Dr Oliver Mathematics
Applied Mathematics: Mechanics or Statistics
Section B
2006 Paper
1 hour

The total number of marks available is 32.
You must write down all the stages in your working.

1. (a) Calculate $\mathbf{A}^{-1}$ where (3)

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 2 & 2 & 1 \end{pmatrix}.$$

(b) Hence solve the system of equations (2)

$$\begin{aligned} x + y &= 1 \\ 2x + 3y + z &= 2 \\ 2x + 2y + z &= 1. \end{aligned}$$

2. Given that (5)

$$y = \ln(1 + \sin x),$$

where $0 < x < \pi$, show that

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{-1}{1 + \sin x}.$$

3. Define

$$S_n = \sum_{r=1}^{n} r^2, \; n \geqslant 1.$$

(a) Write down formulae for S_n and S_{2n+1}. (2)

(b) Obtain a formula for (1)

$$2^2 + 4^2 + \ldots + (2n)^2.$$

4. Solve the differential equation (5)

$$\cos^2 x \frac{\mathrm{d}y}{\mathrm{d}x} = y,$$

given that $y > 0$ and that $y = 2$ when $x = 0$.

5. Use the substitution $u = 1 + x^2$ to obtain (5)

$$\int \frac{x^3}{\sqrt{1+x^2}}\,\mathrm{d}x.$$

6. (a) Evaluate (4)

$$\int_0^1 x\mathrm{e}^{2x}\,\mathrm{d}x.$$

(b) Use part (a) to evaluate (3)

$$\int_0^1 x^2\mathrm{e}^{2x}\,\mathrm{d}x.$$

(c) Hence obtain (2)

$$\int_0^1 (3x^2+2x)\mathrm{e}^{2x}\,\mathrm{d}x.$$