1}^2 \frac{x}{(1 + x^2)^2} dx.$$

OR

12. A curve is such that

$$y = \frac{Ax^2 + B}{x^2 - 2},$$

where A and B are constants.

(a) Show that

$$\frac{dy}{dx} = -\frac{2x(2A + B)}{(x^2 - 2)^2}. \quad (4)$$

It is given that $y = -3$ and $\frac{dy}{dx} = -10$ when $x = 1$.

(b) Find the value of A and of B . (3)

(c) Using your values of A and B , find the coordinates of the stationary point on the curve, and determine the nature of this stationary point. (4)