Dr Oliver Mathematics
GCSE Mathematics
2020 June Paper 2H: Calculator
1 hour 30 minutes

The total number of marks available is 80.
You must write down all the stages in your working.

1. (a) Simplify (1)

$$(x^3)^5.$$

Solution

$$(x^3)^5 = \underline{\underline{x^{15}}}.$$

(b) Expand and simplify (2)

$$4(x+3)+7(4-2x).$$

Solution

$$\begin{aligned} 4(x+3)+7(4-2x) &= 4x+12+28-14x \\ &= \underline{\underline{-10x+40}}. \end{aligned}$$

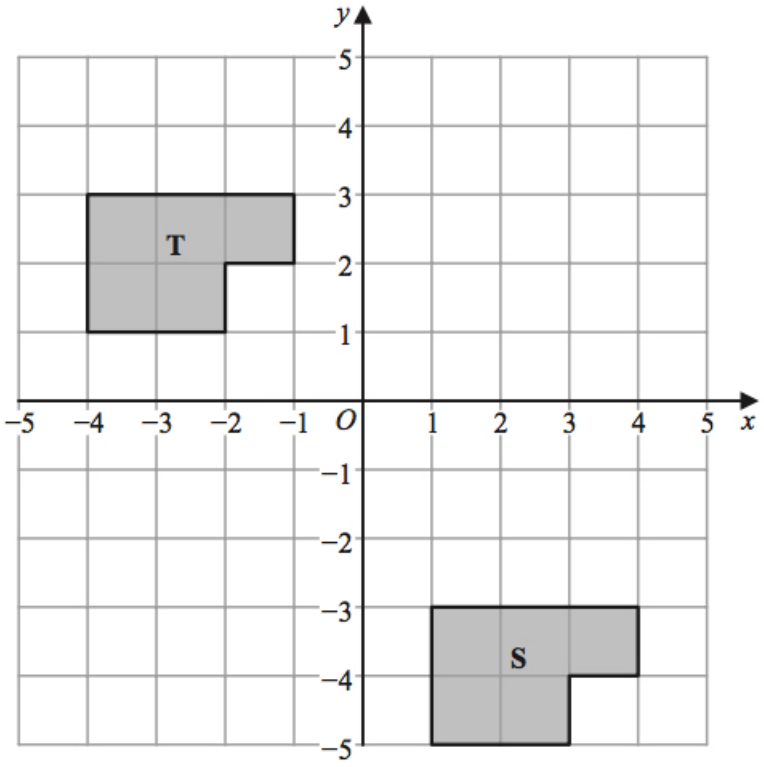
(c) Factorise fully (2)

$$15x^3+3x^2y.$$

Solution

$$15x^3+3x^2y = \underline{\underline{3x^2(5x+y)}}.$$

2. Describe fully the single transformation that maps shape **S** onto shape **T**. (2)



Solution

Take the upper left-hand of **S**: it is a translation of

$$\begin{pmatrix} -4-1 \\ 3-(-3) \end{pmatrix} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}.$$

3. The length of a football pitch is 90 metres, correct to the nearest metre. (2)

   Complete the error interval for the length of the football pitch.

Solution

$$89.5 \text{ m} \leqslant \text{length} < 90.5 \text{ m}.$$

4. Festival A will be in a rectangular field with an area of 80 000 m^2.
   The greatest number of people allowed to attend Festival A is 425.

Festival B will be in a rectangular field 700 m by 2 000 m.
The greatest number of people allowed to attend Festival B is 6 750.

The area per person allowed for Festival B is greater than the area per person allowed for Festival A.

(a) How much greater? (4)
Give your answer correct to the nearest whole number.

Solution

Festival A: the area per person is

$$\frac{80\,000}{425} = 188\tfrac{4}{17}.$$

Festival B: the area per person is

$$\frac{700 \times 2\,000}{6\,750} = 207\tfrac{11}{27}.$$

Finally,

$$\begin{aligned} \text{difference} &= 207\tfrac{11}{27} - 188\tfrac{4}{17} \\ &= 19.172\ldots \\ &= \underline{\underline{19 \text{ (nearest whole number)}}}. \end{aligned}$$

Callum says,

"300 cm^2 is the same as 3 m^2 because there are 100 cm in 1 m so you divide by 100."

Callum's method is wrong.

(b) Explain why (1)

Solution

He has lost a power of 100:

$$\begin{aligned} 3\ \text{m}^2 &= 3 \times 1\ \text{m} \times 1\ \text{m} \\ &= 3 \times 100\ \text{cm} \times 100\ \text{cm} \\ &= \underline{\underline{30\,000\ \text{cm}^2}}. \end{aligned}$$

5. The points L, M, and N are such that LMN is a straight line. (4)

- The coordinates of L are $(-3, 1)$.
- The coordinates of M are $(4, 9)$.

Given that

$$LM : MN = 2 : 3,$$

find the coordinates of N.

Solution

Well,

$$2 + 3 = 5$$

and

$$\begin{aligned}
\overrightarrow{ON} &= \overrightarrow{OL} + \overrightarrow{LN} \\
&= \overrightarrow{OL} + \tfrac{5}{2}\overrightarrow{LM} \\
&= \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \tfrac{5}{2}\begin{pmatrix} 4-(-3) \\ 9-1 \end{pmatrix} \\
&= \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \tfrac{5}{2}\begin{pmatrix} 7 \\ 8 \end{pmatrix} \\
&= \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 17\frac{1}{2} \\ 20 \end{pmatrix} \\
&= \begin{pmatrix} 14\frac{1}{2} \\ 21 \end{pmatrix};
\end{aligned}$$

hence, $\underline{\underline{N(14\frac{1}{2}, 21)}}$.

6. A new phone cost £679. (3)
The value of the phone decreases at a rate of 4% per year.

Work out the value of the phone at the end of 3 years.

Solution

$$\begin{aligned}\text{Value} &= 679 \times (1 - 0.04)^3 \\ &= 679 \times (0.96)^3 \\ &= 600.735\,744 \text{ (FCD)} \\ &= \underline{\underline{£600.74 \text{ (nearest penny)}}}.\end{aligned}$$

7. In Spain, Sam pays 27 euros for 18 litres of petrol. (4)
In Wales, Leo pays £40.80 for 8 gallons of the same type of petrol.

- 1 euro = £0.85.
- 4.5 litres = 1 gallon.

Sam thinks that petrol is cheaper in Spain than in Wales.

Is Sam correct?
You must show how you get your answer.

Solution

Spain:

$$\frac{27 \text{ euros}}{18 \text{ litres}} = 1\tfrac{1}{2}.$$

Wales:

$$\begin{aligned}\frac{£40.80}{8 \text{ gallons}} &= \frac{\frac{40.80}{0.85} \text{ euros}}{8 \times 4.5 \text{ litres}} \\ &= \frac{48 \text{ euros}}{36 \text{ litres}} \\ &= 1\tfrac{1}{3}.\end{aligned}$$

No, Sam is <u>wrong</u>.

8. Use your calculator to work out (2)

$$\frac{\sqrt[3]{1.57^4 + \tan 60^\circ}}{7.2^{\frac{1}{2}}}.$$

Give your answer correct to 3 significant figures.

Solution

$$\frac{\sqrt[3]{1.57^4 + \tan 60^\circ}}{7.2^{\frac{1}{2}}} = 0.739\,337\,940\,3 \text{ (FCD)}$$
$$= \underline{\underline{0.739 \text{ (3 sf)}}}.$$

9. A box in the shape of a cuboid is placed on a horizontal floor. (3)

The box exerts a force of 180 newtons on the floor.
The box exerts a pressure of 187.5 newtons/m^2 on the floor.

The face in contact with the floor is a rectangle of length 1.2 metres and width x metres.

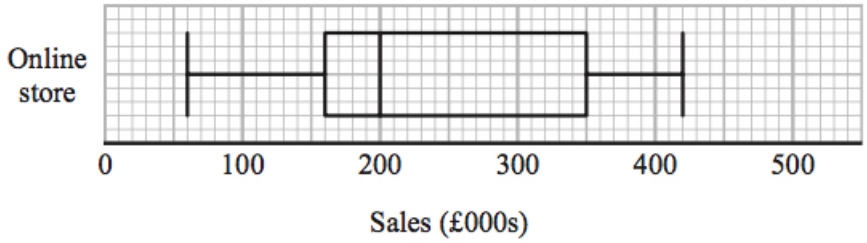
Work out the value of x.

Solution

Well,

$$\text{pressure} = \frac{\text{force}}{\text{area}} \Rightarrow 187.5 = \frac{180}{1.2 \times x}$$
$$\Rightarrow x = \frac{180}{1.2 \times 187.5}$$
$$\Rightarrow \underline{\underline{x = 0.8}}.$$

10. The box plot shows information about the sales, in thousands of pounds (£000s), of an online store each month.



Andrew says, "Three quarters of the given data lies between 160 000 and 350 000 because these are the values of the lower quartile and the upper quartile."

Andrew is wrong.

(a) Explain why. (1)

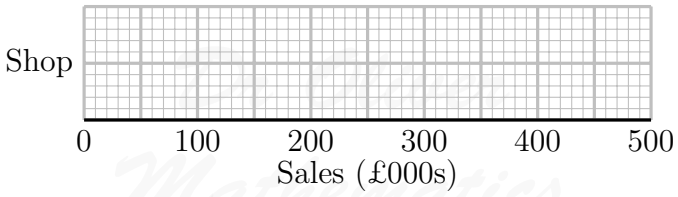
Solution

Andrew really needs to say half: "Half of the given data lies between 160 000 and 350 000 because these are the values of the lower quartile and the upper quartile."

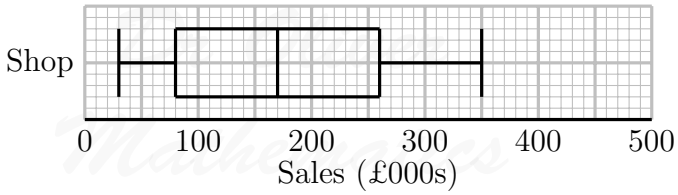
The table shows information about the sales, in £000s, in a shop each month.

	Sales (£000s)
Least value	30
Lower quartile	80
Median	170
Upper quartile	260
Greatest value	350

(b) On the grid below, draw a box plot for this information. (2)



Solution



(c) Compare the distribution of the sales of the online store with the distribution of the sales in the shop. (2)

Solution

Average
Since the median for Online store (200) is higher than the median for Shop (170), the Online shop earned more on average.

Spread
Since the range for the Online store ($420 - 60 = 360$) is larger than the range for the Shop ($350 - 30 = 330$), the earnings were more consistent in the Shop.
OR
Since the IQR for Online shop ($350 - 160 = 190$) is larger than the IQR for Shop ($260 - 80 = 180$), the earnings were more consistent in the Shop.

11. Kieron has 13 workers he can use for a job. (3)

He knows that 6 workers would take $14\frac{1}{2}$ days to complete this job.

Show that Kieron has enough workers to finish this job in less than 7 days.

Solution
Now,

$$\begin{aligned} 6 \text{ workers} &\leftrightarrow 14\tfrac{1}{2} \text{ days}, \\ 1 \text{ worker} &\leftrightarrow 14\tfrac{1}{2} \times 6 \\ &\leftrightarrow 87 \text{ days, and} \\ 13 \text{ workers} &\leftrightarrow \frac{87}{13} \\ &\leftrightarrow 6\tfrac{9}{13}; \end{aligned}$$

hence, Kieron has enough workers to finish this job in less than 7 days.

12. The equation of the line L_1 is (2)

$$y = 2x + 3.$$

The equation of the line L_2 is

$$5y - 10x + 4 = 0.$$

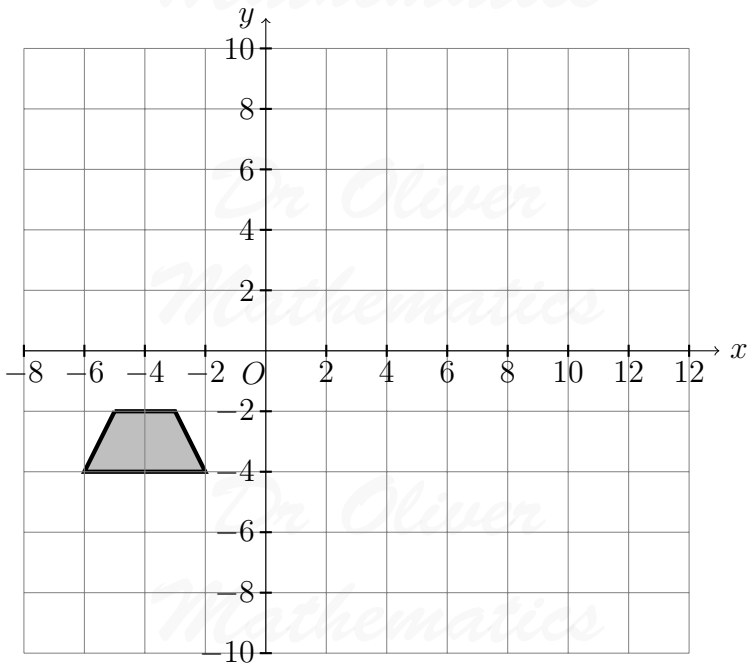
Show that these two lines are parallel.

Solution

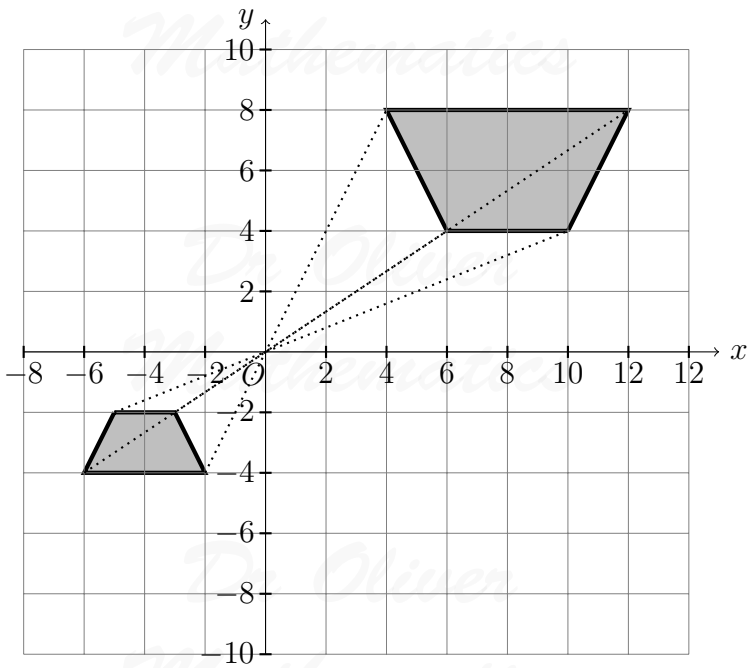
$$5y - 10x + 4 = 0 \Rightarrow 5y = 10x - 4$$
$$\Rightarrow y = 2x - \tfrac{4}{5};$$

they have same gradient (2) so, hence, two lines are parallel.

13. Enlarge the shaded shape by scale factor -2 with centre of enlargement $(0, 0)$. (2)



Solution



14. Saffron wants to work out an estimate for the total number of fish in a lake. (3)

On Friday, Saffron catches 180 fish from the lake.
She puts a tag on each of these fish and puts them back into the lake.

On Saturday, Saffron catches 305 fish from the same lake.
She finds that 45 of the 305 fish are tagged.

Work out an estimate for the total number of fish in the lake.

Solution

Let x be an estimate for the total number of fish in the lake. Now,

$$\frac{x}{180} = \frac{305}{45} \Rightarrow x = \frac{305 \times 180}{45}$$
$$\Rightarrow \underline{\underline{x = 1\,220}}.$$

15. The ratio of Marta's hourly pay to Khalid's hourly pay is 6 : 5. (4)

Both Marta and Khalid get an increase of £1.50 in their hourly pay.

The ratio of Marta's hourly pay to Khalid's hourly pay after this increase is $13 : 11$.

Work out the hourly pay before the increase for Marta and for Khalid.

Solution

Well,

$$13 : 11 = 13x : 11x$$

for some x. Now, before the increase,

$$\begin{aligned} 13x - 1.5 : 11x - 1.5 = 6 : 5 \Rightarrow \frac{13x - 1.5}{11x - 1.5} &= \frac{6}{5} \\ \Rightarrow 5(13x - 1.5) &= 6(11x - 1.5) \\ \Rightarrow 65x - 7.5 &= 66x - 9 \\ \Rightarrow x &= 1.5. \end{aligned}$$

So, **before the increase**, Marta's hourly pay is

$$13(1.5) - 1.5 = \underline{\underline{£18}}$$

and Khalid's hourly pay

$$11(1.5) - 1.5 = \underline{\underline{£15}}.$$

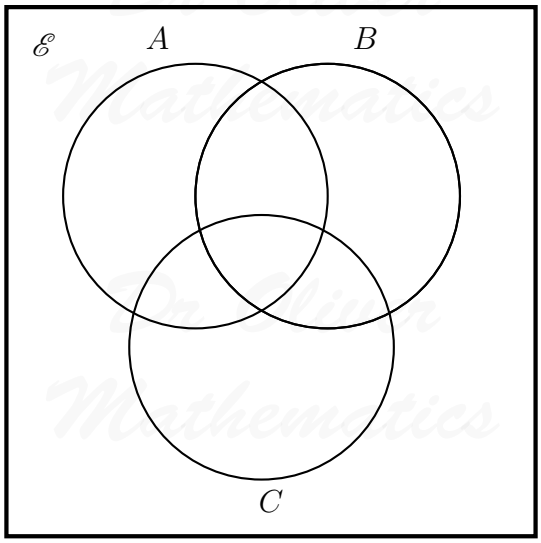
16. A shop manager wants to advertise special offers on social media platforms.

The manager asks 100 customers which of type A, type B, or type C they use.

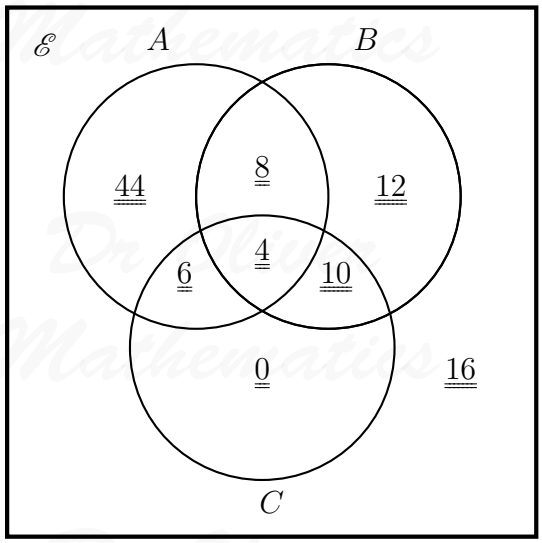
Of these customers,

- 4 use all three types,
- 16 do not use any of type A, type B, or type C,
- 8 use both type A and type B, but not type C,
- 14 use both type B and type C,
- 62 in total use type A, and
- all 20 who use type C also use at least one of type A and type B.

(a) Complete the Venn diagram for this information. (4)



Solution



One of the customers is chosen at random.

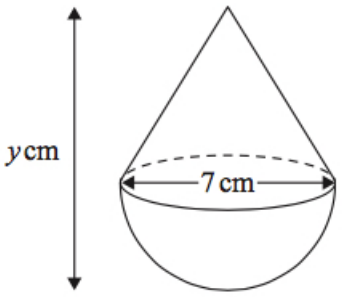
Given that this customer uses type A,

(b) find the probability that this customer also uses type B. (2)

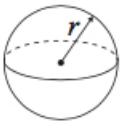
Solution

$$\text{Probability} = \frac{8+4}{62} = \frac{12}{62} = \frac{6}{31}.$$

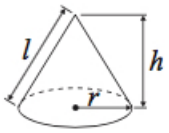
17. A solid cone is joined to a solid hemisphere to make the solid **T** shown below.



Volume of sphere $= \frac{4}{3}\pi r^3$



Volume of cone $= \frac{1}{3}\pi r^2 h$



The diameter of the base of the cone is 7 cm.
The diameter of the hemisphere is 7 cm.

The total volume of **T** is 120π cm^3.
The total height of **T** is y cm.

(a) Calculate the value of y. (4)
Give your answer correct to 3 significant figures.

Solution

Well, the length of the hemisphere is

$$\frac{7}{2} = 3.5 \text{ cm}$$

and that makes

$$\text{volume of the hemisphere} = \tfrac{1}{2} \times \tfrac{4}{3} \times \pi 3.5^3 = \tfrac{343}{12}\pi.$$

Now, the height of the cone is $(y - 3.5)$ and

$$\text{volume of the cone} = \tfrac{1}{3} \times \pi \times 3.5^2 \times (y - 3.5) = \tfrac{49}{12}(y - 3.5)\pi.$$

Finally,

$$\begin{aligned}
&\text{volume of the hemisphere} + \text{volume of the cone} = 120\pi \\
\Rightarrow\quad & \tfrac{343}{12}\pi + \tfrac{49}{12}(y - 3.5)\pi = 120\pi \\
\Rightarrow\quad & \tfrac{49}{12}(y - 3.5) = 120 - \tfrac{343}{12} \\
\Rightarrow\quad & y - 3.5 = \tfrac{12}{49}(120 - \tfrac{343}{12}) \\
\Rightarrow\quad & y = \tfrac{12}{49}(120 - \tfrac{343}{12}) + 3.5 \\
\Rightarrow\quad & y = 25.887\,755\,102 \text{ (FCD)} \\
\Rightarrow\quad & \underline{\underline{y = 25.9 \text{ cm (3 sf)}}}.
\end{aligned}$$

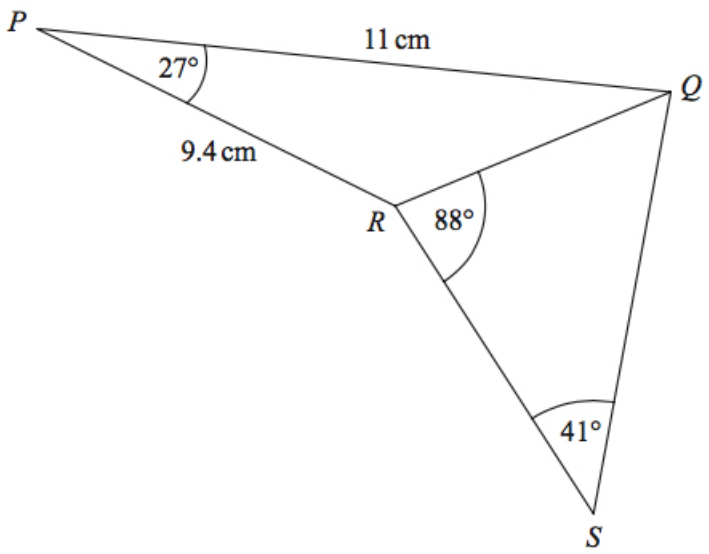
The diameter of the base of the cone and the diameter of the hemisphere are both increased by the same amount.
Assuming the total volume of **T** does not change,

(b) explain the effect this would have on your answer to part (a). (1)

Solution

E.g., in that case, y would be smaller and the height would <u>decrease</u>.

18. PQR and QRS are triangles. (4)



Calculate the length of QS.
Give your answer correct to 3 significant figures.

You must show all your working.

Solution

Cosine rule:

$$\begin{aligned} & QR^2 = PR^2 + PQ^2 - 2 \times PR \times PQ \times \cos QPR \\ \Rightarrow \quad & QR^2 = 9.4^2 + 11^2 - 2 \times 9.4 \times 11 \times \cos 27^\circ \\ \Rightarrow \quad & QR = 5.009\,975\,129 \text{ (FCD)}. \end{aligned}$$

Now,

$$\begin{aligned} \angle QRS + \angle RSQ + \angle SQR = 180 &\Rightarrow 88 + 41 + \angle SQR = 180 \\ &\Rightarrow 129 + \angle SQR = 180 \\ &\Rightarrow \angle SQR = 51^\circ \end{aligned}$$

and now the sine rule:

$$\begin{aligned} \frac{QS}{\sin QRS} = \frac{QR}{\sin QSR} &\Rightarrow \frac{QS}{\sin 88^\circ} = \frac{5.009\ldots}{\sin 41^\circ} \\ &\Rightarrow QS = \frac{5.009\ldots \sin 88^\circ}{\sin 41^\circ} \\ &\Rightarrow QS = 7.631\,818\,124 \text{ (FCD)} \\ &\Rightarrow \underline{\underline{QS = 7.63 \text{ cm (3 sf)}}}. \end{aligned}$$

19. The functions g and h are such that

$$\mathrm{g}(x) = \sqrt[3]{2x-5} \text{ and } \mathrm{h}(x) = \frac{1}{x}.$$

(a) Find g(16). (1)

Solution

$$\begin{aligned} \mathrm{g}(16) &= \sqrt[3]{2(16)-5} \\ &= \sqrt[3]{27} \\ &= \underline{\underline{3}}. \end{aligned}$$

(b) Find $\mathrm{h}\,\mathrm{g}^{-1}(x)$. (3)
Give your answer in terms of x in its simplest form.

Solution

Well,

$$\begin{aligned} y = \sqrt[3]{2x-5} &\Rightarrow y^3 = 2x - 5 \\ &\Rightarrow y^3 + 5 = 2x \\ &\Rightarrow x = \tfrac{1}{2}(y^3 + 5); \end{aligned}$$

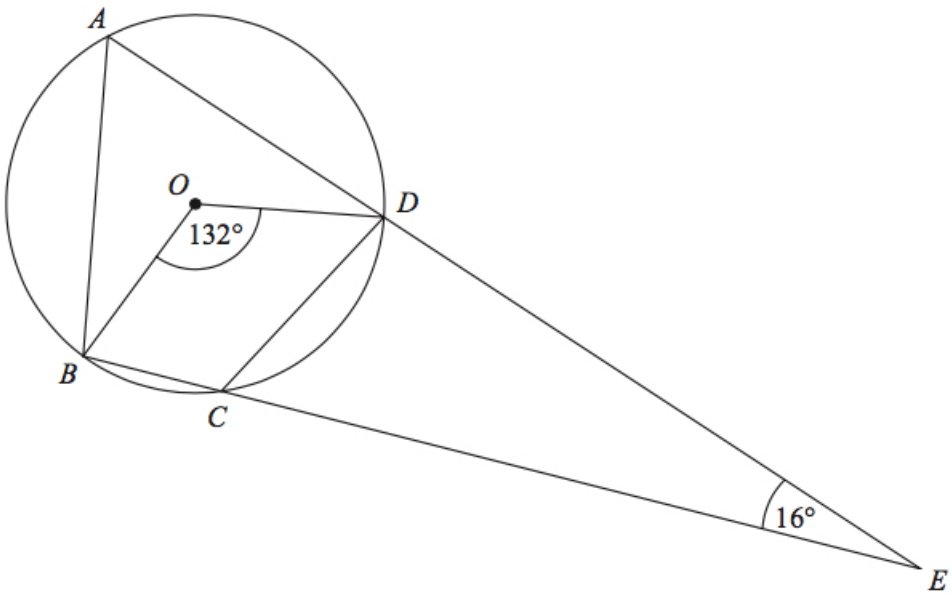
hence,

$$\mathrm{g}^{-1}(x) = \frac{x^3 + 5}{2}.$$

Finally,

$$\begin{aligned} \mathrm{h}\,\mathrm{g}^{-1}(x) &= \mathrm{h}(\mathrm{g}^{-1}(x)) \\ &= \mathrm{h}\left(\frac{x^3+5}{2}\right) \\ &= \underline{\underline{\frac{2}{x^3+5}}}. \end{aligned}$$

20. A, B, C, and D are points on the circumference of a circle, centre O. ADE and BCE are straight lines. (4)



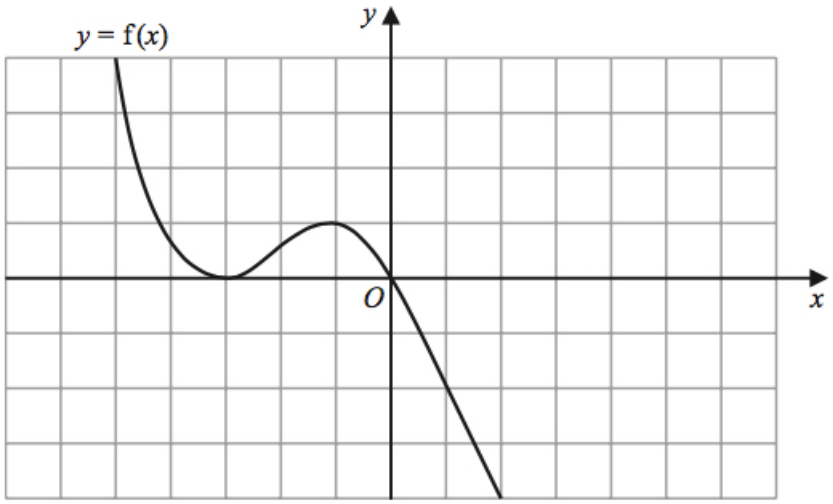
Work out the size of angle CDE.

Give a reason for each stage of your working.

Solution

$\angle BAD = \frac{132}{2} = 66^\circ$ (angle at the centre is twice the angle at the circumference)
$\angle BCD = 180 - 66 = 114^\circ$ (opposite angles in a cyclic quadrilateral)
$\angle DCE = 180 - 114 = 66^\circ$ (supplementary angles)
$\angle CDE = 180 - 66 - 16 = \underline{\underline{98^\circ}}$ (completing the triangle)

21. The graph of $y = \mathrm{f}(x)$ is shown on the grid below.

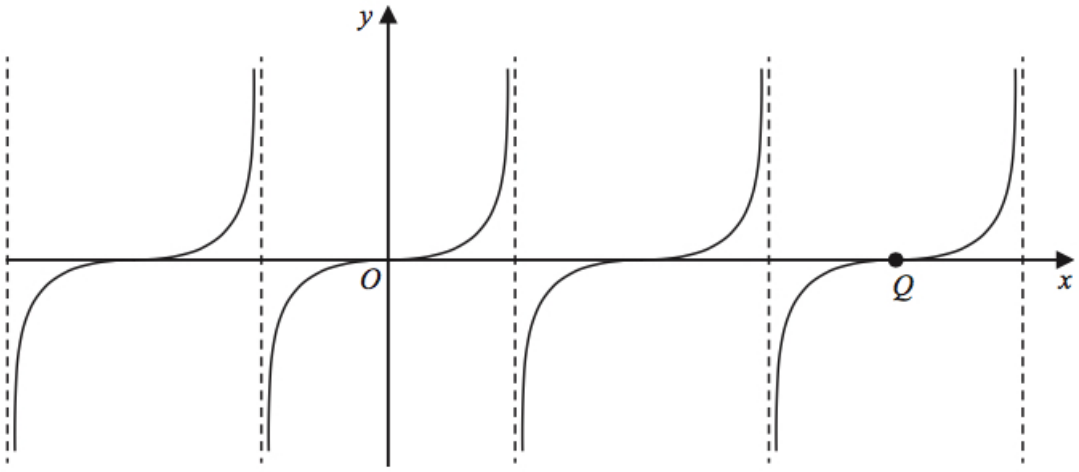


(a) On the grid, sketch the graph of $y = \mathrm{f}(-x)$. (1)

Solution

Well, it goes through $(-5, -4)$, $(-3, 0)$, $(-1, -1)$, $(2, 4)$, ...

Here is a sketch of the graph of $y = \tan x^\circ$.



The graph of $y = \tan x^\circ$ is translated to give the graph of $y = \mathrm{g}(x)$.

Following the translation the point Q, shown on the graph above, moves to point R.

Point R has coordinates $(90, -5)$.

(b) Find an expression for $\mathrm{g}(x)$ in terms of x. (2)

Solution

It moves to left

$$360 - 90 = 270$$

and it is

$$\underline{\underline{\mathrm{g}(x) = \tan(x + 270)^\circ - 5}}.$$

22. Find algebraically the set of values of x for which (5)

$$x^2 - 49 > 0 \textbf{ and } 5x^2 - 31x - 72 > 0.$$

Solution

Difference of two squares:

$$\begin{aligned} x^2 - 49 > 0 &\Rightarrow (x)^2 - 7^2 > 0 \\ &\Rightarrow (x - 7)(x + 7) > 0 \end{aligned}$$

	$x<-7$	$x=-7$	$-7<x<7$	$x=7$	$x>7$
$x+7$	$-$	0	$+$	$+$	$+$
$x-7$	$-$	$-$	$-$	0	$+$
$(x+7)(x-7)$	$+$	0	$-$	0	$+$

$$\Rightarrow x<-7 \text{ or } x>7.$$

Now, $\underline{5x^2-31x-72>0}$:

$$\left.\begin{array}{ll}\text{add to:} & -31\\ \text{multiply to:} & (+5)\times(-72)=-360\end{array}\right\}\text{hmm...}$$

Oh, well: we will try quadratic formula:
$a=5$, $b=-31$, and $c=-72$:

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{31\pm\sqrt{(-31)^2-4(5)(-72)}}{2\times 5}\\&=\frac{31\pm\sqrt{2\,401}}{10}\\&=\frac{31\pm 49}{10}\\&=-1.8,8.\end{aligned}$$

	$x<-1.8$	$x=-1.8$	$-1.8<x<8$	$x=8$	$x>8$
$x+1.8$	$-$	0	$+$	$+$	$+$
$x-8$	$-$	$-$	$-$	0	$+$
$5x^2-31x-72$	$+$	0	$-$	0	$+$

Hence,

$$5x^2-31x-72>0\Rightarrow x<-1.8 \text{ or } x>8.$$

Now, draw a number line and look at the areas where they are both positive:

Finally, the solution set is

$$x < -7 \text{ or } x > 8.$$