Dr Oliver Mathematics
GCSE Mathematics
2003 June Paper 6H: Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. The diagram shows a cylinder with a height of 10 cm and a radius of 4 cm.

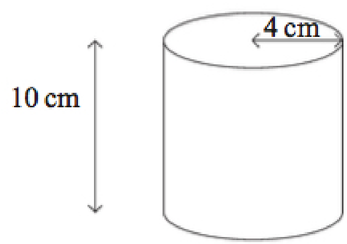


Diagram **NOT** accurately drawn

(a) Calculate the volume of the cylinder. (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}\text{Volume} &= \pi \times 4^2 \times 10 \\ &= 502.654\,824\,6 \text{ (FCD)} \\ &= \underline{\underline{503 \text{ cm}^3 \text{ (3 sf)}}}.\end{aligned}$$

The length of a pencil is 13 cm.
The pencil cannot be broken.

(b) Show that this pencil cannot fit inside the cylinder. (3)

Solution

$$\begin{aligned}\text{Length} &= \sqrt{8^2 + 10^2} \\ &= 12.806\,248\,47 \text{ (FCD)};\end{aligned}$$

hence, <u>this pencil cannot fit inside the cylinder</u> because it is longer the hypontuse.

2. (a) Express the following numbers as products of their prime factors. (4)

(i) 60,

Solution

$$\begin{array}{r|l} & 60 \\ \hline 2 & 30 \\ \hline 2 & 15 \\ \hline 3 & 5 \\ \hline 5 & 1 \\ \hline \end{array}$$

Hence

$$\begin{aligned} 60 &= 2 \times 2 \times 3 \times 5 \\ &= \underline{\underline{2^2 \times 3 \times 5}}. \end{aligned}$$

(ii) 96.

Solution

$$\begin{array}{r|l} & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 2 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

Hence

$$\begin{aligned} 96 &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \\ &= \underline{\underline{2^5 \times 3}}. \end{aligned}$$

(b) Find the Highest Common Factor of 60 and 96. (1)

Solution

$$\text{HCF}(60, 96) = 2^2 \times 3 = \underline{\underline{12}}.$$

(c) Work out the Lowest Common Multiple of 60 and 96. (2)

Solution

$$\text{LCM}(60, 96) = 2^5 \times 3 \times 5 = \underline{\underline{480}}.$$

3. A garage keeps records of the costs of repairs to its customers' cars.
The table gives information about the costs of all repairs which were less than £250 in one week.

Cost (£C)	Frequency
$0 < C \leqslant 50$	4
$50 < C \leqslant 100$	8
$100 < C \leqslant 150$	7
$150 < C \leqslant 200$	10
$200 < C \leqslant 250$	11

(a) Find the class interval in which the median lies. (2)

Solution

Cost (£C)	Frequency	Cost (£C)	Cum. Freq.
$0 < C \leqslant 50$	4	$0 < C \leqslant 50$	4
$50 < C \leqslant 100$	8	$0 < C \leqslant 100$	$4 + 8 = 12$
$100 < C \leqslant 150$	7	$0 < C \leqslant 150$	$12 + 7 = 19$
$150 < C \leqslant 200$	10	$0 < C \leqslant 200$	$19 + 10 = 29$
$200 < C \leqslant 250$	11	$0 < C \leqslant 250$	$29 + 11 = 40$

The median is in

$$\frac{40 + 1}{2} = 20\tfrac{1}{2} \text{ position}$$

which makes the median $\underline{\underline{150 < C \leqslant 200}}$.

There was only one further repair that week, not included in the table.
That repair cost £1000.
Dave says, "The class interval in which the median lies will change."

(b) Is Dave correct?
Explain your answer. (1)

Solution

The median is in

$$\frac{41+1}{2} = 21 \text{ position}$$

which makes the median $150 < C \leqslant 200$; no, Dave is incorrect.

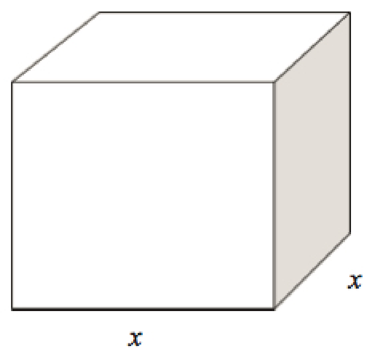
The garage also sells cars.
It offers a discount of 20% off the normal price for cash.
Dave pays £5 200 cash for a car.

(c) Calculate the normal price of the car. (3)

Solution

$$\frac{5\,200}{0.8} = \text{£}6\,500.$$

4. A cuboid has a square base of side x cm.



The height of the cuboid is 1 cm more than the length x cm.
The volume of the cuboid is 230 cm^3.

(a) Show that (2)

$$x^3 + x^2 = 230.$$

Solution

$$x \times x \times (x+1) = 230 \Rightarrow x^3 + x^2 = 230,$$

as required.

The equation

$$x^3 + x^2 = 230$$

has a solution between $x = 5$ and $x = 6$.

(b) Use a trial and improvement method to find this solution. (4)
Give your answer correct to 1 decimal place.
You must show all your working.

Solution
You must be in TABLE mode; on my calculator (Casio fx-991) it is Mode 3.
F(X)= and you type in $X^3 + X^2$; then you press [=].
Start? and you enter 5; then you press [=].
End? and you enter 6; then you press [=].
Step? and enter 0.05 – 1 decimal place divided by 2; then you press [=].

x	f(x)	Comment
5.8	228.75	too low
5.85	234.42	too high

Clearly,

$$5.8 < x < 5.85$$

and the answer is

$$\underline{\underline{x = 5.8 \text{ (1 dp)}}}.$$

5. The diagram shows a semi-circle. (3)
The diameter of the semi-circle is 15 cm.

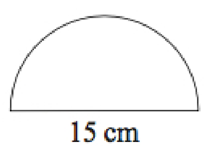


Diagram **NOT** accurately drawn

Calculate the area of the semi-circle.
Give your answer correct to 3 significant figures.

Solution

The radius is 7.5 cm and the area of the semi-circle is

$$\begin{aligned}\tfrac{1}{2} \times \pi \times 7.5^2 &= 88.357\,293\,38 \text{ (FCD)} \\ &= \underline{\underline{88.4 \text{ cm (3 sf)}}}.\end{aligned}$$

6. A straight line has equation

$$y = \tfrac{1}{2}x + 1.$$

The point P lies on the straight line.
P has a y-coordinate of 5.

(a) Find the x-coordinate of P. (2)

Solution

$$\begin{aligned}\tfrac{1}{2}x + 1 = 5 &\Rightarrow \tfrac{1}{2}x = 4 \\ &\Rightarrow \underline{\underline{x = 8}}.\end{aligned}$$

(b) Write down the equation of a different straight line that is parallel to $y = \frac{1}{2}x + 1$. (1)

Solution
E.g., $\underline{\underline{y = \frac{1}{2}x + 2}}$, $\underline{\underline{y = \frac{1}{2}x - 1}}$.

(c) Rearrange $y = \frac{1}{2}x + 1$ to make x the subject. (2)

Solution

$$\begin{aligned}y = \tfrac{1}{2}x + 1 &\Rightarrow y - 1 = \tfrac{1}{2}x \\ &\Rightarrow \underline{\underline{x = 2(y - 1)}}.\end{aligned}$$

7. Solve (4)

$$\begin{aligned}2x - 3y &= 11 \\ 5x + 2y &= 18.\end{aligned}$$

Solution

$$2x - 3y = 11 \quad (1)$$
$$5x + 2y = 18 \quad (2)$$

$$2 \times (1): \quad 4x - 6y = 22 \quad (3)$$
$$3 \times (2): \quad 15x + 6y = 54 \quad (4)$$

(3) + (4):

$$19x = 78 \Rightarrow \underline{\underline{x = 4}}$$
$$\Rightarrow 20 + 2y = 18$$
$$\Rightarrow 2y = -2$$
$$\Rightarrow \underline{\underline{y = -1}}.$$

8. BE is parallel to CD.

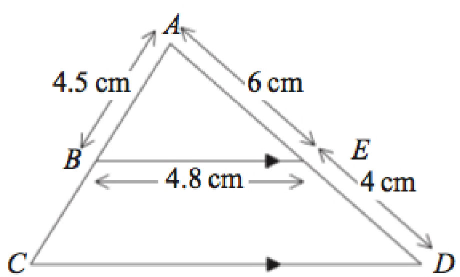


Diagram **NOT** accurately drawn

$AE = 6$ cm.
$ED = 4$ cm.
$AB = 4.5$ cm.
$BE = 4.8$ cm.

(a) Calculate the length of CD. (2)

Solution

$$\begin{aligned} CD &= \left(\frac{4+6}{6}\right) \times 4.8 \\ &= \frac{5}{3} \times 4.8 \\ &= \underline{\underline{8 \text{ cm}}}. \end{aligned}$$

(b) Calculate the perimeter of the trapezium $EBCD$. (2)

Solution

$$\begin{aligned} BC &= \left(\frac{4}{6}\right) \times 4.5 \\ &= 3 \end{aligned}$$

and the perimeter of the trapezium $EBCD$ is

$$3 + 4.8 + 4 + 8 = \underline{\underline{19.8 \text{ cm}}}.$$

9. (3)

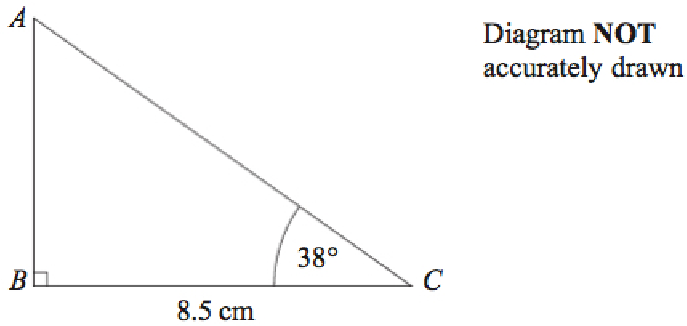
$$y^2 = \frac{ab}{a+b}.$$

$a = 3 \times 10^8$.
$b = 2 \times 10^7$.
Find y.
Give your answer in standard form correct to 2 significant figures.

Solution

$$\begin{aligned} y^2 = \frac{(3 \times 10^8) \times (2 \times 10^7)}{(3 \times 10^8) + (2 \times 10^7)} &\Rightarrow y^2 = \frac{6 \times 10^{15}}{3.2 \times 10^8} \\ &\Rightarrow y = 4330.127\,019 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{y = 4.3 \times 10^3 \text{ (2 sf)}}}. \end{aligned}$$

10. The diagram shows triangle ABC. (3)



$BC = 8.5$ cm.
Angle $ABC = 90°$.
Angle $ACB = 38°$.
Work out the length of AB.
Give your answer correct to 3 significant figures.

Solution

$$\text{opp} = \text{adj} \times \tan \Rightarrow AB = 8.5 \tan 38°$$
$$\Rightarrow AB = 6.640\,927\,825 \text{ (FCD)}$$
$$\Rightarrow \underline{\underline{AB = 6.64 \text{ cm (3 sf)}}}.$$

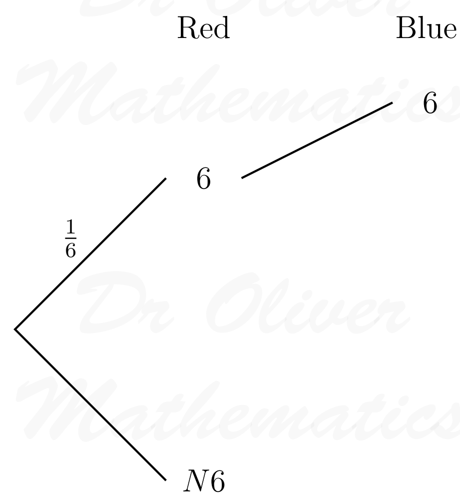
11. Julie does a statistical experiment.
She throws a dice 600 times.
She scores six 200 times.

(a) Is the dice fair? (1)
Explain your answer.

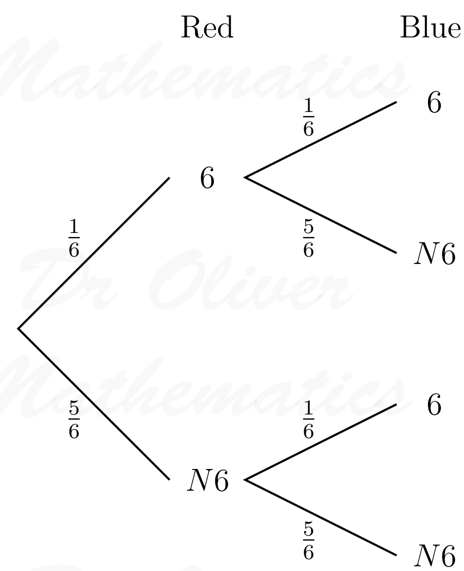
Solution
She gets twice the number of sixes; as she rolls the dice 600 times, the dice is unfair.

Julie then throws a fair red dice once and a fair blue dice once.

(b) Complete the probability tree diagram to show the outcomes. (3)
Label clearly the branches of the probability tree diagram.
The probability tree diagram has been started in the space below.



Solution



(c) (i) Julie throws a fair red dice once and a fair blue dice once. (5)
Calculate the probability that Julie gets a six on both the red dice and the blue dice.

Solution

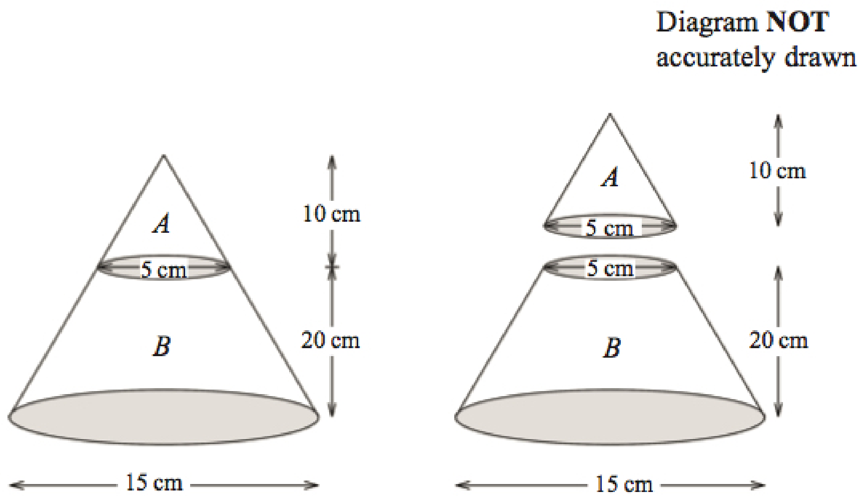
$$\frac{1}{6} \times \frac{1}{6} = \underline{\underline{\frac{1}{36}}}.$$

(ii) Calculate the probability that Julie gets at least one six.

Solution

$$1 - \left(\tfrac{5}{6}\right)^2 = 1 - \tfrac{25}{36} = \underline{\underline{\tfrac{11}{36}}}.$$

12. The diagram represents a large cone of height 30 cm and base diameter 15 cm.



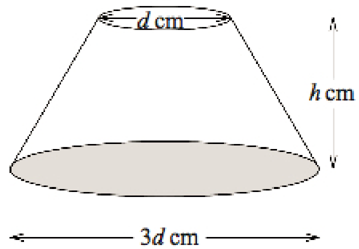
The large cone is made by placing a small cone A of height 10 cm and base diameter 5 cm on top of a frustum B.

(a) Calculate the volume of the frustum B. (3)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned}
\text{Volume} &= \tfrac{1}{3} \times \pi \times 7.5^2 \times 30 - \tfrac{1}{3} \times \pi \times 2.5^2 \times 10 \\
&= 562\tfrac{1}{2}\pi - 20\tfrac{5}{6}\pi \\
&= 541\tfrac{2}{3}\pi \\
&= 1\,701.696\,021 \text{ (FCD)} \\
&= \underline{\underline{1\,700 \text{ cm}^3 \text{ (3 sf)}}}.
\end{aligned}$$

The diagram shows a frustum.



The diameter of the base is $3d$ cm and the diameter of the top is d cm.
The height of the frustum is h cm.
The formula for the curved surface area, S cm^2, of the frustum is

$$S = 2\pi d\sqrt{h^2 + d^2}.$$

(b) Rearrange the formula to make h the subject. (3)

Solution

$$\begin{aligned} S = 2\pi d\sqrt{h^2 + d^2} &\Rightarrow \frac{S}{2\pi d} = \sqrt{h^2 + d^2} \\ &\Rightarrow \left(\frac{S}{2\pi d}\right)^2 = h^2 + d^2 \\ &\Rightarrow \frac{S^2}{(2\pi d)^2} - d^2 = h^2 \\ &\Rightarrow \frac{S^2 - 4\pi^2 d^4}{(2\pi d)^2} = h^2 \\ &\Rightarrow h = \frac{\sqrt{S^2 - 4\pi^2 d^4}}{2\pi d}. \end{aligned}$$

Two mathematically similar frustums have heights of 20 cm and 30 cm.
The surface area of the smaller frustum is 450 cm^2.

(c) Calculate the surface area of the larger frustum. (2)

Solution

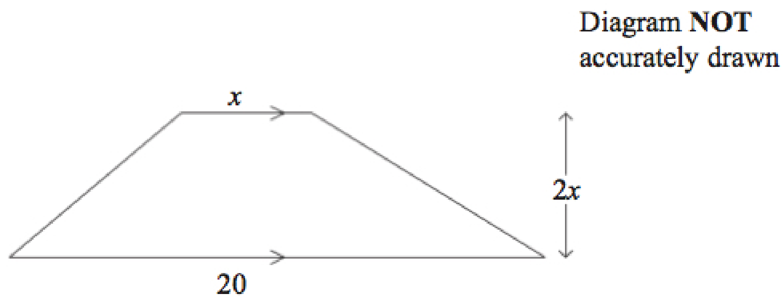
The length scale ratio (LSR) is $\frac{3}{2}$ which makes the area scale ratio (ASR) is

$$(\tfrac{3}{2})^2 = \tfrac{9}{4}.$$

Finally, the surface area of the larger frustum is

$$450 \times \tfrac{9}{4} = \underline{\underline{1\,012.5 \text{ cm}^2}}.$$

13. The diagram shows a trapezium.



The measurements on the diagram are in centimetres.
The lengths of the parallel sides are x cm and 20 cm.
The height of the trapezium is $2x$ cm.
The area of the trapezium is 400 cm^2.

(a) Show that (2)

$$x^2 + 20x = 400.$$

Solution

$$\tfrac{1}{2} \times 2x \times (x + 20) = 400 \Rightarrow x(x + 20) = 400$$
$$\Rightarrow \underline{\underline{x^2 + 20x = 400}},$$

as required.

(b) Find the value of x. (3)
Give your answer correct to 3 decimal places.

Solution

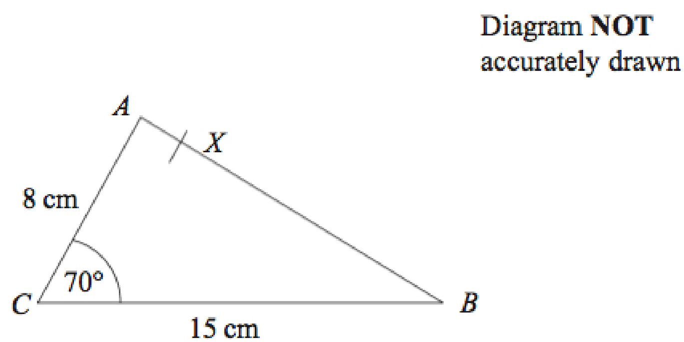
$$x^2 + 20x = 400 \Rightarrow x^2 + 20x - 400 = 0.$$

$a = 1$, $b = 20$, and $c = -400$:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-20 \pm \sqrt{20^2 - 4 \times 1 \times (-400)}}{2} \\ &= \frac{-20 \pm \sqrt{2\,000}}{2} \\ &= -32.360\,679\,77 \text{ or } 12.360\,679\,77 \text{ (FCD)} \\ &= \underline{\underline{12.361 \text{ cm (3 dp)}}} \end{aligned}$$

as $x > 0$.

14. In triangleABC, $AC = 8$ cm, $CB = 15$ cm, and angle $ACB = 70°$.



(a) Calculate the area of triangle ABC. (2)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} \text{Area} &= \tfrac{1}{2} \times 8 \times 15 \times \sin 70° \\ &= 56.381\,557\,25 \text{ (FCD)} \\ &= \underline{\underline{56.4 \text{ cm}^2 \text{ (3 sf)}}}. \end{aligned}$$

X is the point on AB such that angle $CXB = 90°$.

(b) Calculate the length of CX. (4)
Give your answer correct to 3 significant figures.

Solution

$$\begin{aligned} & c^2 = a^2 + b^2 - 2ab\cos C \\ \Rightarrow\ & c^2 = 8^2 + 15^2 - 2 \times 8 \times 15 \times \cos 70^\circ \\ \Rightarrow\ & c = 14.384\,546\,07 \text{ (FCD)}. \end{aligned}$$

Now,

$$\begin{aligned} & \tfrac{1}{2} \times CX \times 14.384\ldots = 56.381\ldots \\ \Rightarrow\ & CX = 7.839\,184\,772 \text{ (FCD)} \\ \Rightarrow\ & \underline{\underline{CX = 7.84 \text{ cm (3 sf)}}}. \end{aligned}$$

15. (a) Show that (3)

$$(2a-1)^2 - (2b-1)^2 = 4(a-b)(a+b-1).$$

Solution

$$\begin{aligned} (2a-1)^2 - (2b-1)^2 &= (4a^2 - 4a + 1) - (4b^2 - 4b + 1) \\ &= 4a^2 - 4a - 4b^2 + 4b \\ &= 4(a^2 - a - b^2 + b) \\ &= 4(a^2 - b^2 - a + b) \\ &= 4[(a+b)(a-b) - (a-b)] \\ &= \underline{\underline{4(a-b)(a+b-1)}}, \end{aligned}$$

as required.

(b) Prove that the difference between the squares of any two odd numbers is a multiple of 8. (3)
(You may assume that any odd number can be written in the form $2r-1$, where r is an integer).

Solution
Let a and $b \in \mathbb{Z}$.

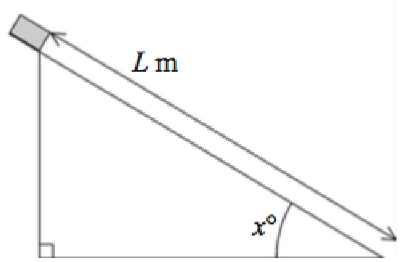
Both even:
If a and b are both even, then $a-b$ is even, so $4(a-b)$ is a multiple of 8.

Both odd:
If a and b are both odd, then $a - b$ is even, so $4(a - b)$ is a multiple of 8.

Both odd:
If there is a mixture of parites, then $a+b-1$ is even, so $4(a+b-1)$ is a multiple of 8.

Hence, the difference between the squares of any two odd numbers is a multiple of 8.

16. Elliot did an experiment to find the value of g m/s^2, the acceleration due to gravity.



He measured the time, T seconds, that a block took to slide L m down a smooth slope of angle x°.
He then used the formula

$$g = \frac{2L}{T^2 \sin x^\circ}$$

to calculate an estimate for g.
$T = 1.3$, correct to 1 decimal place.
$L = 4.5$, correct to 2 decimal places.
$x = 30$, correct to the nearest integer.

(a) Calculate the lower bound and the upper bound for the value of g. (4)
Give your answers correct to 3 decimal places.

Solution

$$1.25 \leqslant T < 1.35,$$

$$4.495 \leqslant L < 4.505,$$

and

$$29.5 \leqslant x < 30.5.$$

Finally,

$$\frac{2 \times 4.495}{1.35^2 \sin 30.5^\circ} < g < \frac{2 \times 4.505}{1.25^2 \sin 29.5^\circ}$$
$$\Rightarrow \quad 9.719\,038\,001 < g < 11.710\,243\,92 \text{ (FCD)}$$
$$\Rightarrow \quad \underline{\underline{9.719 < g < 11.710 \text{ (3 dp)}}}.$$

(b) Use your answers to part (a) to write down the value of g to a suitable degree of accuracy. (1)
Explain your reasoning.

Solution

Degree of accuracy	Lower bound	Upper bound
1 sig fig	10	10
2 sig fig	9.7	12

The value of g is $\underline{\underline{10}}$.

17.

$$x = 2^p \text{ and } y = 2^q.$$

(a) Express in terms of x and/or y, (3)

(i) 2^{p+q},

Solution

$$2^{p+q} = 2^p \times 2^q = \underline{\underline{xy}}.$$

(ii) 2^{2q},

Solution

$$2^{2q} = (2^q)^2 = \underline{\underline{y^2}}.$$

(iii) 2^{p-1}.

Solution

$$2^{p-1} = \tfrac{1}{2} \times 2^p = \underline{\underline{\tfrac{1}{2}x}}.$$

$$xy = 32$$
$$2xy^2 = 32.$$

(b) Find the value of p and the value of q. (2)

Solution

Divide the second one by the first:

$$\begin{aligned} 2y = 1 &\Rightarrow y = \tfrac{1}{2} \\ &\Rightarrow \underline{\underline{p = -1}} \\ &\Rightarrow x = 64 \\ &\Rightarrow \underline{\underline{q = 6}} \end{aligned}$$

18. For all values of x and m,

$$x^2 - 2mx = (x - m)^2 - k.$$

(a) Express k in terms of m. (2)

Solution

$$\begin{aligned} (x - m)^2 - k &= (x^2 - 2mx + m^2) - k \\ &= x^2 - 2mx + (m^2 - k); \end{aligned}$$

hence, $\underline{\underline{k = m^2}}$.

The expression $x^2 - 2mx$ has a minimum value as x varies.

(b) (i) Find the minimum value of $x^2 - 2mx$. (3)
Give your answer in terms of m.

Solution

The minimum value is $\underline{\underline{-m^2}}$.

(ii) State the value of x for which this minimum value occurs.
Give your answer in terms of m.

Solution

This minimum value occurs when $\underline{\underline{x = m}}$.

19. The probability that Betty will be late for school tomorrow is 0.05. (2)
The probability that Colin will be late for school tomorrow is 0.06.
The probability that both Betty and Colin will be late for school tomorrow is 0.011.
Fred says that the events "Betty will be late tomorrow" and "Colin will be late tomorrow" are independent.
Justify whether Fred is correct or not.

Solution

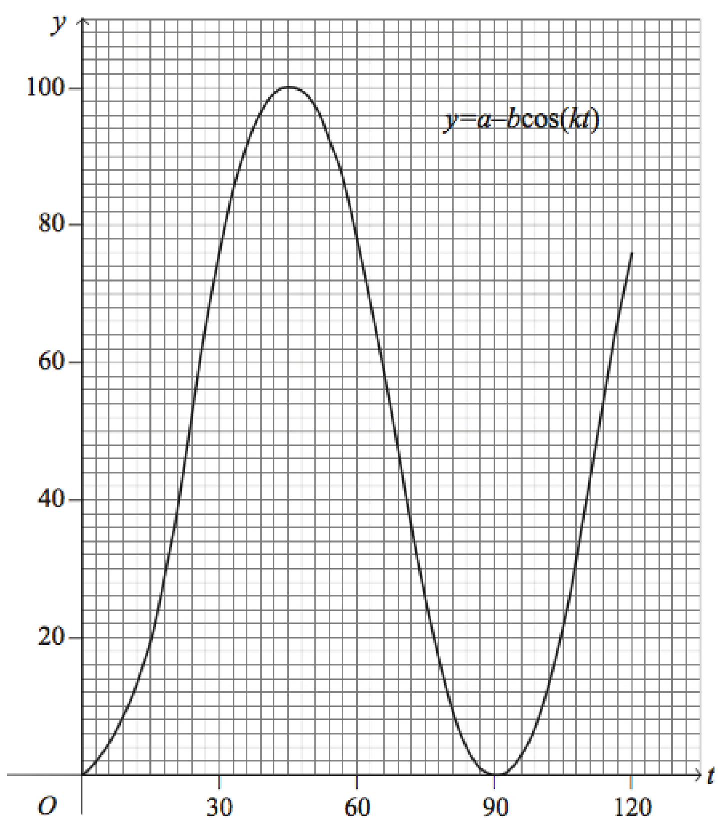
$$\begin{aligned} 0.05 \times 0.06 &= 0.003 \\ &\neq 0.011; \end{aligned}$$

hence, they are not independent.

20. The graph of (3)

$$y = a - b\cos(kt)$$

for values of t between 0° and 120°, is drawn on the grid.



Use the graph to find an estimate for the value of

(a) a,

Solution

$\underline{\underline{a = 50}}$.

(b) b,

Solution

$\underline{\underline{b = 50}}$.

(c) k.

Solution

$\underline{\underline{k = 4}}$.