Dr Oliver Mathematics
GCSE Mathematics
2007 November Paper 5H: Non-Calculator
2 hours

The total number of marks available is 100.
You must write down all the stages in your working.

1. (a) Work out (3)

$$2\tfrac{3}{4} + 3\tfrac{2}{3}.$$

Give your answer as a fraction in its simplest form.

Solution

$$\begin{aligned} 2\tfrac{3}{4} + 3\tfrac{2}{3} &= 5 + \tfrac{9}{12} + \tfrac{8}{12} \\ &= 5 + \tfrac{17}{12} \\ &= \underline{\underline{6\tfrac{5}{12}}}. \end{aligned}$$

(b) (i) Which of these fractions can be written as a recurring decimal? (2)

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5}$$

Solution
$\underline{\underline{\tfrac{1}{3}}}$.

(ii) Explain your answer.

Solution
$\frac{1}{2} = 0.5$, $\frac{1}{4} = 0.25$, and $\frac{1}{5} = 0.2$ but $\frac{1}{3} = 0.\dot{3}$.

2. The cost of hiring a car can be worked out using this rule.

$$\text{Cost} = £90 + 50\text{p per mile}.$$

The cost of hiring a car and driving m miles is C pounds.

(a) Complete the formula for C in terms of m. (2)

Solution

$$\underline{\underline{C = 90 + 0.5m}}.$$

Zara hired a car.
The cost is £240.

(b) How many miles did Zara drive? (3)

Solution

$$\begin{aligned} 90 + 0.5m = 240 &\Rightarrow 0.5m = 150 \\ &\Rightarrow \underline{\underline{m = 300 \text{ miles}}}. \end{aligned}$$

3. (a) Work out the Highest Common Factor (HCF) of 24 and 64. (2)

Solution

$$\begin{array}{r|l} & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 2 & 3 \\ \hline 3 & 1 \\ \hline \end{array}$$

and

$$\begin{array}{r|l} & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline 2 & 1 \\ \hline \end{array}$$

Hence,

$$24 = 2^3 \times 3 \text{ and } 64 = 2^6$$

and

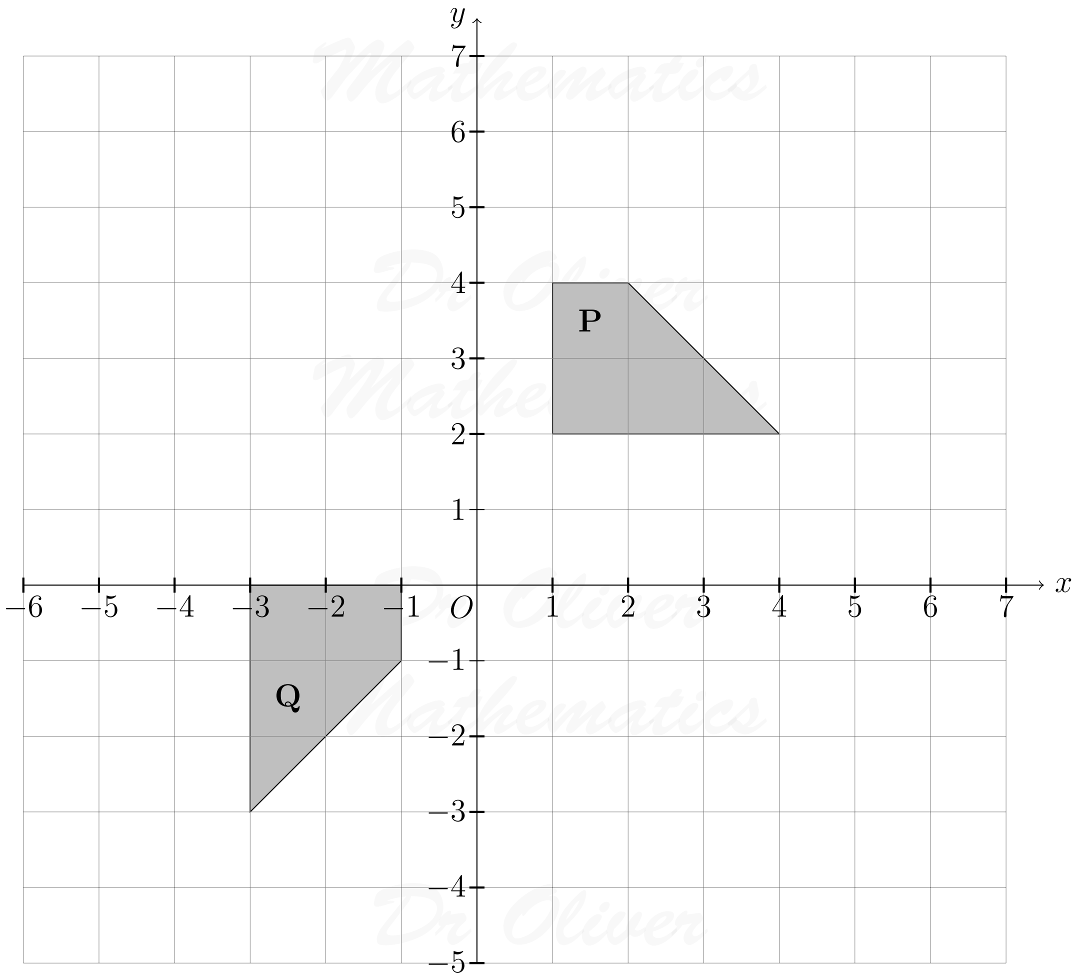
$$\text{HCF} = 2^3 = \underline{\underline{8}}.$$

(b) Work out the Lowest Common Multiple (LCM) of 24 and 64. (2)

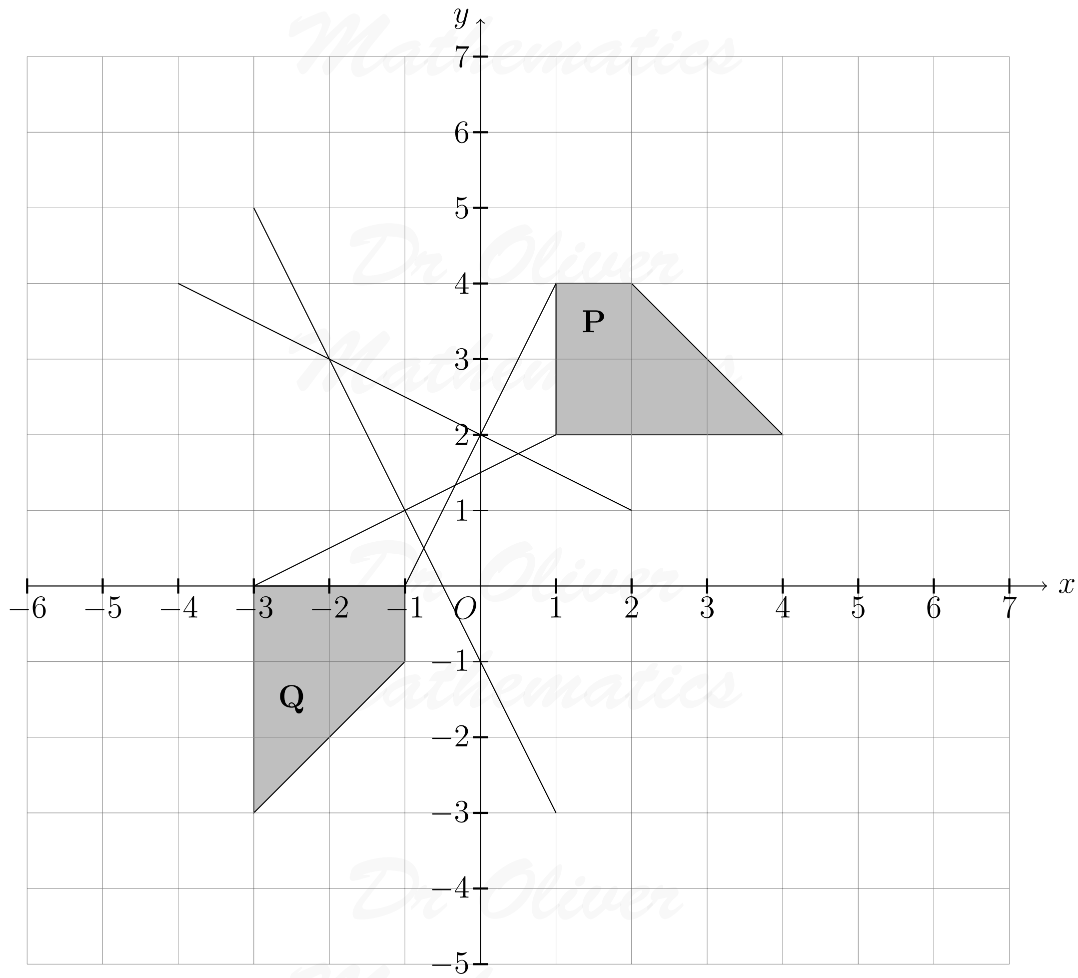
Solution

$$\text{LCM} = 2^6 \times 3 = \underline{\underline{192}}.$$

4. Describe fully the single transformation that will map shape **P** onto shape **Q**. (3)



Solution



It is a $\underline{\underline{\text{rotation}}}$, about $\underline{\underline{(-2, 3)}}$, $\underline{\underline{90^\circ \text{ clockwise}}}$.

5. Lillian, Max, and Nazia share a sum of money in the ratio $2 : 3 : 5$. (3)
Nazia receives £60.
Work out how much money Lillian receives.

Solution

Lillian receives

$$\frac{2}{5} \times 60 = \underline{\underline{£24}}.$$

6. Here are the first four terms of a number sequence.

$$2 \quad 7 \quad 12 \quad 17$$

(a) Work out the 10th term of this number sequence. (2)

Solution
Let the
$$n\text{th term} = an + b.$$
Now,
$$n\text{th term} = 5n - 3$$
and
$$10\text{th term} = 5 \times 10 - 3 = \underline{\underline{47}}.$$

Here are the first five terms of another number sequence.

$$-4 \quad -1 \quad 2 \quad 5 \quad 8$$

(b) (i) Find, in terms of n, an expression for the nth term of this number sequence. (3)

Solution
Let the
$$n\text{th term} = an + b.$$

$$\begin{array}{ccccccc} -4 & & -1 & & 2 & & 8 \\ & 3 & & 3 & & 3 & \end{array}$$

$$\begin{array}{ccccccc} a+b & & 2a+b & & 3a+b & & 4a+b \\ & a & & a & & a & \end{array}$$

We compare terms:
$$a = 3$$
and
$$\begin{aligned} a + b = -4 &\Rightarrow 3 + b = -4 \\ &\Rightarrow b = -7. \end{aligned}$$
Hence,
$$n\text{th term} = \underline{\underline{3n - 7}}.$$

(ii) Find **two** numbers that are in both number sequences.

Solution

First sequence:

$$2 \quad 7 \quad 12 \quad 17 \quad 22 \quad 27 \quad 32 \quad 37 \quad 42$$

Second sequence:

$$-4 \quad -1 \quad 2 \quad 5 \quad 8 \quad 11 \quad 14 \quad 17$$

Hence, e.g.,

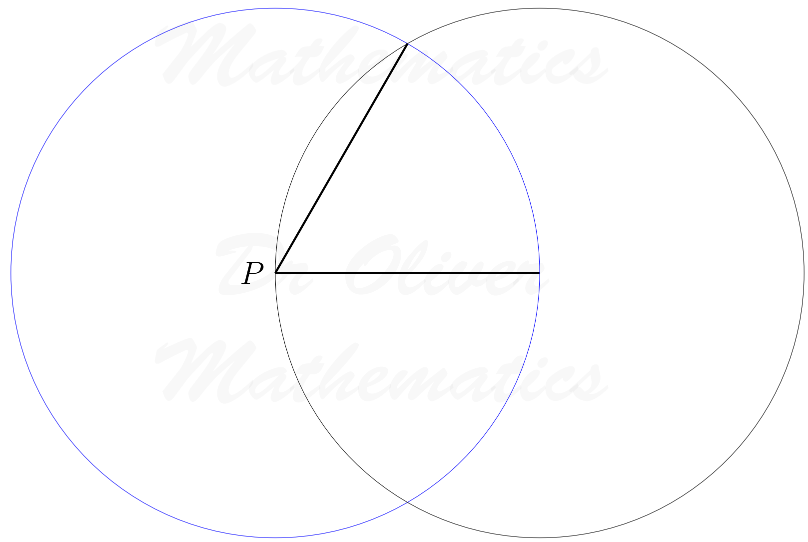
$$\underline{\underline{\text{2 and 17}}}.$$

7. Use ruler and compasses to construct an angle of 30° at P. (3)
You must show all your construction lines.

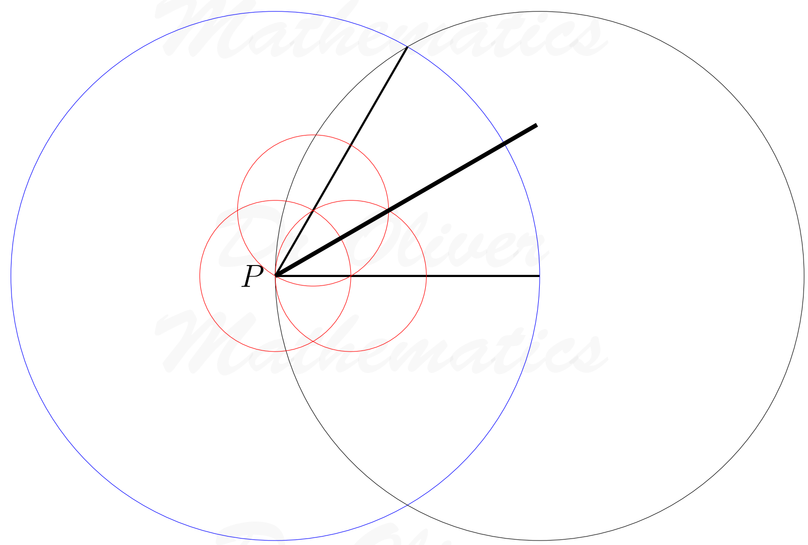
P

Solution

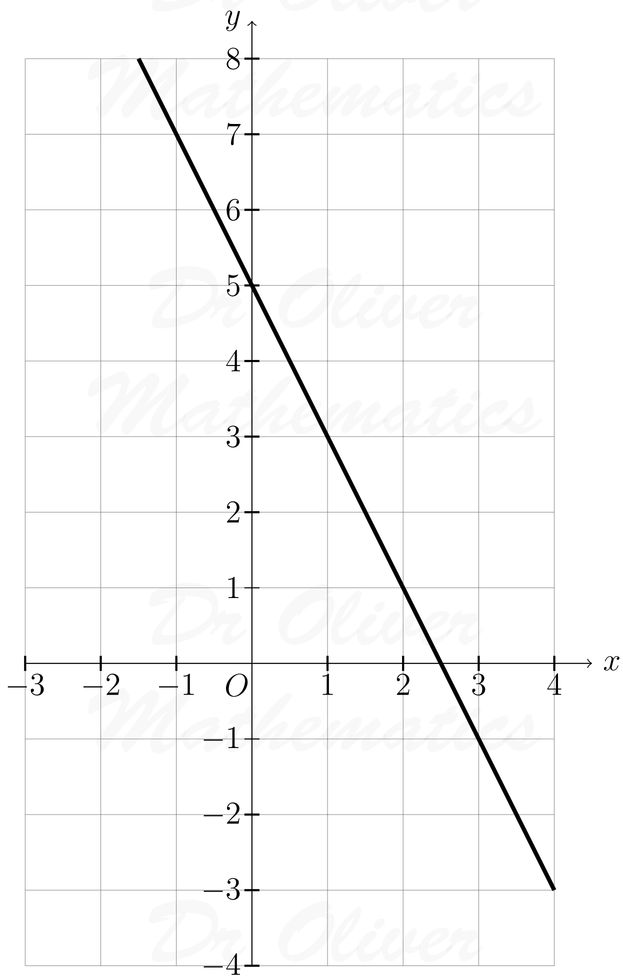
First, we construct an angle of 60° at P.



Second, we construct a bisector of that angle.



8. The straight line $y + 2x = 5$ has been drawn on the grid.



(a) Complete this table of values for $y = 2x - 1$. (2)

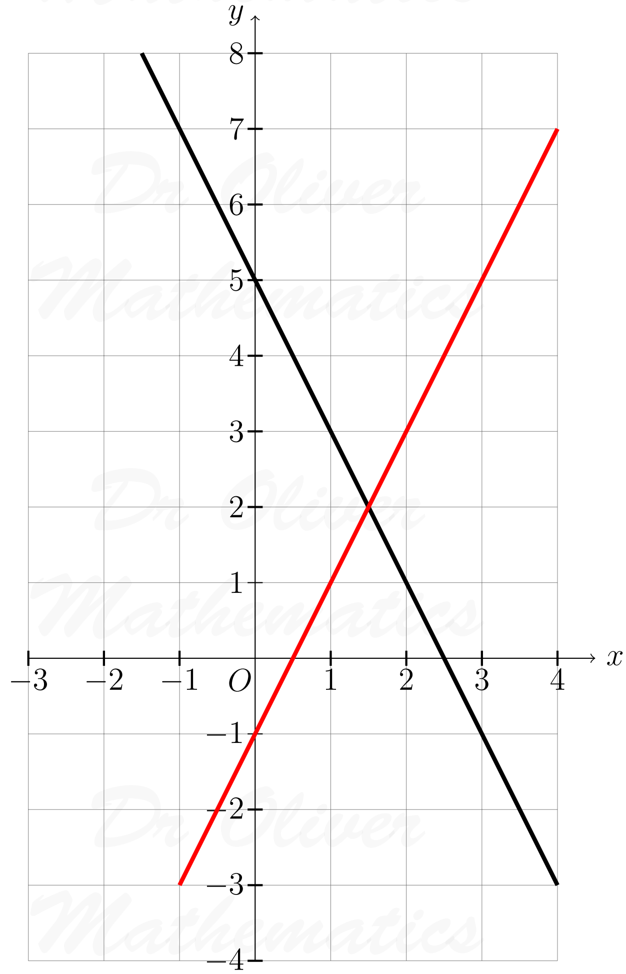
x	-1	0	1	2	3	4
y		-1		3	5	

Solution

x	-1	0	1	2	3	4
y	$\underline{\underline{-3}}$	-1	$\underline{\underline{1}}$	3	5	$\underline{\underline{7}}$

(b) On the grid, draw the graph of $y = 2x - 1$. (2)

Solution



(c) Use your diagram to solve the simultaneous equations (2)

$$y + 2x = 5$$
$$y = 2x - 1.$$

Solution

$\underline{\underline{x = 1.5, y = 2}}$.

9. (a) Factorise completely $3a^2 - 6a$. (2)

Solution

$$3a^2 - 6a = \underline{\underline{3a(a-2)}}.$$

(b) Make q the subject of the formula $P = 2q + 10$. (2)

Solution

$$\begin{aligned} P = 2q + 10 &\Rightarrow 2q = P - 10 \\ &\Rightarrow \underline{\underline{q = \frac{P-10}{2}}}. \end{aligned}$$

(c) Expand and simplify $(y+3)(y-4)$. (2)

Solution

$\times$	y	$+3$
y	y^2	$+3y$
-4	$-4y$	-12

Hence,

$$(y+3)(y-4) = \underline{\underline{y^2 - y - 12}}.$$

(d) Factorise $4p^2 - 9q^2$. (2)

Solution

$$\begin{aligned} 4p^2 - 9q^2 &= (2p)^2 - (3q)^2 \\ &= \underline{\underline{(2p-3q)(2p+3q)}}. \end{aligned}$$

10. (a) (i) Write 7 900 in standard form. (2)

Solution

$$7\,900 = \underline{\underline{7.9 \times 10^3}}.$$

(ii) Write 0.000 35 in standard form.

Solution

$$0.000\,35 = \underline{\underline{3.5 \times 10^{-4}}}.$$

(b) Work out (2)

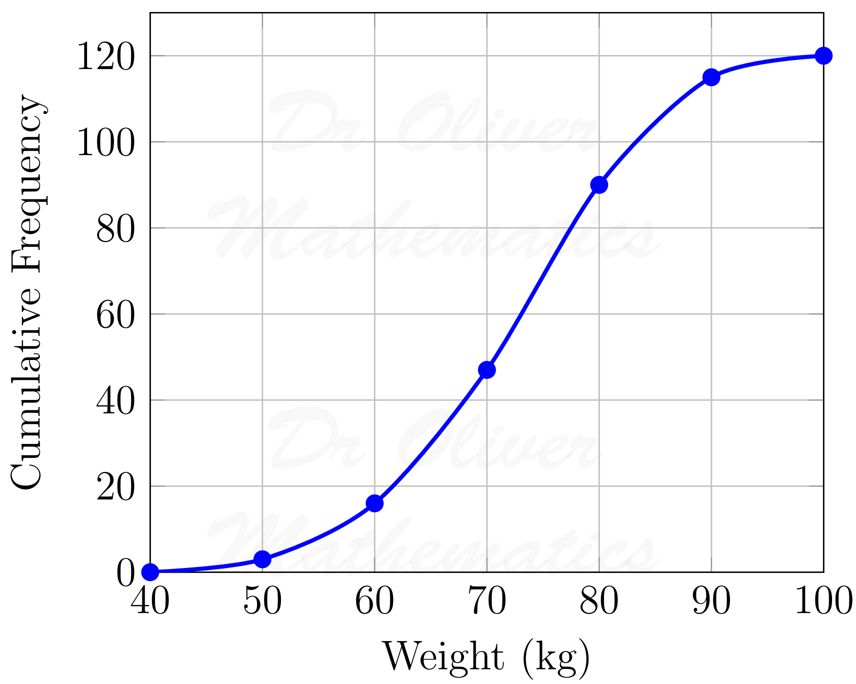
$$\frac{4 \times 10^3}{8 \times 10^{-5}}.$$

Give your answer in standard form.

Solution

$$\begin{aligned}\frac{4 \times 10^3}{8 \times 10^{-5}} &= 0.5 \times 10^8 \\ &= \underline{\underline{5 \times 10^7}}.\end{aligned}$$

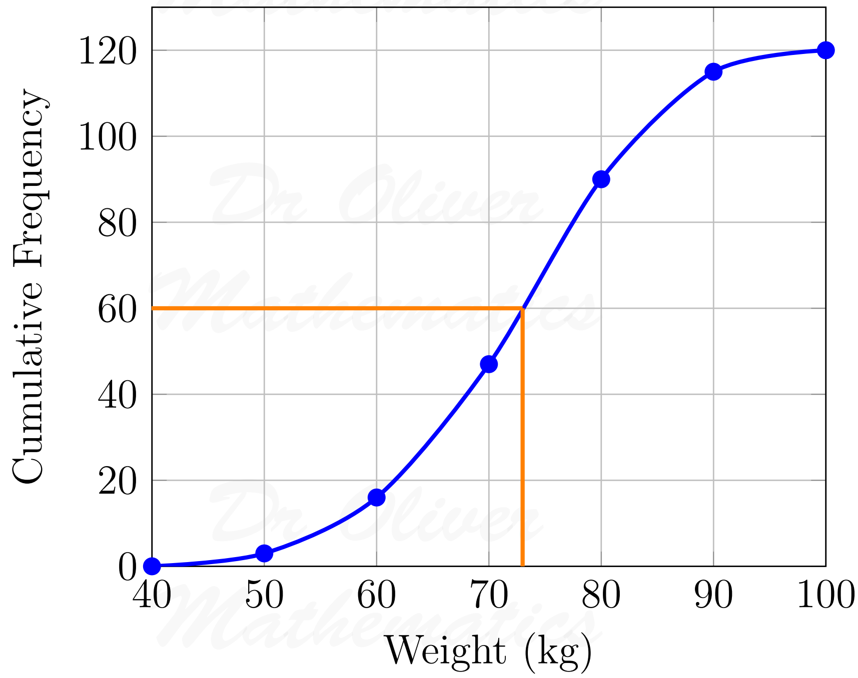
11. Here is the cumulative frequency curve of the weights of 120 girls at Mayfield Secondary School. (3)



Use the cumulative frequency curve to find an estimate for the

(a) median weight,

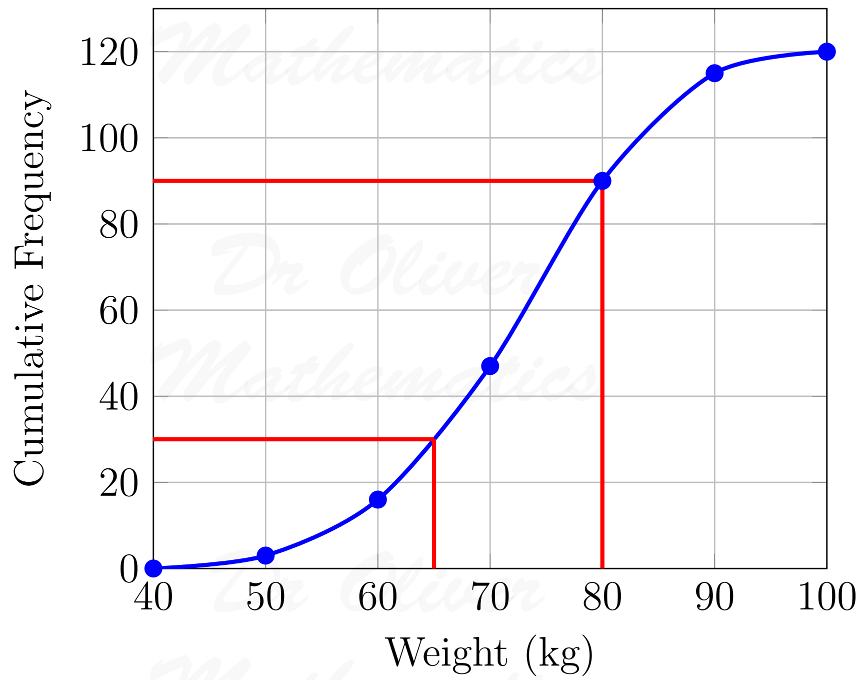
Solution



Correct read-off: approximately 73 kg.

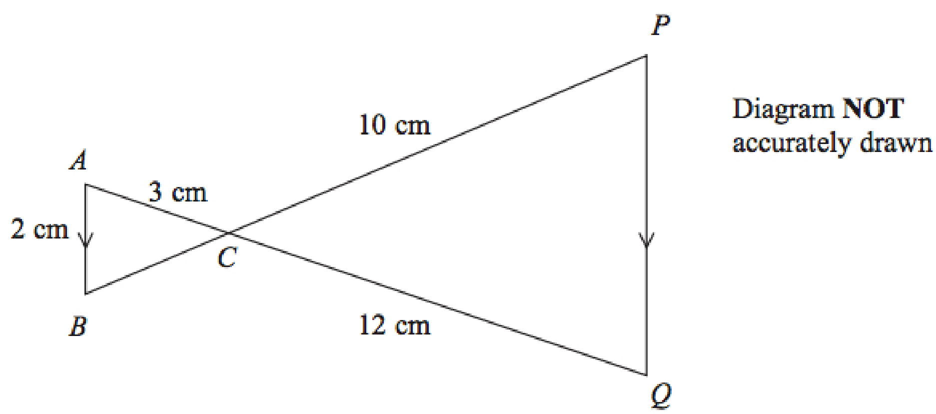
(b) interquartile range of the weights.

Solution



Correct read-off: approximately $80 - 65 = \underline{\underline{15 \text{ kg}}}$.

12. ACQ and BCP are straight lines.
AB is parallel to PQ.



$AB = 2$ cm.
$AC = 3$ cm.
$CQ = 12$ cm.
$CP = 10$ cm.

(a) Work out the length of PQ. (2)

Solution

$$\frac{PQ}{AB} = \frac{CQ}{AC} \Rightarrow \frac{PQ}{2} = \frac{12}{3}$$
$$\Rightarrow PQ = 4 \times 2$$
$$\Rightarrow \underline{\underline{PQ = 8 \text{ cm}}}.$$

(b) Work out the length of BP. (3)

Solution

$$\frac{BC}{CP} = \frac{AC}{CQ} \Rightarrow \frac{BC}{10} = \frac{1}{4}$$
$$\Rightarrow BC = 2\tfrac{1}{2}$$
$$\Rightarrow \underline{\underline{BP = 12\tfrac{1}{2} \text{ cm}}}.$$

13. Sarah wants to survey students in her school about which vegetables they eat.
These vegetables are on the menu in the school canteen.

Carrots Peas Cauliflower Broccoli Swede

(a) Design a suitable question she could use for a questionnaire to find out which of these vegetables each student eats. (2)

Solution

E.g., In the school canteen, did you eat the following least week?
Tick all the appropriate box(es).

Carrots	Peas	Cauliflower	Broccoli	Swede	Did not eat
☐	☐	☐	☐	☐	☐

There are 800 students in Sarah's school.
Sarah selects 50 students at random.
30 of these 50 students eat carrots.

(b) Work out an estimate for the number of students in Sarah's school who eat carrots. (2)

Solution

$$\frac{30}{50} \times 800 = \frac{2\,400}{5} = \underline{\underline{480 \text{ students}}}.$$

14. $-6 \leqslant 2y < 5$. (3)
y is an integer.
Write down all the possible values of y.

Solution

$$-6 \leqslant 2y < 5 \Rightarrow -3 \leqslant y < 2\tfrac{1}{2}$$

and so the value are $\underline{\underline{-3, -2, -1, 0, 1, 2}}$.

15. A cuboid has length 3 cm, width 4 cm, and height 12 cm. (3)

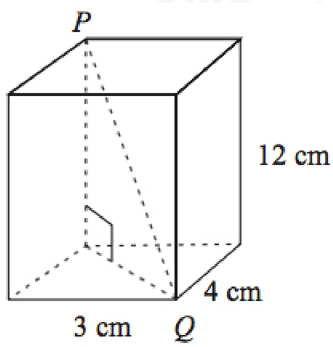


Diagram **NOT** accurately drawn

Work out the length of PQ.

Solution

$$\begin{aligned} PQ &= \sqrt{3^2 + 4^2 + 12^2} \\ &= \sqrt{9 + 16 + 144} \\ &= \sqrt{169} \\ &= \underline{\underline{13 \text{ cm}}}. \end{aligned}$$

16. (a) Simplify $(a^2)^4$. (1)

Solution

$$(a^2)^4 = \underline{\underline{a^8}}.$$

$2^{30} \div 8^9 = 2^x$.

(b) Work out the value of x. (2)

Solution

$$\begin{aligned} 2^{30} \div 8^9 &= \frac{2^{30}}{(2^3)^9} \\ &= \frac{2^{30}}{2^{27}} \\ &= 2^3; \end{aligned}$$

hence, $\underline{\underline{x = 3}}$.

17. Here are the equations of 5 straight lines.

P	$y = 2x + 5$	**Q**	$y = -2x + 5$
R	$y = x + 5$	**S**	$y = -\frac{1}{2}x + 6$
T	$y = \frac{1}{2}x + 1$		

(a) Write down the letter of the line that is parallel to $y = x + 6$. (1)

Solution

$\underline{\underline{\textbf{R}}}$.

(b) Write down the letter of the line that is perpendicular to $y = 2x - 1$. (1)

Solution

$\underline{\underline{\textbf{S}}}$.

(c) Find the coordinates of the point where the line $y = 2x + 5$ cuts the (2)

(i) y-axis,

Solution
$\underline{\underline{(0, 5)}}$.

(ii) x-axis

Solution
$\underline{\underline{(-2\frac{1}{2}, 0)}}$.

18. Here are the first 4 lines of a number pattern. (4)

1 + 2 + 3 + 4	=	(4 × 3) – (2 × 1)
2 + 3 + 4 + 5	=	(5 × 4) – (3 × 2)
3 + 4 + 5 + 6	=	(6 × 5) – (4 × 3)
4 + 5 + 6 + 7	=	(7 × 6) – (5 × 4)

n is the first number in the nth line of the number pattern.
Show that the above number pattern is true for the four consecutive integers n, $(n+1)$, $(n+2)$, and $(n+3)$.

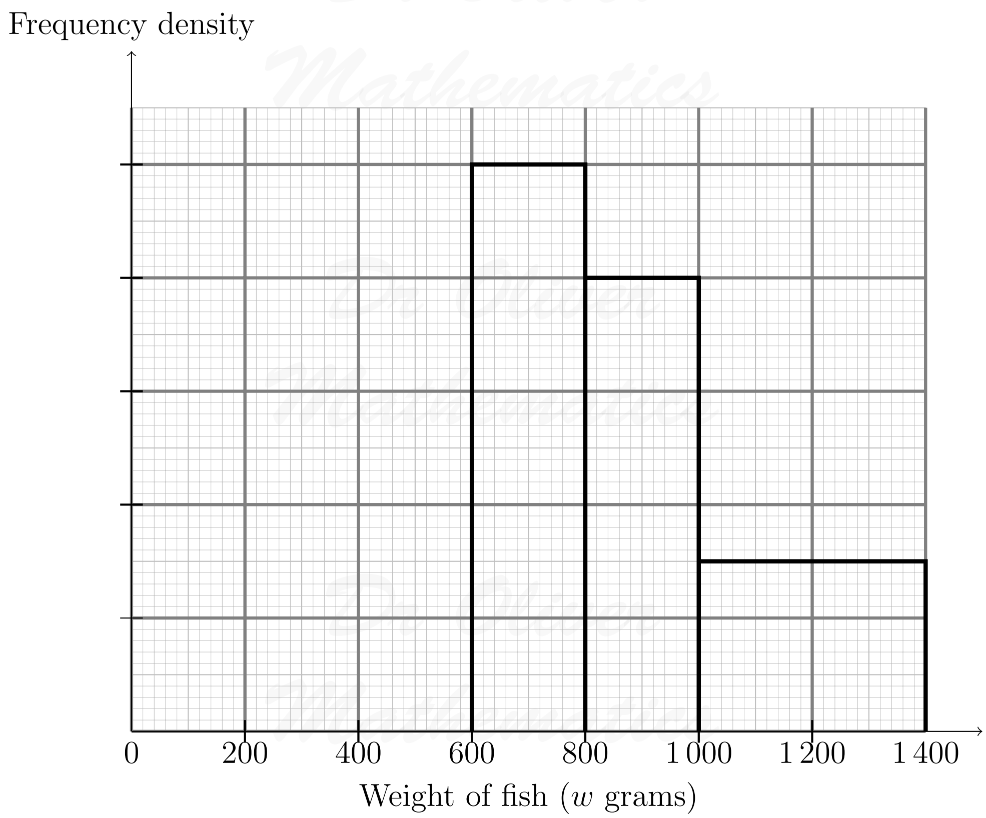
Solution

$$\begin{aligned}(n+3)(n+2)-(n+1)n &= (n^2+5n+6)-(n^2+n)\\ &= 4n+6\\ &= \underline{\underline{n+(n+1)+(n+2)+(n+3)}},\end{aligned}$$

as required.

19. The unfinished table and histogram show information about the weight, w grams, of fish that Alan caught each day.

Weight (w grams)	Frequency
$0 < w \leqslant 400$	8
$400 < w \leqslant 600$	5
$600 < w \leqslant 800$	10
$800 < w \leqslant 1\,000$	
$1\,000 < w \leqslant 1\,400$	



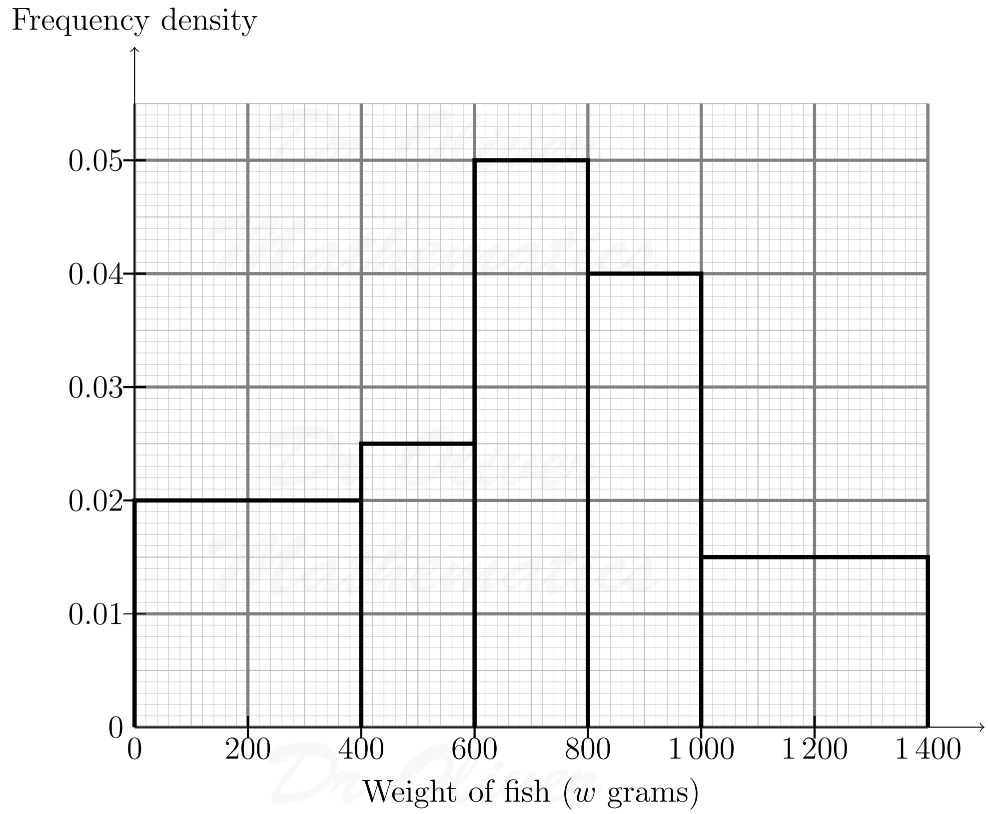
(a) Use the information in the histogram to complete the table. (2)

Solution

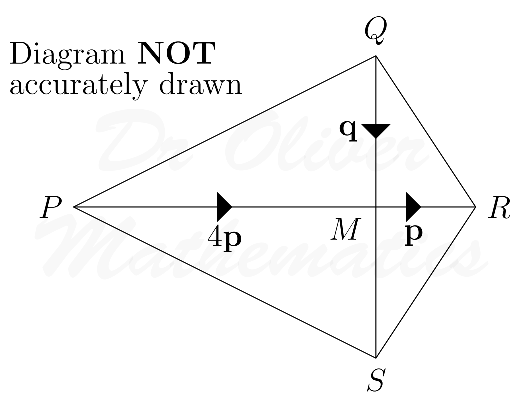
Weight (w grams)	Frequency	Width	Frequency density
$0 < w \leqslant 400$	8	400	$\frac{8}{400} = 0.02$
$400 < w \leqslant 600$	5	200	$\frac{5}{200} = 0.025$
$600 < w \leqslant 800$	10	200	$\frac{10}{200} = 0.05$
$800 < w \leqslant 1\,000$	$\underline{\underline{8}}$	200	$\frac{8}{200} = 0.04$
$1\,000 < w \leqslant 1\,400$	$\underline{\underline{6}}$	400	$\frac{6}{400} = 0.015$

(b) Use the information in the table to complete the histogram. (2)

Solution



20. $PQRS$ is a kite.



The diagonals PR and QS intersect at M.
$\overrightarrow{PM} = 4\mathbf{p}$.

$\overrightarrow{QM} = \mathbf{q}$.
$\overrightarrow{MR} = \mathbf{p}$.
$\overrightarrow{QM} = \overrightarrow{MS}$.

(a) Find expressions, in terms of $\mathbf{p}$ and/or $\mathbf{q}$ for (4)

(i) $\overrightarrow{PR}$,

Solution

$$\begin{aligned} \overrightarrow{PR} &= \overrightarrow{PM} + \overrightarrow{MR} \\ &= 4\mathbf{p} + \mathbf{p} \\ &= \underline{\underline{5\mathbf{p}}}. \end{aligned}$$

(ii) $\overrightarrow{QS}$,

Solution

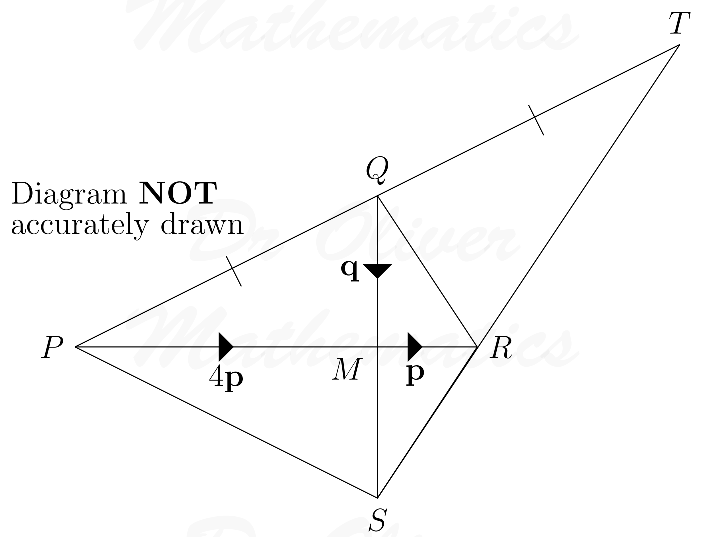
$$\begin{aligned} \overrightarrow{QS} &= \overrightarrow{QM} + \overrightarrow{MS} \\ &= \mathbf{q} + \mathbf{q} \\ &= \underline{\underline{2\mathbf{q}}}. \end{aligned}$$

(iii) $\overrightarrow{PQ}$.

Solution

$$\begin{aligned} \overrightarrow{PS} &= \overrightarrow{PM} + \overrightarrow{MQ} \\ &= \underline{\underline{4\mathbf{p} - \mathbf{q}}}. \end{aligned}$$

SR and PQ are extended to meet at point T.



Q is the midpoint of PT.

(b) Find $\overrightarrow{RT}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. (4)

Solution

$$\begin{aligned}\overrightarrow{RT} &= \overrightarrow{RM} + \overrightarrow{MQ} + \overrightarrow{QT} \\ &= -\mathbf{p} - \mathbf{q} + 4\mathbf{p} - \mathbf{q} \\ &= \underline{\underline{3\mathbf{p} - 2\mathbf{q}}}.\end{aligned}$$

21. The volumes of two mathematically similar solids are in the ratio 27 : 125. (3)
The surface area of the smaller solid is 36 cm^2.
Work out the surface area of the larger solid.

Solution

$$\begin{aligned}\text{VSF} = \frac{125}{27} &\Rightarrow \text{VSF} = \frac{5^3}{3^3} \\ &\Rightarrow \text{LSF} = \frac{5}{3} \\ &\Rightarrow \text{ASF} = \frac{5^2}{3^2},\end{aligned}$$

and

$$36 \times \frac{25}{9} = 4 \times 25 = \underline{\underline{100 \text{ cm}^2}}.$$

22. Solve the equation (4)

$$\frac{3}{x+3} - \frac{4}{x-3} = \frac{5x}{x^2-9}.$$

Solution

$$\begin{aligned}\frac{3}{x+3} - \frac{4}{x-3} = \frac{5x}{x^2-9} &\Rightarrow \frac{3(x-3)-4(x+3)}{(x+3)(x-3)} = \frac{5x}{x^2-9} \\ &\Rightarrow 3(x-3) - 4(x+3) = 5x \\ &\Rightarrow 3x - 9 - 4x - 12 = 5x \\ &\Rightarrow -21 = 6x \\ &\Rightarrow \underline{\underline{x = -3\tfrac{1}{2}}}.\end{aligned}$$

23. The table shows the number of boys and the number of girls in each year group at Springfield Secondary School. (2)
There are 500 boys and 500 girls in the school.

Year group	Number of boys	Number of girls
7	100	100
8	150	50
9	100	100
10	50	150
11	100	100
Total	500	500

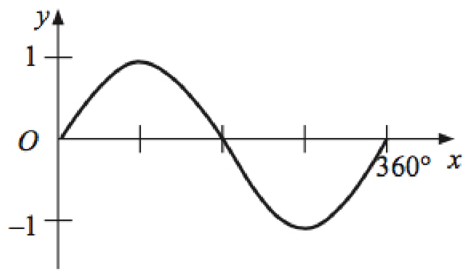
Azez took a stratified sample of 50 girls, by year group.
Work out the number of Year 8 girls in his sample.

Solution

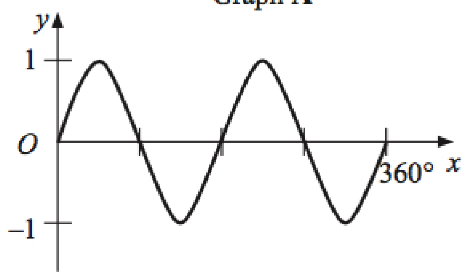
The number of Year 8 girls in his sample is

$$50 \times \frac{50}{500} = \underline{\underline{5}}.$$

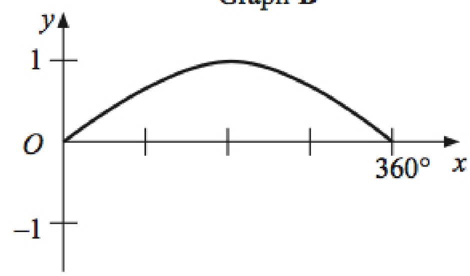
24. Here is the graph of $y = \sin x$, where $0^\circ \leqslant x \leqslant 360^\circ$. (4)



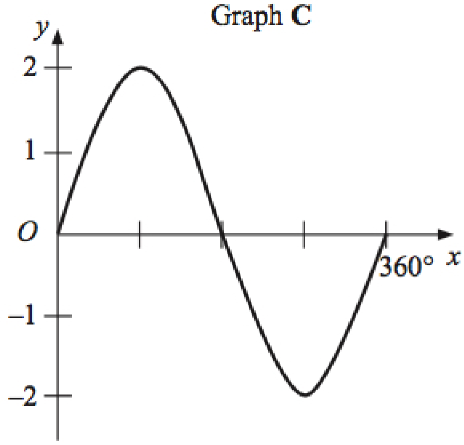
Graph **A**



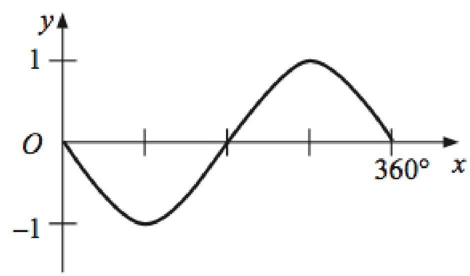
Graph **B**



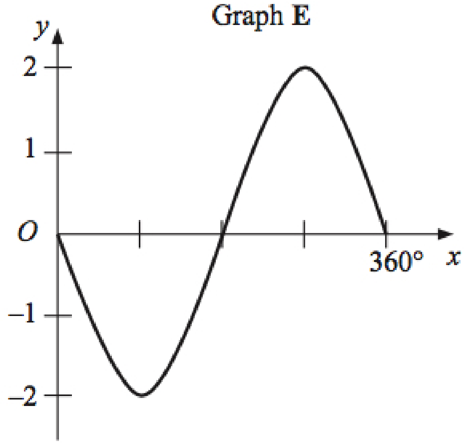
Graph **C**



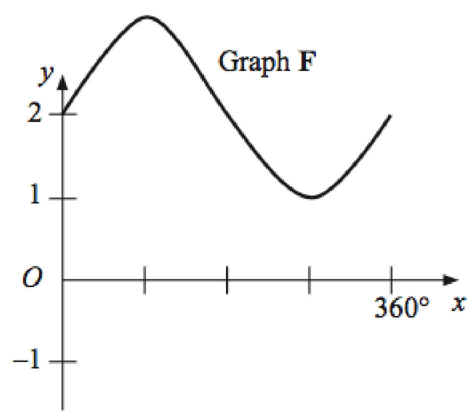
Graph **D**



Graph **E**



Graph **F**



Match each of the graphs **A**, **B**, **C**, **D**, **E**, and **F** to the equations in the table.

Equation	Graph
$y = 2\sin x$	
$y = -\sin x$	
$y = \sin 2x$	
$y = \sin x + 2$	
$y = \sin \frac{1}{2}x$	
$y = -2\sin x$	

Solution

Equation	Graph
$y = 2\sin x$	$\underline{\underline{\mathbf{C}}}$
$y = -\sin x$	$\underline{\underline{\mathbf{D}}}$
$y = \sin 2x$	$\underline{\underline{\mathbf{A}}}$
$y = \sin x + 2$	$\underline{\underline{\mathbf{F}}}$
$y = \sin \frac{1}{2}x$	$\underline{\underline{\mathbf{B}}}$
$y = -2\sin x$	$\underline{\underline{\mathbf{E}}}$