Dr Oliver Mathematics
Applied Mathematics: Sequences

The total number of marks available is 25.
You must write down all the stages in your working.

1. Define
$$S_n = \sum_{r=1}^{n} r^2,\ n \geqslant 1.$$
   (a) Write down formulae for S_n and S_{2n+1}. (2)
   (b) Obtain a formula for (1)
$$2^2 + 4^2 + \ldots + (2n)^2.$$

2. (a) Use the standard formulas for (3)
$$\sum_{r=1}^{n} r \text{ and } \sum_{r=1}^{n} r^2$$
   to show that
$$\sum_{r=1}^{n}(6r^2 - r) = \tfrac{1}{2}n(n+1)(4n+1).$$
   (b) Hence evaluate (2)
$$\sum_{r=5}^{10}(6r^2 - r).$$

3. (a) Find the value of N for which (3)
$$\sum_{r=1}^{N} r = 210.$$
   (b) Evaluate (2)
$$\sum_{r=1}^{N} r^2$$
   for this value of N.

4. (a) State (2)
$$\sum_{r=1}^{n} r \text{ and } \sum_{r=1}^{n} r^3$$
   in terms of n.

(b) Hence show that (2)

$$\sum_{r=1}^{n}(r^3 - 3r) = \frac{n(n+1)(n-2)(n+3)}{4}.$$

(c) Use the above result to evaluate (2)

$$\sum_{r=5}^{15}(r^3 - 3r).$$

5. Evaluate (2)

$$\sum_{r=1}^{80} 3r^2.$$

6. (a) Express (1)

$$\log_a 2 + \log_a 4 + \log_a 8$$

in the form

$$p \log_a 2,$$

where p is a constant.

(b) Hence evaluate (3)

$$\sum_{r=1}^{100} \log_a 2^r,$$

giving your answer in the form

$$q \log_a 2,$$

where q is a constant.