Dr Oliver Mathematics
Mathematics: National Qualifications N5
2019 Paper 2: Calculator
1 hour 50 minutes

The total number of marks available is 60.
You must write down all the stages in your working.

1. A charity distributed 80 000 emergency packages during 2018. (3)
   This number is expected to increase by 15% each year.
   Calculate how many emergency packages the charity expects to distribute in 2021.

Solution

$$\begin{aligned}\text{Emergency packages} &= 80\,000 \times (1.15)^3 \\ &= \underline{\underline{121\,670}}.\end{aligned}$$

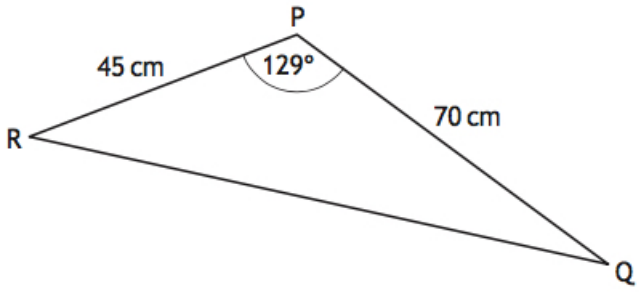
2. Find $|\mathbf{p}|$, the magnitude of vector (2)

$$\mathbf{p} = \begin{pmatrix} 6 \\ 27 \\ -18 \end{pmatrix}.$$

Solution

$$\begin{aligned}|\mathbf{p}| &= \sqrt{6^2 + 27^2 + (-18)^2} \\ &= \sqrt{36 + 729 + 324} \\ &= \sqrt{1\,089} \\ &= \underline{\underline{33}}.\end{aligned}$$

3. The diagram shows triangle PQR. (2)



- $PR = 45$ centimetres,
- $PQ = 70$ centimetres, and
- angle $QPR = 129°$.

Calculate the area of triangle PQR.

Solution

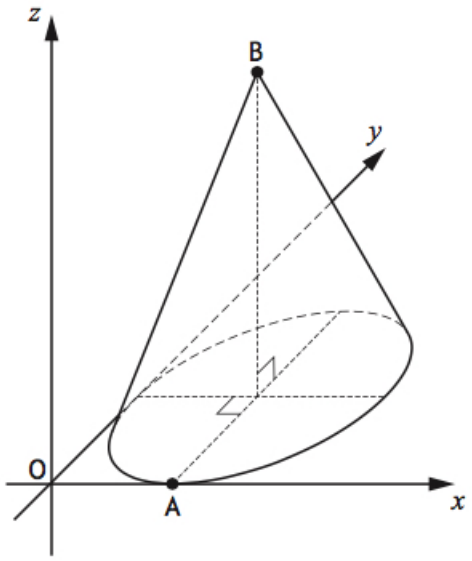
$$\begin{aligned}\text{Area} &= \tfrac{1}{2} \times 45 \times 70 \times \sin 129° \\ &= 1\,224.004\,889 \text{ (FCD)} \\ &= \underline{\underline{1\,220 \text{ cm}^2 \text{ (3 sf)}}}.\end{aligned}$$

4. A sesame seed weighs 3.6×10^{-6} kilograms. (2)
The weight of a poppy seed is 8% of the weight of a sesame seed.
Calculate the weight of a poppy seed in kilograms.
Give your answer in scientific notation.

Solution

$$\begin{aligned}\text{Weight} &= \tfrac{8}{100} \times (3.6 \times 10^{-6}) \\ &= \underline{\underline{2.88 \times 10^{-7} \text{ kg}}}.\end{aligned}$$

5. The diagram shows a cone with diameter 6 units and height 8 units. (2)



- The x-axis and the y-axis are tangents to the base.
- A is the point of contact between the base and the x-axis.
- B is directly above the centre of the base.

Write down the coordinates of A and B.

Solution

$\underline{\underline{A(3,0,0)}}$ and $\underline{\underline{B(3,3,8)}}$

6. Solve the equation (3)

$$3x^2 + 9x - 2 = 0.$$

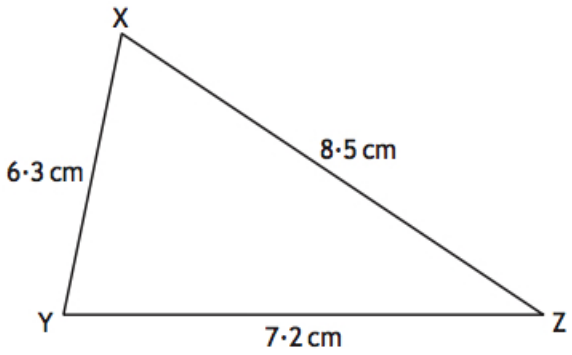
Give your answers correct to 1 decimal place.

Solution

$a = 3$, $b = 9$, and $c = -2$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-9 \pm \sqrt{9^2 - 4 \times 3 \times (-2)}}{2 \times 3} \\ &= \frac{-9 \pm \sqrt{105}}{6} \\ &= -3.207\,825\,128,\ 0.207\,825\,127\,7 \text{ (FCD)} \\ &= \underline{\underline{-3.2,\ 0.2 \text{ (1 dp)}}}. \end{aligned}$$

7. Triangle XYZ is shown below. (3)



Calculate the size of the smallest angle in triangle XYZ.

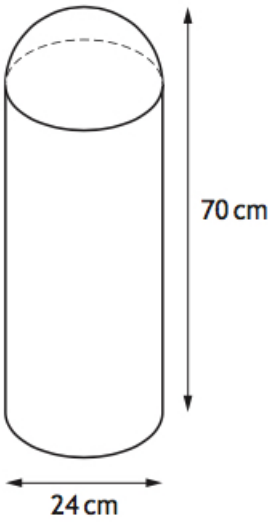
Solution

The smallest angle is opposite the smallest side:

$$\begin{aligned} \cos XZY = \frac{8.5^2 + 7.2^2 - 6.3^2}{2 \times 8.5 \times 7.2} &\Rightarrow \cos XZY = \tfrac{211}{306} \\ &\Rightarrow \angle XZY = 46.406\,096\,61 \text{ (FCD)} \\ &\Rightarrow \angle XZY = \underline{\underline{46.4^\circ \text{ (3 sf)}}}. \end{aligned}$$

8. A traffic bollard is in the shape of a cylinder with a hemisphere on top. (5)
The bollard has

- diameter 24 centimetres and
- height 70 centimetres.



Calculate the volume of the bollard.
Give your answer correct to 3 significant figures.

Solution

The radius of traffic bollard is

$$\frac{24}{2} = 12 \text{ cm}.$$

Well,

$$\begin{aligned}
\text{volume} &= (\tfrac{2}{3} \times \pi \times 12^3) + [\pi \times 12^2 \times (70 - 12)] \\
&= 1\,152\pi + 8\,352\pi \\
&= 9\,504\pi \\
&= 29\,857.696\,58 \text{ (FCD)} \\
&= \underline{\underline{29\,900 \text{ cm}^3 \text{ (3 sf)}}}.
\end{aligned}$$

9. Georgie had her roof repaired. (3)
She was charged an extra 2.5% for late payment.
She had to pay a total of £977.85.
Calculate how much she would have **saved** if she had paid on time.

Solution

$$977.85 = \text{old cost} \times 1.025 \Rightarrow \text{old cost} = \frac{977.85}{1.025}$$
$$= 954$$

and the amount she would have saved is

$$977.85 - 954 = \underline{\underline{£23.85}}.$$

10. Express (2)

$$x^2 + 10x - 15$$

in the form

$$(x + p)^2 + q.$$

Solution

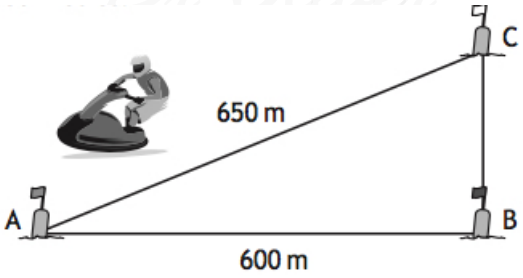
$$x^2 + 10x - 15 = (x^2 + 10x + 25) - 15 - 25$$
$$= \underline{\underline{(x + 5)^2 - 40}};$$

hence,

$$\underline{\underline{p = 5 \text{ and } q = -40}}.$$

11. The diagram shows the course for a jet-ski race. (4)
The course is indicated by markers A, B, and C.
The total length of the course is 1500 metres.

- B is 600 metres from A,
- C is 650 metres from A, and
- C is due north of B



Determine whether B is due east of A.
Justify your answer.

Solution

Suppose $\triangle ABC$ **was** right-angled. Then,

$$\begin{aligned} BC &= \sqrt{650^2 - 600^2} \\ &= \sqrt{422\,500 - 360\,000} \\ &= \sqrt{62\,500} \\ &= 250; \end{aligned}$$

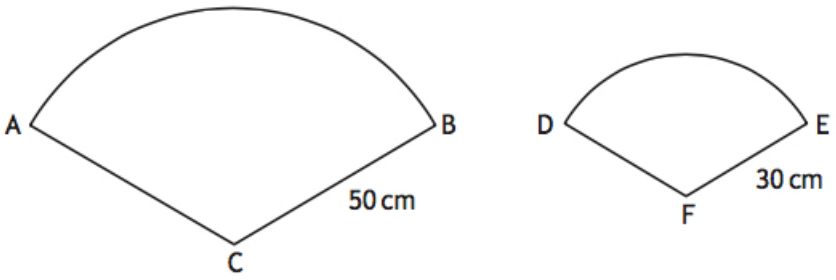
hence, the length is

$$650 + 600 + 250 = 1\,500 \text{ m}$$

and B is due east of A.

12. In the diagram,

- ABC is a sector of a circle, centre C and
- DEF is a sector of a circle, centre F.



The sectors are mathematically similar.
The area of the larger sector, ABC, is 2 750 square centimetres.

(a) Calculate the area of the smaller sector, DEF. (3)

Solution

The length scale factor (LSF) is

$$\frac{30}{50} = \tfrac{3}{5}$$

and the area scale factor (ASF) is

$$(\tfrac{3}{5})^2 = \tfrac{9}{25}.$$

Finally,

$$\begin{aligned} \text{area of } DEF &= \tfrac{9}{25} \times 2\,750 \\ &= \underline{\underline{990 \text{ square centimetres}}}. \end{aligned}$$

(b) Calculate the size of angle ACB. (3)

Solution

$$\begin{aligned} 2\,750 = \frac{\angle ACB}{360} \times \pi \times 50^2 &\Rightarrow \angle ACB = \frac{2\,750 \times 360}{\pi \times 50^2} \\ &\Rightarrow \angle ACB = 126.050\,714\,9 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{\angle ACB = 126^\circ \text{ (3 sf)}}}. \end{aligned}$$

13. Find an expression for the gradient of the line joining point $A(6, 9)$ to point $B(4p, 4p^2)$. Give your answer in its simplest form. (3)

Solution

$$\text{Gradient} = \frac{4p^2 - 9}{4p - 6}$$

add to: 0
multiply to: $(+4) \times (-9) = -36$ $\Big\}$ $-6,\ +6$

$$\begin{aligned} &= \frac{4p^2 + 6p - 6p - 9}{4p - 6} \\ &= \frac{2p(2p+3) - 9(2p+3)}{4p - 6} \\ &= \frac{(2p+3)(2p-3)}{2(2p-3)} \\ &= \underline{\underline{\frac{2p+3}{2}}}. \end{aligned}$$

14. Solve the equation (3)

$$5\cos x^\circ + 2 = 1,\ 0 \leqslant x < 360.$$

Solution

$$\begin{aligned} 5\cos x^\circ + 2 = 1 &\Rightarrow 5\cos x^\circ = -1 \\ &\Rightarrow \cos x^\circ = -\tfrac{1}{5} \\ &\Rightarrow x = 101.536\,959,\ 258.463\,041 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x = 102,\ 258 \text{ (3 sf)}}}. \end{aligned}$$

15. Express (3)

$$\frac{4}{x-2} - \frac{3}{x+5},\ x \neq 2,\ x \neq -5$$

as a single fraction in its simplest form.

Solution

$$\frac{4}{x-2} - \frac{3}{x+5} = \frac{4(x+5)-3(x-2)}{(x-2)(x+5)}$$
$$= \frac{4x+20-3x+6}{(x-2)(x+5)}$$
$$= \underline{\underline{\frac{x+26}{(x-2)(x+5)}}}.$$

16. Simplify (3)

$$\frac{a^4 \times 3a}{\sqrt{a}}.$$

Solution

$$\frac{a^4 \times 3a}{\sqrt{a}} = \frac{3a^5}{a^{\frac{1}{2}}}$$
$$= \underline{\underline{3a^{\frac{9}{2}}}}.$$

17. Expand and simplify (2)

$$(\sin x^\circ + \cos x^\circ)^2.$$

Show your working.

Solution

$\times$	$\sin x^\circ$	$+\cos x^\circ$
$\sin x^\circ$	$\sin^2 x^\circ$	$+\sin x^\circ \cos x^\circ$
$+\cos x^\circ$	$+\sin x^\circ \cos x^\circ$	$+\cos^2 x^\circ$

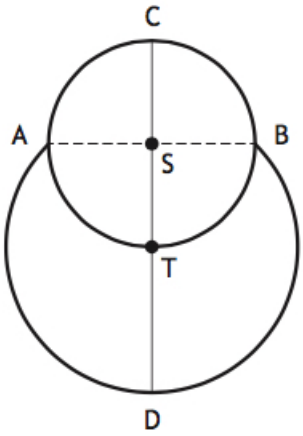
$$(\sin x^\circ + \cos x^\circ)^2 = \sin^2 x^\circ + 2\sin x^\circ \cos x^\circ + \cos^2 x^\circ$$
$$= (\sin^2 x^\circ + \cos^2 x^\circ) + 2\sin x^\circ \cos x^\circ$$
$$= \underline{\underline{1 + 2\sin x^\circ \cos x^\circ}},$$

as

$$\sin^2 x^\circ + \cos^2 x^\circ = 1.$$

18. The picture shows a cartoon snowman. (4)

The diagram below represents the snowman.

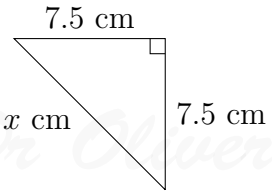


- The head is a small circle, centre S, with diameter 15 centimetres.
- The body is part of a larger circle, centre T.
- The point T lies on the circumference of the small circle.
- The points A and B lie on the circumferences of both circles

Calculate CD, the height of the snowman.

Solution

Let x cm be the radius of DT.



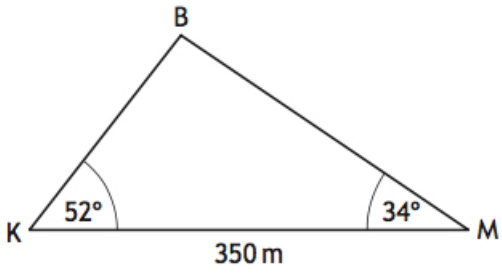
Now,

$$\begin{aligned} x &= \sqrt{7.5^2 + 7.5^2} \\ &= 7.5\sqrt{2} \end{aligned}$$

and

$$\begin{aligned} CD &= DT + TS + SC \\ &= 7.5\sqrt{2} + 7.5 + 7.5 \\ &= 7.5\sqrt{2} + 15 \\ &= 25.606\,601\,72 \text{ (FCD)} \\ &= \underline{\underline{25.6 \text{ cm (3 sf)}}}. \end{aligned}$$

19. Katy and Mona are looking up at a hot-air balloon. (5)
In the diagram below, K, M, and B represent the positions of Katy, Mona, and the balloon respectively.



- The angle of elevation of the balloon from Katy is 52°.
- The angle of elevation of the balloon from Mona is 34°.
- Katy and Mona are 350 metres apart on level ground.

Calculate the height of the hot-air balloon above the ground.

Solution

$$\begin{aligned}\angle KBM &= 180 - (52 + 34)\\ &= 180 - 86\\ &= 94^\circ\end{aligned}$$

and

$$\begin{aligned}\frac{KB}{\sin KMB} = \frac{KM}{\sin KBM} &\Rightarrow \frac{KB}{\sin 34^\circ} = \frac{350}{\sin 94^\circ}\\ &\Rightarrow KB = \frac{350 \sin 34^\circ}{\sin 94^\circ}\\ &\Rightarrow KB = 196.195\,438\,4 \text{ (FCD)}.\end{aligned}$$

Now, let BX vertical, i.e., KBX is a right-angled triangle with $\angle KXB = 90^\circ$. Next,

$$\begin{aligned}\sin 52^\circ = \frac{BX}{196.195\ldots} &\Rightarrow BX = \sin 52^\circ \times 196.195\ldots\\ &\Rightarrow BX = 154.604\,115\,3 \text{ (FCD)}\\ &\Rightarrow \underline{\underline{BX = 155 \text{ m (3 sf)}}}.\end{aligned}$$