Dr Oliver Mathematics
Cambridge O Level Additional Mathematics
2004 November Paper 2: Calculator
2 hours

The total number of marks available is 80.
You must write down all the stages in your working.

1. Given that (4)

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ -5 & 4 \end{pmatrix},$$

find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$\begin{aligned} 2x + 3y + 4 &= 0 \\ -5x + 4y + 13 &= 0. \end{aligned}$$

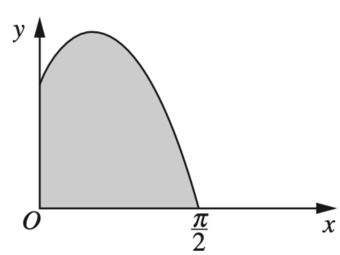
2. Given that (4)

$$\sqrt{a + b\sqrt{3}} = \frac{13}{4 + \sqrt{3}},$$

where a and b are integers, find, without using a calculator, the value of a and b

3. The diagram shows part of the curve (5)

$$y = 3 \sin 2x + 4 \cos x.$$



Find the area of the shaded region, bounded by the curve, and the coordinate axes.

4. Find the values of k for which the line (5)

$$y = x + 2$$

meets the curve

$$y^2 + (x + k)^2 = 2.$$

5. Solve the equation (6)

$$\log_{16}(3x - 1) = \log_4(3x) + \log_4(0.5).$$

6. Given that

$$x = 3\sin\theta - 2\cos\theta \text{ and } y = 3\cos\theta + 2\sin\theta,$$

   (a) find the value of the acute angle θ for which $x = y$, (3)

   (b) show that (3)

$$x^2 + y^2$$

   is constant for all values of θ.

7. Given that (7)

$$6x^3 + 5ax - 12a$$

leaves a remainder of -4 when divided by $(x - a)$, find the possible values of a.

8. A motor boat travels in a straight line across a river which flows at 3 ms^{-1} between straight parallel banks 200 m apart. (7)

The motor boat, which has a top speed of 6 ms^{-1} in still water, travels directly from a point A on one bank to a point B, 150 m downstream of A, on the opposite bank.

Assuming that the motor boat is travelling at top speed, find, to the nearest second, the time it takes to travel from A to B.

9. In order that each of the equations (7)

$$y = ab^x \quad y = Ax^k \quad px + qy = xy,$$

where a, b, A, k, p, and q are unknown constants, may be represented by a straight line, they each need to be expressed in the form

$$Y = mX + c,$$

where X and Y are each functions of x and/or y, and m and c are constants.

	Y	X	m	c
$y = ab^x$				
$y = Ax^k$				
$px + qy = xy$				

Complete the following table and insert in it an expression for Y, X, m, and c for each case.

10. The function f is defined by

$$f : x \mapsto |x^2 - 8x + 7|$$

for the domain $3 \leqslant x \leqslant 8$.

(a) By first considering the stationary value of the function (4)

$$x \mapsto x^2 - 8x + 7,$$

show that the graph of $y = f(x)$ has a stationary point at $x = 4$ and determine the nature of this stationary point.

(b) Sketch the graph of $y = f(x)$. (2)

(c) Find the range of f. (2)

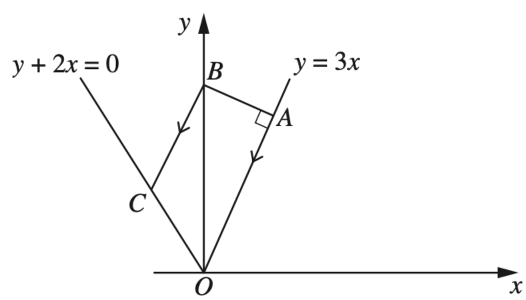
The function g is defined by

$$g : x \mapsto |x^2 - 8x + 7|$$

for the domain $3 \leqslant x \leqslant k$.

(d) Determine the largest value of k for which g^{-1} exists. (1)

11. The diagram shows a trapezium $OABC$, where O is the origin. (10)



The equation of OA is

$$y = 3x$$

and the equation of OC is

$$y + 2x = 0.$$

The line through A perpendicular to OA meets the y-axis at B and BC is parallel to AO.

Given that the length of OA is $\sqrt{250}$ units, calculate the coordinates of A, of B, and of C.

EITHER

12. A particle, travelling in a straight line, passes a fixed point O on the line with a speed of 0.5 ms^{-1}.

The acceleration, a ms^{-2}, of the particle, t s after passing O, is given by

$$a = 1.4 - 0.6t.$$

(a) Show that the particle comes instantaneously to rest when $t = 5$. (4)

(b) Find the total distance travelled by the particle between $t = 0$ and $t = 10$. (6)

OR

13. Each member of a set of curves has an equation of the form

$$y = ax + \frac{b}{x^2},$$

where a and b are integers.

(a) For the curve where $a = 3$ and $b = 2$, find the area bounded by the curve, the x-axis, and the lines $x = 2$ and $x = 4$. (4)

Another curve of this set has a stationary point at $(2, 3)$.

(b) Find the value of a and of b in this case and determine the nature of the stationary point. (6)