# Dr Oliver Mathematics
# GCSE Mathematics
# 2023 November Paper 3H: Calculator
# 1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

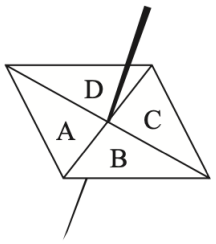
1. (a) Write 468 000 in standard form. (1)

**Solution**
$468\,000 = \underline{\underline{4.68 \times 10^5}}$.

(b) Write $5.037 \times 10^{-4}$ as an ordinary number. (1)

**Solution**
$5.037 \times 10^{-4} = \underline{\underline{0.000\,503\,7}}$.

2. Here is a biased spinner. (2)




The table shows the probabilities that when the spinner is spun it will land on $A$, on $B$, on $C$, and on $D$.

| Letter | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| Probability | 0.4 | 0.21 | 0.32 | 0.07 |

Luka will spin the spinner 200 times.

Work out an estimate for the number of times the spinner will land on $A$.

**Solution**

Well,

$$200 \times 0.4 = \underline{\underline{80}}.$$

3. Seija works at a weather station.
The table gives information about the temperature, $T$°C at midday for each of 50 cities in the UK on Tuesday.

| Temperature ($T$°C) | Frequency |
|---|---|
| $10 < T \leqslant 15$ | 2 |
| $15 < T \leqslant 20$ | 8 |
| $20 < T \leqslant 25$ | 13 |
| $25 < T \leqslant 30$ | 21 |
| $30 < T \leqslant 35$ | 6 |

(a) Calculate an estimate for the mean temperature. (3)

**Solution**

Well,

| Temperature ($T$°C) | Frequency | Midpoint | Midpoint×Frequency |
|---|---|---|---|
| $10 < T \leqslant 15$ | 2 | 12.5 | $2 \times 12.5 = 25$ |
| $15 < T \leqslant 20$ | 8 | 17.5 | $8 \times 17.5 = 140$ |
| $20 < T \leqslant 25$ | 13 | 22.5 | $13 \times 22.5 = 292.5$ |
| $25 < T \leqslant 30$ | 21 | 27.5 | $21 \times 27.5 = 577.5$ |
| $30 < T \leqslant 35$ | 6 | 32.5 | $6 \times 32.5 = 195$ |
| Total | 50 | | 1 230 |

and so

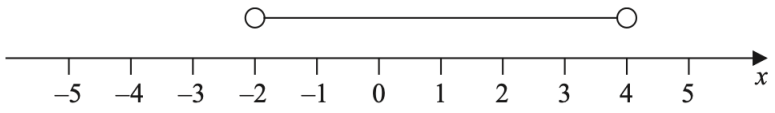
$$\begin{aligned} \text{mean} &= \frac{\sum fx}{\sum f} \\ &\approx \frac{1\,230}{50} \\ &= \underline{\underline{24.6^\circ\text{C}}}. \end{aligned}$$

Seija says, "The median temperature is 22.5°C because 22.5 is the middle number in the middle group."

(b) Is Seija correct? (1)
Give a reason for your answer.

**Solution**
No: the median is in the $25 < T \leqslant 30$ category

4. Jenna is asked to show the inequality $-3 < x \leqslant 4$ on a number line.
Here is her answer.




(a) Write down two mistakes Jenna has made. (2)

**Solution**
E.g., she has plotted the left-hand circle at $-2$, rather at $-3$,
she has not shaded the plotted the right-hand circle at 4.

(b) Work out the greatest integer that satisfies the inequality (2)

$$5y - 7 < 16.$$

**Solution**

$$5y - 7 < 16 \Rightarrow 5y < 23$$
$$\Rightarrow y < 4\tfrac{3}{5}.$$

Hence, the greatest integer is $\underline{\underline{4}}$.

5. Ali buys packs of balloons and boxes of pencils. (3)

There are 30 balloons in each pack.
There are 24 pencils in each box.

Ali buys exactly the same number of balloons and pencils.

Work out how many packs of balloons and how many boxes of pencils she could have bought.
You must show all your working.

**Solution**

Well,

| | |
|---|---|
| | 30 |
| 2 | 15 |
| 3 | 5 |
| 5 | 1 |

and

| | |
|---|---|
| | 24 |
| 2 | 12 |
| 2 | 6 |
| 2 | 3 |
| 3 | 1 |

and so

$$30 = 2 \times 3 \times 5 \text{ and } 24 = 2^3 \times 3.$$

So the HCF is

$$2^3 \times 3 \times 5 = 120.$$

Finally, he buys

$$\frac{120}{30} = \underline{\underline{4 \text{ packs of balloons}}}$$

and

$$\frac{120}{24} = \underline{\underline{5 \text{ packs of pencils.}}}$$

6. A company orders a large number of plates from a factory. (2)

It would take 30 hours to make all the plates using 4 machines.

How many machines are needed to make all the plates in 6 hours?

**Solution**

Inverse proportion:

$$4 \text{ machines} \leftrightarrow 30 \text{ hours} \Leftrightarrow 1 \text{ machine} \leftrightarrow 120 \text{ hours}$$
$$\Leftrightarrow 6 \text{ machines} \leftrightarrow \underline{\underline{20 \text{ hours}}}.$$

7. Riley travelled by car and by aeroplane. (3)

He travelled 143 miles by car at an average speed of 55 miles per hour.
Riley then travelled for 5 hours and 20 minutes by aeroplane.

Work out, in hours and minutes, Riley's total travelling time.

**Solution**

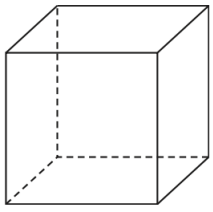
By car,

$$\frac{143}{55} = 2.6 \text{ hours}$$
$$= 2 \text{ hours } 36 \text{ minutes}.$$

Finally,

$$\text{total travelling time} = \text{by car} + \text{by aeroplane}$$
$$= 2 \text{ hours } 36 \text{ minutes} + 5 \text{ hours } 20 \text{ minutes}$$
$$= \underline{\underline{7 \text{ hours } 56 \text{ minutes}}}.$$

8. The diagram shows a solid cube placed on a horizontal table. (3)



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The pressure on the table due to the cube is 3.5 newtons/cm$^2$.
The force exerted by the cube on the table is 504 newtons.

Show that the total surface area of the cube is less than 900 cm$^2$.

**Solution**

Now,

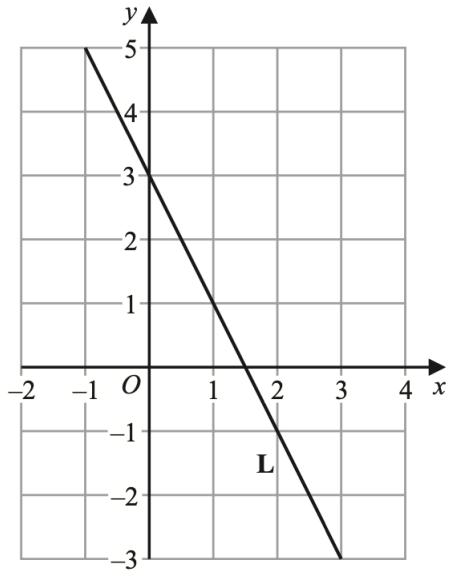
$$\text{pressure} = \frac{\text{force}}{\text{area}} \Rightarrow 3.5 = \frac{504}{\text{surface area}}$$
$$\Rightarrow \text{surface area} = \frac{504}{3.5}$$
$$\Rightarrow \text{surface area} = 144 \text{ cm}^2$$
$$\Rightarrow \text{total surface area} = 6 \times 144$$
$$\Rightarrow \text{total surface area} = 864 \text{ cm}^2.$$

Next,

$$864 < 900$$

so the total surface area of the cube is less than 900 cm$^2$.

9. The line $L$ is shown on the grid. (3)




Find an equation for $L$.

**Solution**

Well, the line $L$ goes through $(0, 3)$ and $(2, -1)$. Now,

$$\begin{aligned} m &= \frac{3-(-1)}{0-2} \\ &= \frac{4}{-2} \\ &= -2 \end{aligned}$$

and an equation for $L$ is

$$y - 3 = -2(x - 0) \Rightarrow \underline{\underline{y = -2x + 3}}.$$

10. Make $m$ the subject of (3)

$$k = p + \frac{2m}{5}.$$

**Solution**

$$\begin{aligned} k = p + \frac{2m}{5} &\Rightarrow k - p = \frac{2m}{5} \\ &\Rightarrow \underline{\underline{m = \frac{5(k-p)}{2}}}. \end{aligned}$$

11. The floor plan of a house is drawn using a scale of $1 : 50$. (3)
On the plan, a room in the house has a floor area of $48$ cm$^2$.

Work out the real area of the floor of this room.
Give your answer in m$^2$.

**Solution**

Well,

$$\begin{aligned} 1 \text{ cm}^2 &= 1 \text{ cm} \times 1 \text{ cm} \\ &= 0.01 \text{ m} \times 0.01 \text{ m} \\ &= 0.000\,1 \text{ m}^2 \end{aligned}$$

and

$$48 \text{ cm}^2 = 0.004\,8 \text{ m}^2.$$

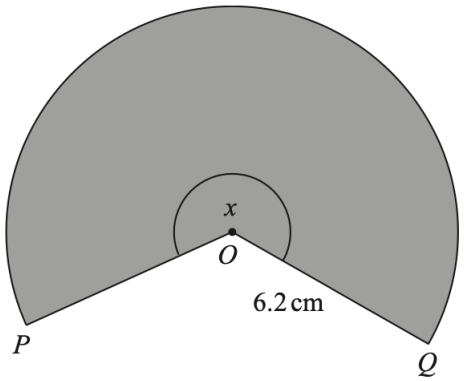
Now, the length scale ratio (LSR) is $1 : 50$ which means the area scale ratio (ASR) is

$$1^2 : 50^2 = 1 : 2\,500.$$

Finally,

$$\begin{aligned} \text{real area of the floor} &= 0.004\,8 \times 2\,500 \\ &= \underline{\underline{12 \text{ m}^2}}. \end{aligned}$$

12. The diagram shows a shaded sector $POQ$ of a circle with centre $O$ and radius 6.2 cm. (2)




The area of the shaded sector is 82.6 $\text{cm}^2$.

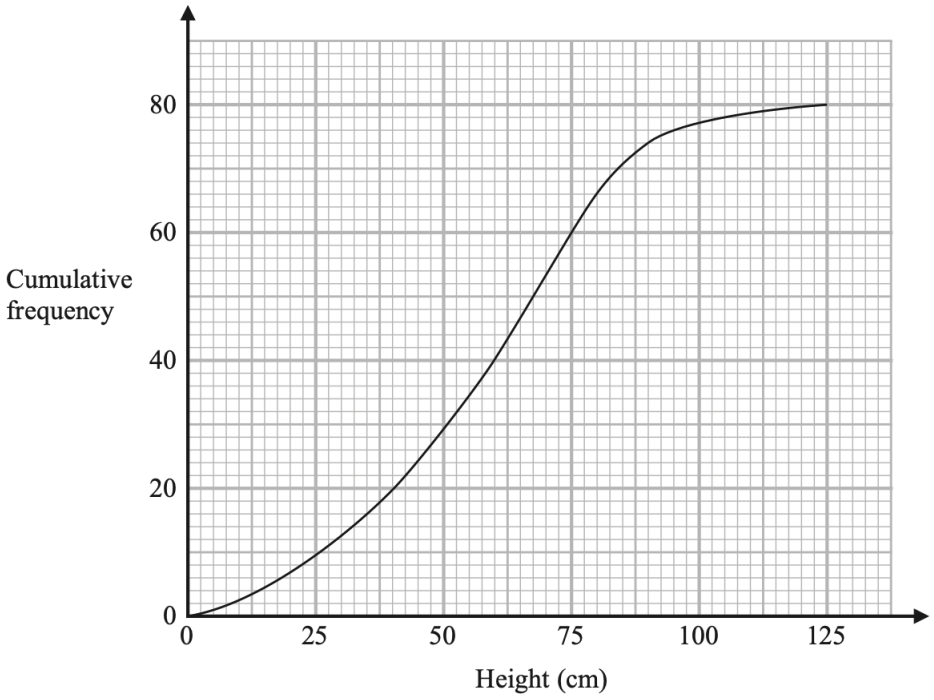
Calculate the size of angle $x$.
Give your answer correct to 3 significant figures.

**Solution**

Well,

$$\begin{aligned} \frac{x}{360} \times \pi \times 6.2^2 = 82.6 \Rightarrow x &= \frac{82.6 \times 360}{\pi \times 6.2^2} \\ \Rightarrow x &= 246.234\,723\,6 \text{ (FCD)} \\ \Rightarrow x &= \underline{\underline{246^\circ \text{ (3 sf)}}}. \end{aligned}$$

13. Alan grew 80 plants of the same type outside. The cumulative frequency graph shows information about the heights, in cm, of these plants.




One of the plants is chosen at random.

(a) Find an estimate for the probability that this plant will have a height greater than 90 cm. (2)

**Solution**

Go up to 90 and then read along the graph: approximately 74.
$80 - 74 = 6$.
Hence,

$$\text{probability} = \frac{6}{80} = \underline{\underline{\frac{3}{40}}}.$$

(b) Use the graph to find an estimate for the median height. (1)

**Solution**

Go up to 40 and then read along the graph: approximately $\underline{\underline{60 \text{ cm}}}$.

(c) Use the graph to find an estimate for the interquartile range of the heights. (2)

**Solution**

$$\begin{aligned}\text{IQR} &= \text{UQ} - \text{LQ}\\ &= 75 - 40\\ &= \underline{\underline{35 \text{ cm}}}.\end{aligned}$$

Alan also grew plants of the same type inside.

The interquartile range of the heights of these plants is 30 cm.

(d) Give one comparison between the distribution of the heights of the plants grown inside with the distribution of the heights of the plants grown outside. (1)

**Solution**

Based on the IQR, the ones outside grew <u>more varied</u>.

14. Here are the first six terms of a quadratic sequence: (2)

$$5 \quad 11 \quad 21 \quad 35 \quad 53 \quad 75.$$

Find an expression, in terms of $n$, for the $n$th term of this sequence.

**Solution**

Let the

$$n\text{th term} = an^2 + bn + c.$$

We only need the second line of differences (why?):

$$\begin{array}{ccccc} 5 & & 11 & & 21 \\ & 6 & & 10 & \\ & & 4 & & \end{array}$$

$$\begin{array}{ccccc} a+b+c & & 4a+2b+c & & 9a+3b+c \\ & 3a+b & & 5a+b & \\ & & 2a & & \end{array}$$

We compare terms:

$$2a = 4 \Rightarrow a = 2,$$

$$3a + b = 6 \Rightarrow 3 \times 2 + b = 6$$
$$\Rightarrow b = 0,$$

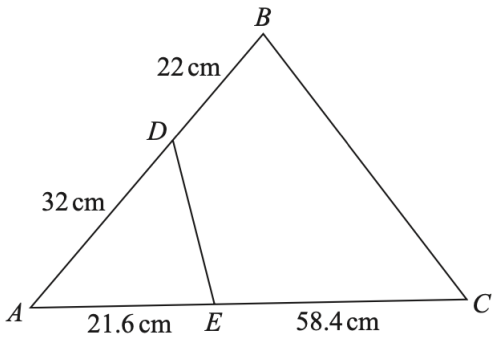
and

$$a + b + c = 5 \Rightarrow 2 + 0 + c = 5$$
$$\Rightarrow c = 3;$$

hence,

$$n\text{th term} = \underline{\underline{2n^2 + 3}}.$$

15. The diagram shows triangle $ABC$ and triangle $AED$. (2)




Show that triangle $ABC$ and triangle $AED$ are similar.

**Solution**

$\triangle ADE$ and $\triangle ABC$ are similar. (Why?) Well,

$$\frac{AD}{AC} = \frac{AD}{AE + EB}$$
$$= \frac{32}{21.6 + 58.4}$$
$$= \frac{32}{80}$$
$$= \tfrac{2}{5}$$

and

$$\begin{aligned}\frac{AE}{AB} &= \frac{AE}{AD+DB}\\ &= \frac{21.6}{32+22}\\ &= \frac{21.6}{54}\\ &= \tfrac{2}{5}.\end{aligned}$$

So,

$$\frac{AD}{AC} = \frac{AE}{AB}$$

so the triangle $ABC$ and triangle $AED$ are similar.
(Note: we do not require reference to the common angle.)

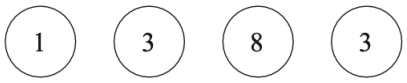
16. Zia has to set a 4-digit security passcode on her phone. (3)

Each digit of the passcode is a number from 1 to 9.
She can use each number more than once.

Zia tells her friend Amber that

- the first digit is a cube number,
- the second digit is a prime number,
- the third digit is greater than 6, and
- the fourth digit is an odd number.

The diagram shows one possible 4-digit passcode.




Amber is going to have one attempt at guessing Zia's passcode.

Work out the probability that Amber guesses Zia's passcode on the first attempt.

**Solution**

Well,

- the first digit: 2 (1 or 8),
- the second digit: 4 (2, 3, 5, or 7),
- the third digit: 3 (7, 8, or 9), and
- the fourth digit: 5 (1, 3, 5, 7, or 9)

Finally,

$$\begin{aligned}\text{probability} &= \frac{1}{2 \times 4 \times 3 \times 5} \\ &= \underline{\underline{\tfrac{1}{120}}}.\end{aligned}$$

17. (a) (i) Write (2)

$$x^2 - 8x + 3$$

in the form

$$(x - a)^2 - b,$$

where $a$ and $b$ are integers.

**Solution**
Well,

$$\begin{aligned}x^2 - 8x + 3 &= (x^2 - 8x + 16) + (3 - 16) \\ &= \underline{\underline{(x - 4)^2 - 13}};\end{aligned}$$

hence, $\underline{\underline{a = 4}}$ and $\underline{\underline{b = 13}}$.

(ii) Hence, write down the coordinates of the turning point on the graph of (1)

$$y = x^2 - 8x + 3.$$

**Solution**
$\underline{\underline{(4, -13)}}$.

(b) Solve (3)

$$7x^2 + 8x - 5 = 0.$$

Give your solutions correct to 3 significant figures.

**Solution**

Quadratic formula: $a = 7$, $b = 8$, and $c = -5$ so

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4 \times 7 \times (-5)}}{2 \times 7} \\ &= \frac{-8 \pm \sqrt{204}}{14} \\ &= -1.591\,632\,633,\ 0.448\,775\,489\,8 \text{ (FCD)} \\ &= \underline{\underline{-1.59,\ 0.449 \text{ (3 sf)}}}. \end{aligned}$$

Alex has to find the solutions of the quadratic equation

$$3k^2 + 10k - 8 = 0.$$

Here is his working and answer.

$$\begin{aligned} (3k - 2)(k + 4) &= 0 \\ k = 2 \text{ or } k &= -4. \end{aligned}$$

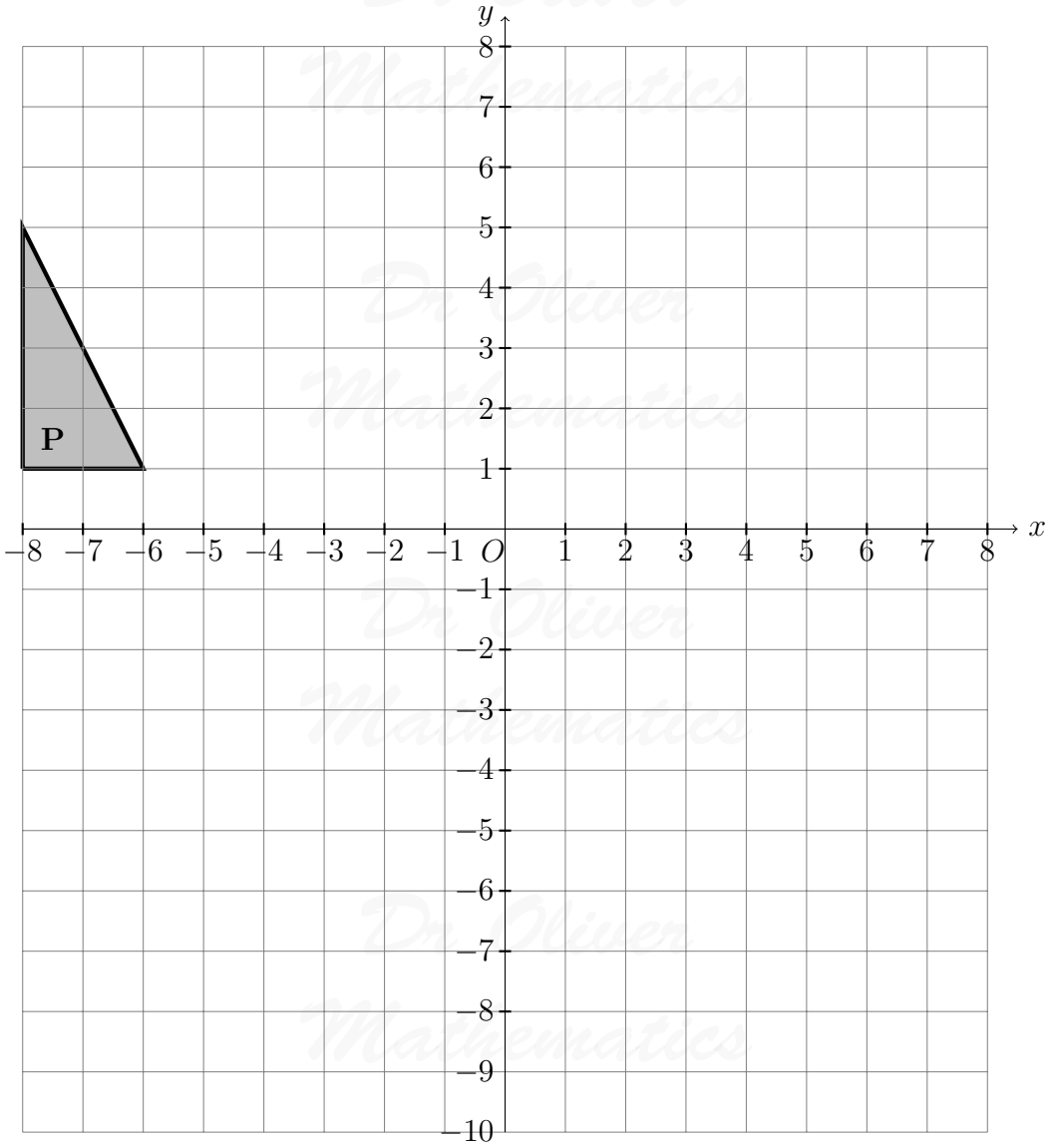
(c) What mistake has Alex made? (1)

**Solution**

E.g.,

$$3k - 2 = 0 \Rightarrow \underline{\underline{k = \tfrac{2}{3}}}.$$

18. Here is a picture.




(a) Enlarge triangle **P** by scale factor $-1\frac{1}{2}$ with centre of enlargement $(-2, -1)$. (2)
Label your image **Q**.

**Solution**

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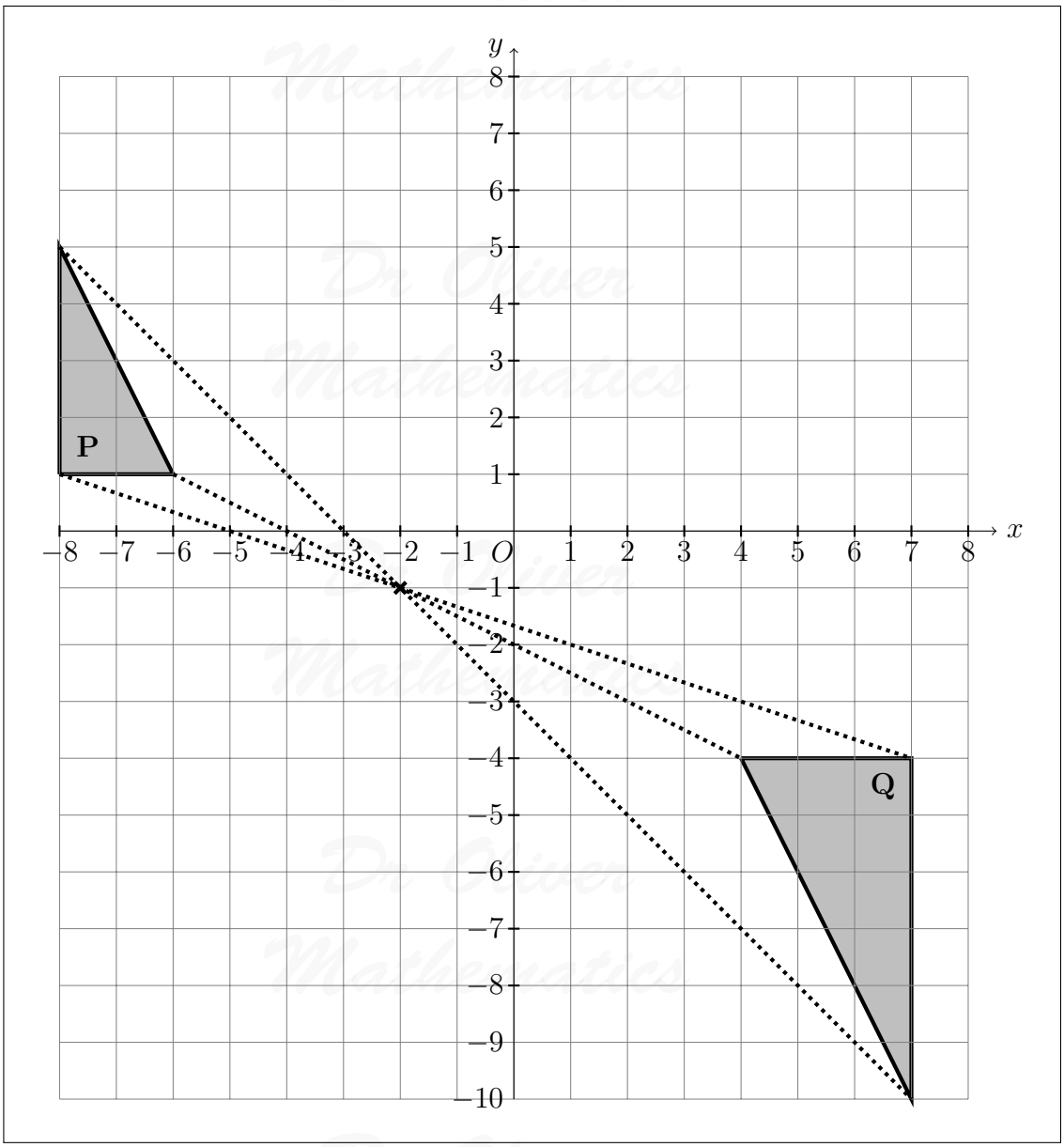


Triangle **P** is transformed by a combined transformation of a rotation of 90° anticlockwise about the origin followed by a translation to give triangle **R**.

Exactly one vertex of triangle **P** is invariant under the combined transformation.

(b) Find one possible column vector for the translation. (1)

**Solution**

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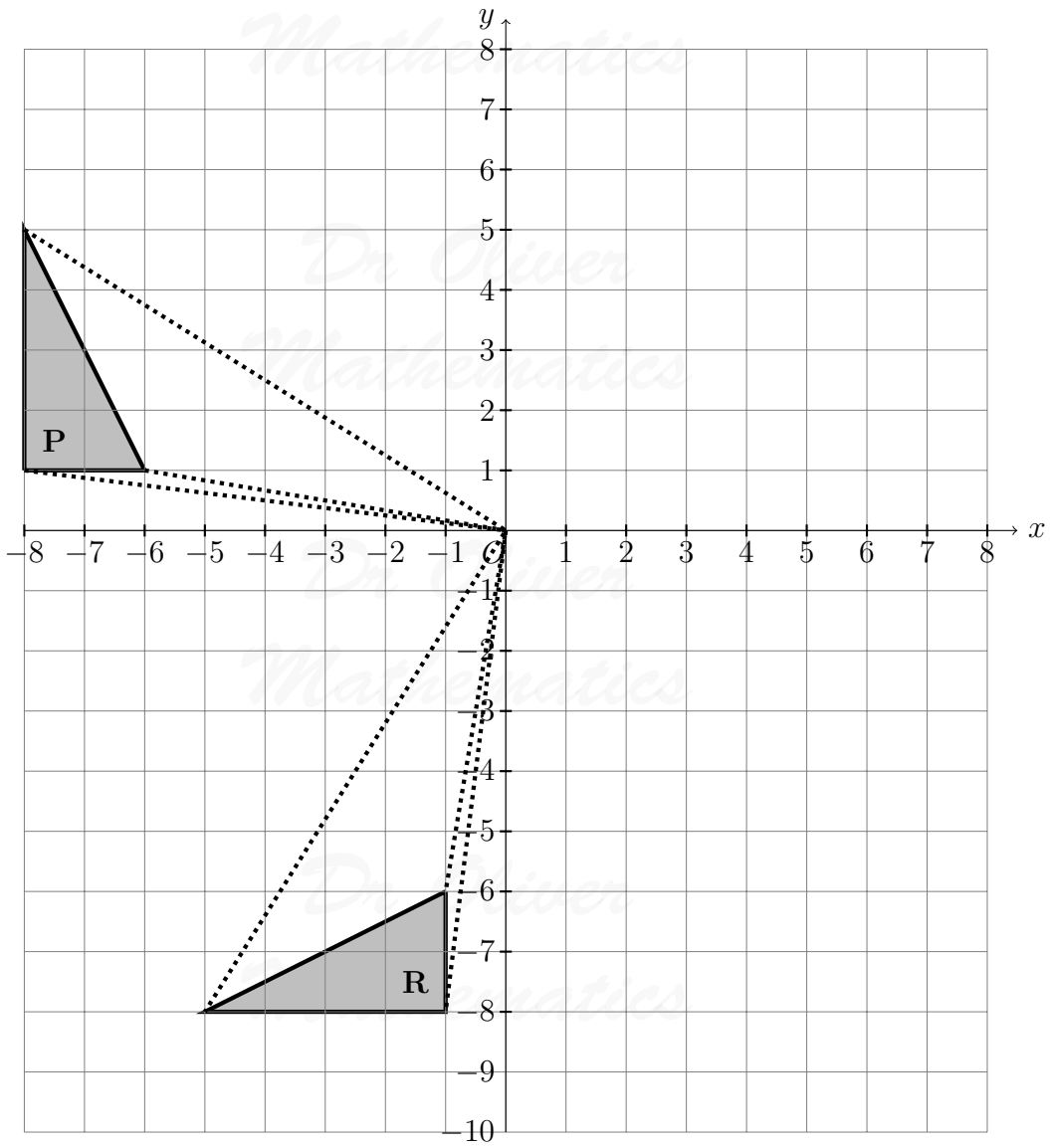


So, a translation by

- $\underline{\underline{\begin{pmatrix} -5 \\ 7 \end{pmatrix}}}$,
- $\underline{\underline{\begin{pmatrix} -7 \\ 9 \end{pmatrix}}}$, or
- $\underline{\underline{\begin{pmatrix} -3 \\ 13 \end{pmatrix}}}$.

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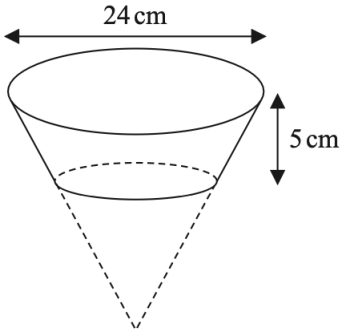
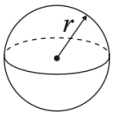
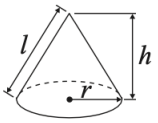
19. The diagram above shows a frustum **F** of a cone. (4)




Volume of sphere = $\frac{4}{3}\pi r^3$

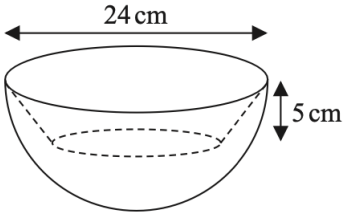



Volume of cone = $\frac{1}{3}\pi r^2 h$




The frustum is made by removing a cone with height 10 cm from a solid cone with height 15 cm and base diameter 24 cm.

The solid **S** is made by removing **F** from a solid hemisphere as shown in the diagram below.

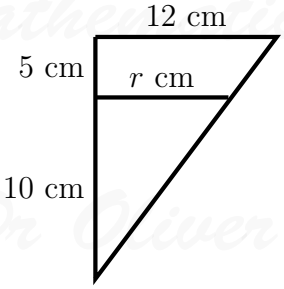



The hemisphere has diameter 24 cm.

Calculate the volume of solid **S**.
Give your answer correct to 3 significant figures.

**Solution**

Let the radius be $r$ cm.



Similar triangles:

$$\frac{r}{10} = \frac{12}{15} \Rightarrow r = \tfrac{4}{5} \times 10$$
$$\Rightarrow r = 8$$

and

$$\begin{aligned}\text{frustum} &= \text{whole cone} - \text{little cone}\\ &= (\tfrac{1}{3} \times \pi \times 12^2 \times 15) - (\tfrac{1}{3} \times \pi \times 8^2 \times 10)\\ &= 720\pi - \tfrac{640}{3}\pi\\ &= \tfrac{1\,520}{3}\pi.\end{aligned}$$

Next,

$$\begin{aligned}\text{hemisphere} &= \tfrac{1}{2} \times \tfrac{4}{3} \times \pi \times 12^3\\ &= 1152\pi.\end{aligned}$$

Finally,

$$\begin{aligned}\text{solid } \mathbf{S} &= \text{hemisphere} - \text{frustum}\\ &= 1152\pi - \tfrac{1\,520}{3}\pi\\ &= \tfrac{1\,5936}{3}\pi\\ &= 2\,027.374\,459 \text{ (FCD)}\\ &= \underline{\underline{2\,030 \text{ cm}^3 \text{ (3 sf)}}}.\end{aligned}$$

20. The turning point on the graph of $y = \mathrm{g}(x)$ has coordinates $(-3, 6)$.

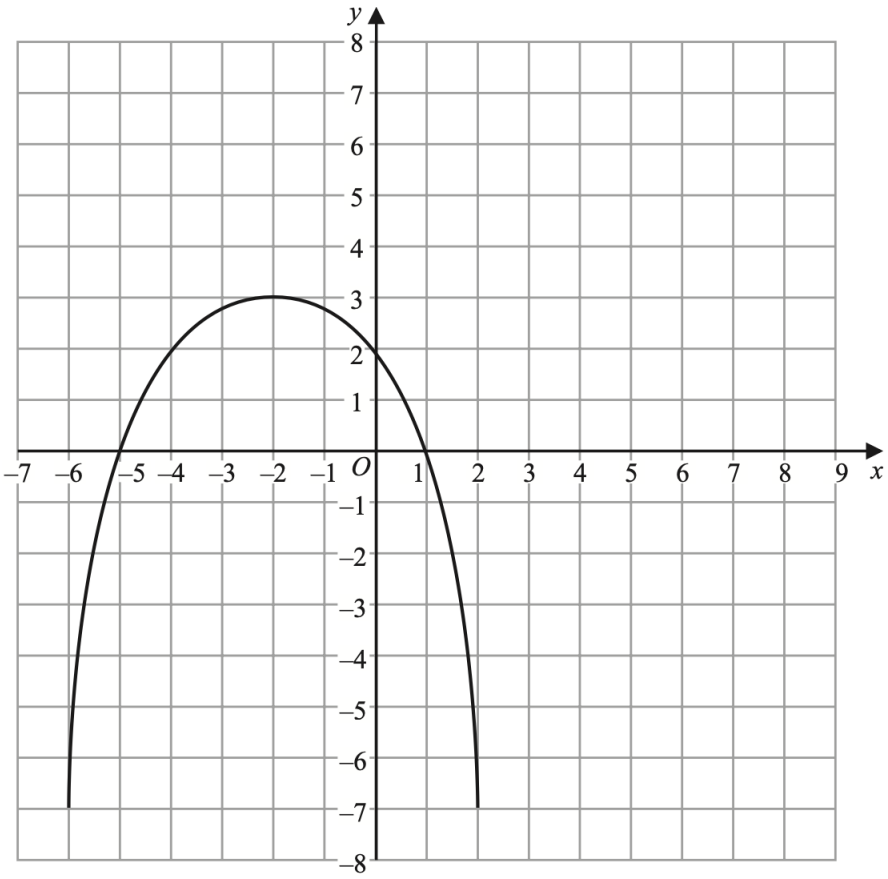
(a) Write down the coordinates of the turning point on the graph of (1)

$$y = \mathrm{g}(x - 7).$$

**Solution**

$\underline{\underline{(4, 6)}}$.

The graph of $y = \mathrm{f}(x)$ is shown on the grid.




(b) On the grid, sketch the graph of (2)

$$y = \mathrm{f}(-x) + 3.$$

**Solution**

$y = \mathrm{f}(-x)$: It goes through $(-2, -7)$, $(-1, 0)$, $(2, 3)$, $(5, 0)$, and $(6, -7)$.
$y = \mathrm{f}(-x) + 3$: It goes through $(-2, -4)$, $(-1, 3)$, $(2, 6)$, $(5, 3)$, and $(6, -4)$.

21. $A$, $B$, $C$, and $D$ are points on the circumference of a circle, centre $O$. (4)

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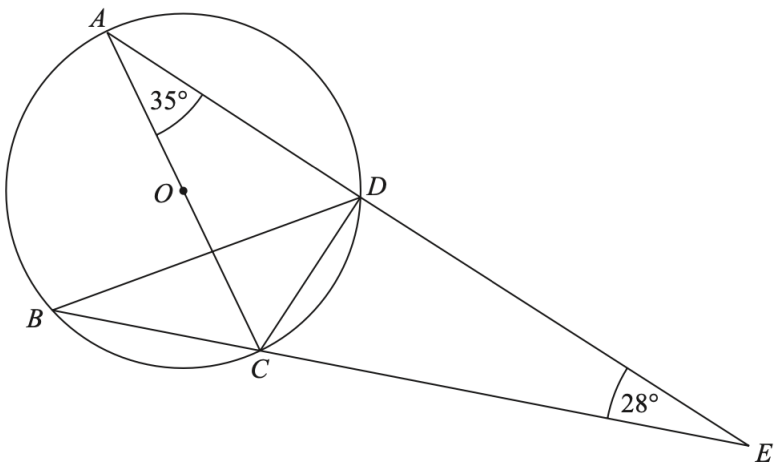



- $AC$ is a diameter of the circle.
- $ADE$ and $BCE$ are straight lines.

Work out the size of angle $BDC$.
Write down any circle theorems that you use.

**Solution**

$\angle ADC = 90^\circ$ (right-angle)
$\angle CBD = 35^\circ$ (angles at the circumference)
$\angle ADB = \angle BCA = x^\circ$ (angles at the circumference)
$\angle ACD = 90 - 35 = 55^\circ$ (completing the triangle)
$\angle CDE = 90^\circ$ (right-angle)
$\angle DCE = 180 - 90 - 28 = 62^\circ$ (completing the triangle)
$\angle ACD = 180 - (35 + 28 + 62) = 55^\circ$ (completing the triangle)
Now, $x^\circ = 180 - (55 + 62) = 63^\circ$.
Finally, $\angle BDC = 90 - 63 = \underline{\underline{27^\circ}}$.

22. Ebony makes some bracelets to sell. (4)

- The materials to make all the bracelets cost £190, correct to the nearest £5.
- Ebony sells all the bracelets for a total of £875, correct to the nearest £5.
- The total time taken to make and sell all these bracelets was 72 hours, correct to the nearest hour.

Ebony uses this method to calculate her hourly rate of pay.

$$\text{Hourly rate of pay} = \frac{\text{total selling price} - \text{total cost of materials}}{\text{total time taken}}$$

The minimum hourly rate of pay for someone of Ebony's age is £8.20.

By considering bounds, determine if Ebony's hourly rate of pay was definitely more than £8.20.
You must show all your working.

**Solution**

Well,

$$\begin{aligned} 187.5 &\leqslant \text{materials} < 192.5, \\ 872.5 &\leqslant \text{sells} < 875.5, \\ 71.5 &\leqslant \text{time} < 72.5. \end{aligned}$$

Now,

$$\begin{aligned} \text{lower bound} &= \frac{\text{lower bound: sales} - \text{upper bound: materials}}{\text{upper bound: time}} \\ &= \frac{872.5 - 192.5}{72.5} \\ &= 9\tfrac{11}{29}. \end{aligned}$$

Hence, Ebony's hourly rate of pay was definitely more than £8.20.

23. Given that (4)

$$\frac{2x^2 + y^2}{4x^2 - y^2} = \frac{43}{11},$$

where $x > 0$ and $y > 0$, find, in its simplest form, the ratio $x : y$.

**Solution**

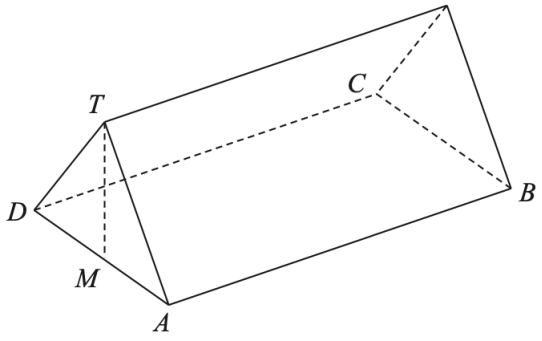
Cross-multiply:

$$\frac{2x^2+y^2}{4x^2-y^2}=\frac{43}{11} \Rightarrow 11(2x^2+y^2)=43(4x^2-y^2)$$
$$\Rightarrow 22x^2+11y^2=172x^2-43y^2$$
$$\Rightarrow 150x^2=54y^2$$
$$\Rightarrow x^2=\tfrac{9}{25}y^2$$
$$\Rightarrow x=\tfrac{3}{5}y$$
$$\Rightarrow 5x=3y,$$

and so

$$x:y=\underline{\underline{3:5}}.$$

24. The diagram shows a triangular prism with a horizontal rectangular base $ABCD$. (4)




- $M$ is the midpoint of $AD$.
- The vertex $T$ of the prism is vertically above $M$.
- $AB = 14.7$ cm.
- $BC = 3.8$ cm.
- $MT = 2.3$ cm.
- $P$ is the point on $AB$ such that $AP : PB = 5 : 2$.

Calculate the size of the angle between $TP$ and the base $ABCD$ of the prism.
Give your answer correct to 1 decimal place.

**Solution**

Well,

$$5 + 2 = 7$$

so

$$\begin{aligned} AP &= \tfrac{5}{7} \times 14.7 \\ &= 10.5 \text{ cm}. \end{aligned}$$

Now,

$$AM = \tfrac{1}{2}BC = 1.9 \text{ cm}.$$

Next, 3-dimensional Pythagoras' theorem:

$$\begin{aligned} TP^2 = TM^2 + AM^2 + AP^2 &\Rightarrow TP^2 = 2.3^2 + 1.9^2 + 10.5^2 \\ &\Rightarrow TP^2 = 119.15 \\ &\Rightarrow TP = \sqrt{119.15}. \end{aligned}$$

Let the required angle be $x^\circ$. Finally,

$$\begin{aligned} \sin = \frac{\text{opp}}{\text{hyp}} &\Rightarrow \sin x^\circ = \frac{2.3}{\sqrt{119.15}} \\ &\Rightarrow x^\circ = 12.163\,840\,04 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{x^\circ = 12.2^\circ \text{ (1 dp)}}}. \end{aligned}$$

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